442

DATE LABEL

~~58~~ 4-5-67

25/4/67

25 SEP 1971

18 DEC 1971

Call No. 537.1 M196S

Date 24.4.52

Account No. 5391

J. & K. UNIVERSITY LIBRARY

This book should be returned on or before the last stamped above. An overdue charges of 6 nP. will be levied for each day, the book is kept beyond that day.

THE KASHMIR SYNDICATE SRINAGAR BOOKS DEPTT.

A SECOND COURSE OF ELECTRICITY

S/C

A SECOND COURSE OF ELECTRICITY

BY

A. E. E. McKENZIE, M.A.
Trinity College, Cambridge

ALLAMA IQBAL LIBRARY
5391

> These fundamental things have got to be simple
> LORD RUTHERFORD

JAMMU & KASHMIR UNIVERSITY
LIBRARY
No. 5391
Date 24-4-52
SRINAGAR

cat
13/2
ST82

CAMBRIDGE
AT THE UNIVERSITY PRESS
1949

THE KASHMIR SYNDICATE
SRINAGAR
BOOKS DEPTT.

PUBLISHED BY
THE SYNDICS OF THE CAMBRIDGE UNIVERSITY PRESS
London Office: Bentley House, N.W. 1
American Branch: New York
Agents for Canada, India, and Pakistan: Macmillan

CHECKED

537.1
M196S

Printed in Great Britain at the University Press, Cambridge
(Brooke Crutchley, University Printer)

CONTENTS

Preface *page* vii

Chapter 1. Charge and the Electrical Field 1

2. Induced Charges and Condensers 17

3. Further Electrostatics 39

4. Magnetism 55

5. Electromagnetism 84

6. Magnetic Materials 102

7. The Electric Current 121

8. Electrical Measurements 141

9. Absolute Measurements. Units and Dimensions 166

10. Electrolysis 184

11. Cells 199

12. Electromagnetic Induction 214

13. Alternating Current 239

14. Thermoelectric, Photoelectric, and Thermionic Effects 251

15. Electromagnetic Waves 276

16. Radio Reception and Transmission 296

17. Atomic Physics 309

Answers to questions 351

Index 355

ILLUSTRATIONS

In addition to the text-figures, of which there are 327, the following portraits have been included:

FARADAY AND KELVIN *page* 25

COULOMB AND AMPÈRE 101

OHM AND JOULE 127

GAUSS AND WEBER 167

GALVANI AND VOLTA 201

MAXWELL AND HERTZ 281

J. J. THOMSON AND RUTHERFORD 315

The portraits of Ampère, Galvani and Volta are from the Library of the Royal Society. Those of Maxwell and Lord Rutherford are from portraits in the National Portrait Gallery. The portrait of Sir J. J. Thomson is taken from W. Nicholson's painting, at Trinity College, Cambridge. That of Coulomb is from the portrait by Emile Lecomte, in the Musée de Versailles. That of Faraday is from a lithograph of 1851 by S. H. Maguire. That of Lord Kelvin is from a photograph dated 1906, at the University Press, Cambridge. The portraits of Weber, Gauss, Hertz, Joule and Ohm are by courtesy of Picture Post Library.

PREFACE

WRITING a book on Electricity is like fitting together the pieces of a jigsaw puzzle most of which interlock with several other pieces. Indeed, the fascination of the subject lies in its close-knit coherence. The traditional presentation has been followed, however, and cross-references inserted liberally. The book may be regarded as a sequel to the author's *Magnetism and Electricity*, reaching a standard approximately that required for University Scholarships, but somewhat exceeding these requirements where the logical development of the subject demanded it. There is some overlap between the two books, notably in magnetometry.

It is widely held that specialisation in sixth forms should be reduced, and there is much to be said for achieving this in physics by a simpler presentation and by the elimination of detail rather than by too much reduction of scope. Thus Gauss's theorem need not be mentioned to beginners in electrostatics except in the simple form that unit charge carries 4π lines of force. Even so the attraction between the plates of a condenser can be deduced very simply, and it is unnecessary to omit the attracted disc electrometer owing to its being too difficult. When the same ground is rapidly covered at the University, it can be done rigidly using Gauss's theorem, and the simple fundamental ideas will be already clear in the minds of the students. Again, electromagnetism can be studied adequately without the aid of the calculus, but the better mathematicians can cover the same ground a second time starting from Laplace's expression. The art of the teacher is to make the first presentation of the fundamental elements of a subject in the simplest possible terms.

A simple account of Maxwell's theory and of electromagnetic waves has been given, since it seemed inartistic not to continue to the ultimate triumph of classical electromagnetic theory, even though it is outside the examination syllabus. A description of radio circuits follows naturally and seems to arouse great interest among all but the 'highbrow' sixth-form pupil. The electronic work function has been introduced because it links up so satisfactorily the electric cell, thermionics and the photoelectric effect.

The last chapter is a survey of atomic physics, the early parts of the subject being treated more fully than the later. Again it seemed desirable to continue to the great achievement, the release of atomic energy.

The view has been adopted that B and H are physically identical, which simplifies dimensions and eliminates μ_0 (and similarly k_0) from many formulae.

The symbols used are those recommended by the Committee for Co-ordination and Guidance in Physics Teaching.

Brief accounts have been given of the life and work of Maxwell, Edison and Marconi. Faraday, the obvious choice to complete a quartet, has already been dealt with in the earlier book.

I am very much indebted to Mr E. S. Shire and to Mr F. Record, who have read through the manuscript and made numerous valuable criticisms and suggestions, to Mr J. C. Hill, who drew the line diagrams from my rough sketches, to the Librarian of the Royal Society for the portraits of Ampère, Galvani and Volta, and to the Council of Trinity College, Cambridge, for permission to use a detail from W. Nicholson's portrait of Sir J. J. Thomson and to Mr J. Stead for checking the answers to the questions.

I should like to express my thanks to the following, who have provided illustrations: Mr K. E. Machin of the Cavendish Laboratory who took the photographs for Figs. 244, 261 and 274; Sir George Simpson and the Superintendent of Kew Observatory for Figs. 45, 47 and 48 taken from the *Proceedings of the Royal Society*; the Radio Corporation of America for Figs. 251 and 252; Messrs Henry Hughes and Son Ltd., Barkingside, Essex, for Figs. 77 and 78; Marconi's Wireless Telegraph Co. Ltd., Chelmsford, for Fig. 266; The Automatic Coil Winder and Electrical Co. Ltd., Douglas St., S.W.1, for Fig. 163; New York Times Photos for Figs. 43 and 131; Fox Photos Ltd. for Fig. 80; Dr W. L. Sumner and Messrs Basil Blackwell for Figs. 270 and 271 taken from *Progress in Science*; Messrs Bell for Fig. 307 taken from *An Introduction to Crystal Analysis*, Bragg; Messrs Allen and Unwin for Fig. 304 taken from *The World and the Atom*, Moller and Rasmussen; Dr W. Wilson and Messrs Chapman and Hall for Fig. 247 taken from *The Cathode Ray Oscillograph in Industry*; Dr C. T. R. Wilson for Fig. 317; Two Cities Films Ltd. for Fig. 231 (*a*) and Gainsborough Pictures Ltd. for Fig. 231 (*b*); and to the Cambridge University Press for Figs. 84, 315, 316, 321, 322, 323 and 326.

I am very grateful to the different Examining Bodies for allowing me to reproduce questions. The following is the key to the acknowledgements made after each question: (O. S.) Oxford Scholarship, (C. S.) Cambridge Scholarship, (O. & C.) Oxford and Cambridge, (O.) Oxford Local, (C.) Cambridge Local, (L.) London, (N.) Northern Universities, (B.) Bristol, (D.) Durham, Higher School Certificates.

A. E. E. M.

CAMBRIDGE
January 1949

CHAPTER 1

CHARGE AND THE ELECTRIC FIELD

THE word electricity was coined by William Gilbert in the reign of Queen Elizabeth. It is derived from the Greek word electron, meaning amber.

The ancient Greeks discovered that amber when rubbed with silk acquires the property of attracting light objects such as pieces of chaff. Gilbert discovered that other substances exhibit the same effect, and that the magnitude of the effect is roughly proportional to the area of the surface rubbed. He was thus led to the idea of a charge of electricity.

About 1745, Du Fay discovered that there are two kinds of electricity. Two ebonite rods when rubbed with fur exert a force of repulsion on each other. Two glass rods rubbed with silk also repel one another. However, an ebonite rod which has been rubbed with fur attracts a glass rod which has been rubbed with silk.

Any substance rubbed with a different substance acquires a charge of electricity, and will be found either to repel charged ebonite and to attract charged glass, or vice versa. Since the two kinds of electricity can neutralise each other's effects, one is called positive and the other negative, the choice as to which is positive being purely arbitrary. Glass rubbed with silk is said to have a positive charge and ebonite rubbed with fur a negative charge.

The law of force between charges may be stated thus: *like charges repel, unlike charges attract.*

Ebonite and glass become charged only in the area in which they have been rubbed but a metal such as brass, on being rubbed in one part, is found to be charged all over. This is explained by the fact that ebonite and glass do not allow electricity to pass through them, whereas metals do. The former are called *insulators* and the latter *conductors.* If the charge acquired by a metal when rubbed is to be retained, the metal must be held with an insulating handle; otherwise the charge will escape through the metal and the human body to the earth. Charges can pass through moisture on the surface of a substance, and in all experiments in this branch of electricity, known as electrostatics, apparatus must be kept dry.

The electroscope

The simplest method of detecting the presence of a charge on a body is by the attraction of a light object. One of the earliest instruments, the pith-ball electroscope, consisted of a small ball of pith, the white substance of very low density found under the bark of an elder tree. If the pith ball is suspended by a silk thread which, when dry, is an excellent insulator, it can be given a permanent charge by touching it with a rubbed rod. It will then serve to detect the sign of a charge; since if, for example, its charge is negative, it will be attracted by a positive charge and repelled by a negative.

A more convenient instrument is the gold-leaf electroscope. This consists of a leaf of thin metal foil suspended close to a vertical metal rod or plate to which it is attached along its upper edge. When the rod and leaf are charged the repulsion between like charges causes the leaf to diverge as in Fig. 1. The leaf is protected from draughts by enclosure in a metal case with glass windows. The rod is usually provided with a metal disc at its upper end and must be insulated from the case with a good insulator such as sulphur or paraffin wax. Note that in Fig. 1 the plug is shaded, a convention which will be used for insulators, while conductors will be left unshaded.

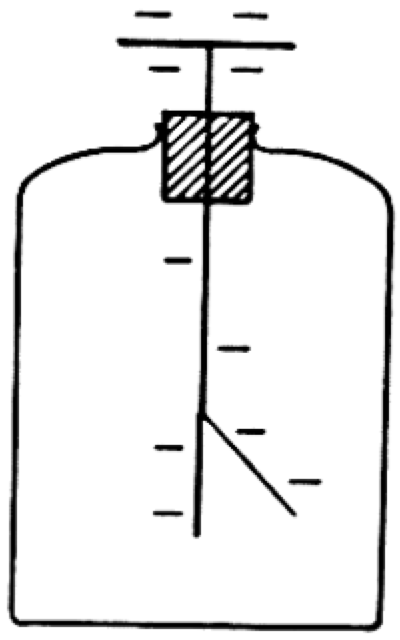


Fig. 1

The instrument may be charged negatively by flicking the disc with fur. Then on bringing near to the disc a negatively charged body the leaf diverges farther. This can be explained by the repulsion of the negative charge on the disc down to the leaf and lower part of the rod. A positively charged body causes the leaf to fall owing to the attraction of negative charge up to the disc.

Coulomb's law

The elementary phenomena of electrostatics were discovered in the seventeenth and first half of the eighteenth centuries. Newton in the seventeenth century laid the foundations of dynamics, and in his theory of gravitation gave to physical science a mathematical theory which was to serve as a model for all future development. Electricity became a quantitative and mathematical science when Coulomb in 1784 applied Newton's methods and concept of force to electric charges.

Coulomb charged two gilded pith balls with like charges of electricity and found how the force of repulsion between them varied with their distance apart. One pith ball was attached to the end of a light insulating rod suspended from a fine silver wire (Fig. 2) and the other was fixed. The repulsion between them was estimated from the twist of the wire. Coulomb discovered that the force is inversely proportional to the square of the distance between the balls, i.e.

$$F \propto \frac{1}{r^2}.$$

If the charge on a pith ball is shared with an exactly similar ball the charge on the original ball is halved. If this is again shared with a third similar ball the charge becomes one-quarter of the original charge, and so on. By using different submultiples of charge on the two balls in the torsion balance, at a fixed distance apart, Coulomb showed that the force between the charges is proportional to the product of their magnitudes.

The balls in Coulomb's experiment must be small compared with their distance apart. Otherwise, owing to repulsion, the charges are not uniformly distributed on the spheres, and it is unjustifiable to take the distance apart of the centres of the spheres as the mean distance apart of the charges.

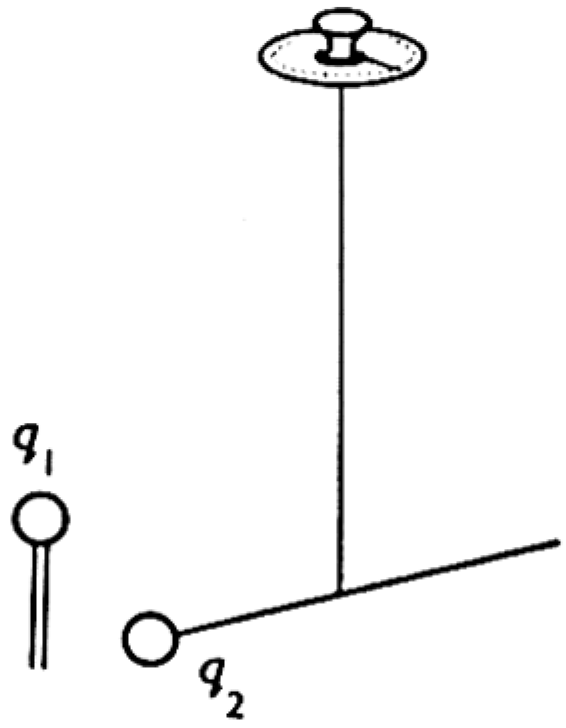


Fig. 2

In the simple theory which follows, this difficulty is avoided by the concept of point charges.

Coulomb's law. The force between two point charges is proportional to their product and inversely proportional to the square of their distance apart, i.e.

$$F \propto \frac{q_1 q_2}{r^2},$$

where q_1 and q_2 are two point charges and r is their distance apart, or

$$F = A\,\frac{q_1 q_2}{r^2},$$

where A is a constant depending on the choice of units for q_1 and q_2.

The c.g.s. electrostatic system of units is based on the three fundamental units, the centimetre, the gramme and the second. The unit of force on this system is the *dyne*, which is the force which gives a mass of 1 g. an acceleration of 1 cm. per sec. per sec. The unit of work is the *erg*, which is the work done when 1 dyne acts through 1 cm.

By a suitable selection of the size of the unit in which charges are reckoned the numbers q_1 and q_2 can be adjusted so that the constant A becomes 1. When this has been done

$$\mathbf{F} = \frac{\mathbf{q_1 q_2}}{\mathbf{r^2}}.$$

The unit of charge is therefore defined as follows:

The electrostatic unit charge when placed 1 *cm. from a similar charge* in vacuo *repels it with a force of* 1 *dyne.* (If the charges are in air the force is reduced by about 0·06%, an error which can be ignored except in the most accurate work.)

The e.s.u. of charge is sometimes called the statcoulomb.

The practical unit of charge is the coulomb,

$$1 \text{ coulomb} = 3 \times 10^9 \text{ e.s.u.} \quad (\text{see p. } 175).$$

The electric field

A charged body exerts an influence in the space around it; it attracts light objects. The charged body is said to create an *electric field* in the space around it.

The electric field can be investigated by scattering certain crystals on an insulating plate placed over the charged body. The crystals set themselves in lines called *lines of electric force* (Fig. 3). These lines represent at any point the direction of the force exerted on a crystal.

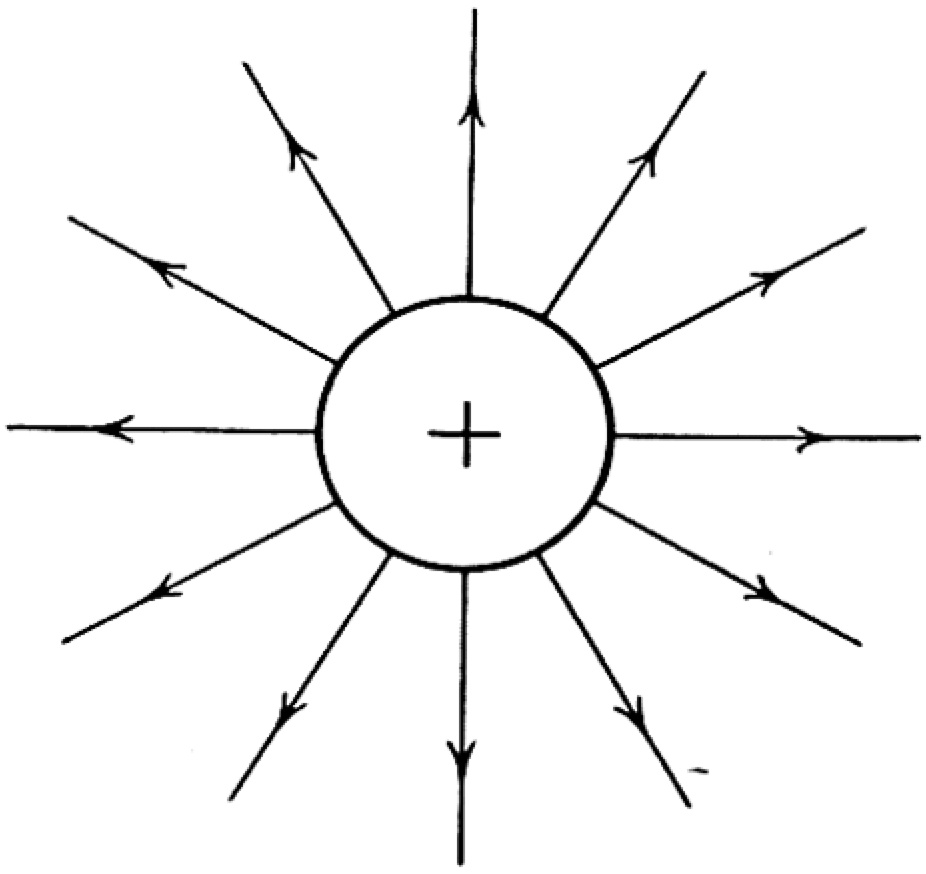


Fig. 3

The strength or intensity of an electric field at a point is defined as the force exerted on a unit positive charge placed there.

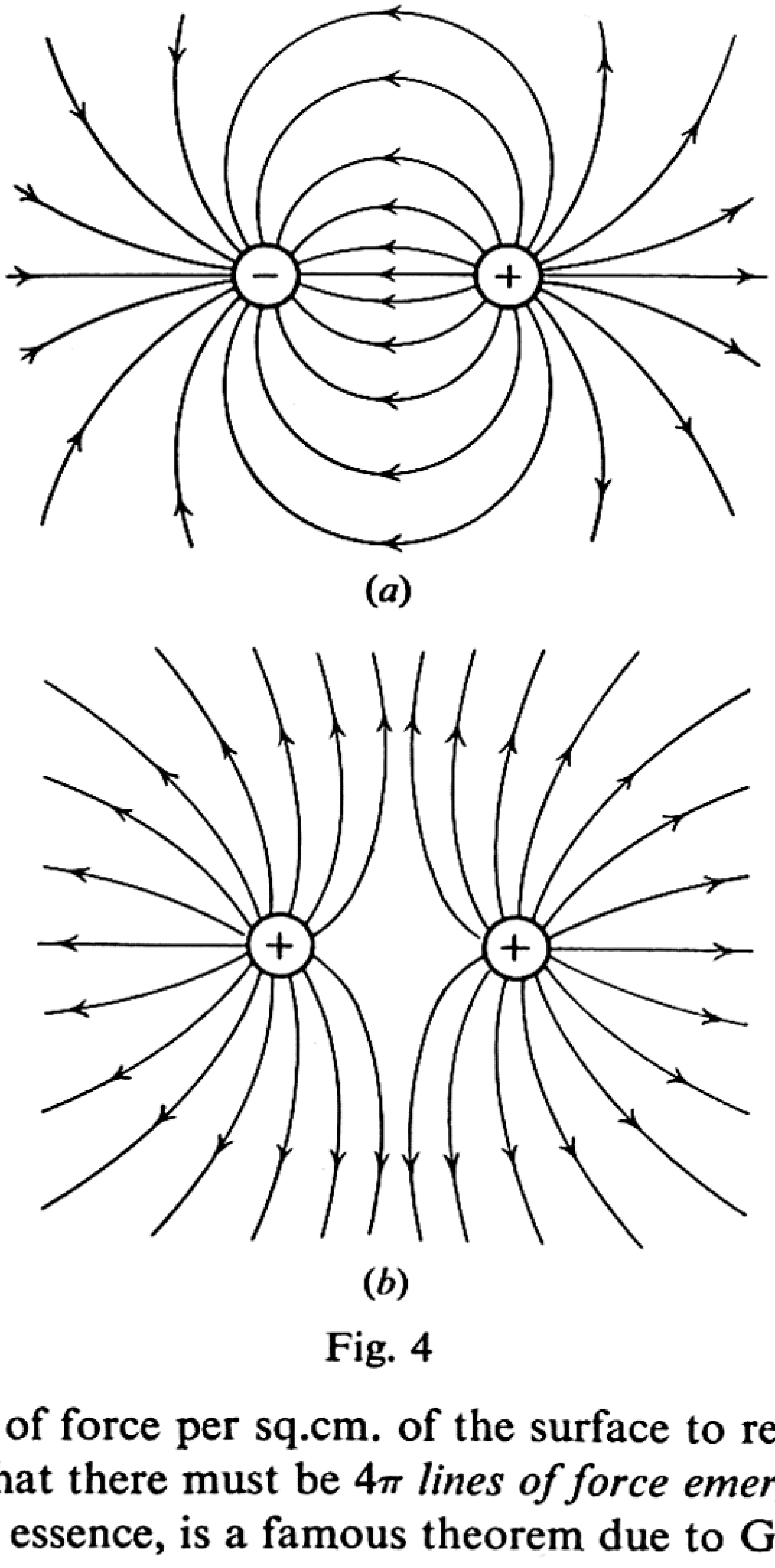
Thus if a charge of q e.s.u. is placed in a field of strength E e.s.u., the force, F dynes, exerted on the charge is given by

$$F = qE.$$

The field due to a point charge q at a distance r from it in a vacuum is, applying Coulomb's law, $\frac{q \times 1}{r^2} = \frac{q}{r^2}$ e.s.u.

It is seen in Fig. 3 that the lines of force due to a charged disc or sphere diverge and become farther apart as the field becomes weaker. The number of lines of force per unit area can be used to represent numerically the strength of the field. *One line per sq.cm. represents a field of unit strength.*

Consider a sphere of radius 1 cm. described round a unit point charge. The area of the surface of the sphere is 4π sq.cm. and the strength of the field at the surface is $\frac{1 \times 1}{1^2} = 1$ e.s.u. of field strength.



(*a*)

(*b*)

Fig. 4

Taking 1 line of force per sq.cm. of the surface to represent the unit field it is clear that there must be 4π *lines of force emerging from a unit charge.* This, in essence, is a famous theorem due to Gauss. It follows from this theorem that *the electric field very near to the surface of a conductor is* $4\pi\sigma$, where σ is the charge per unit area of surface on that side (σ is called the surface density of the charge).

A field in which the lines of force are parallel and equally spaced is called a *uniform* field, and the strength of the field is everywhere constant. This contrasts with the field due to a point charge where the lines diverge and the field weakens, according to the inverse square of the distance.

If we imagine, as did Faraday, that the lines of force are in tension and exert a lateral pressure, we can form, from Fig. 4, a mental picture of the nature of the attraction and repulsion of unlike and like charges.

We take the positive direction of a line of force as the direction in which a positive charge tends to move; this is the positive direction of the electric field and is indicated in diagrams by an arrow.

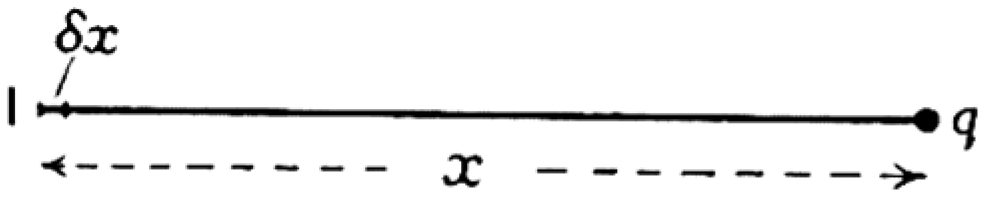


Fig. 5

Potential

The strength of an electric field is a vector quantity; it has direction as well as magnitude, and can be represented by a straight line. Hence the field due to two or more point charges is calculated by applying the parallelogram law for the addition of the vectors.

A concept which is often more convenient is that of *electric potential,* since potential is a scalar quantity, having magnitude but no direction, and hence potentials can be summed by straightforward, algebraic addition.

A body when raised above the surface of the earth is said to acquire potential energy because it can do work in falling; if free to move in a gravitational field it will fall to the position in which its potential energy is zero. Similarly, a charged body in an electric field has potential energy, and it will tend to move to those parts of the field where its potential energy is smaller. When a unit positive charge is repelled by another positively charged body and moves away, its potential energy decreases. Its potential energy will be zero when it is completely away from the influence of the charged body, i.e. at infinity.

The *electric potential* at a point is therefore defined as *the work done in bringing up a unit positive charge from infinity to the point.*

The units of electric potential are ergs per statcoulomb, or statvolts. On the practical system, which will be used in later chapters, the unit of potential is the volt. *A potential difference of* 1 *volt is said to exist between two points if* 1 *joule of work is done when a charge of* 1 *coulomb moves from one point to the other.*

Potential due to a point charge

It can be shown that the potential at a point distant r from a point charge q, in air (or more strictly *in vacuo*), is $\mathbf{q/r}$. To deduce this expression the work done in bringing up a unit charge from infinity must be calculated, bearing in mind that the force becomes continuously greater as the unit charge approaches q.

When the distance of the unit charge from q is x (Fig. 5)

$$\text{Force on unit charge} = \frac{q \times 1}{x^2} \text{ by Coulomb's law.}$$

Move the charge a minute distance δx nearer to q so that the force is not appreciably increased,

$$\text{Work done} = -\frac{q}{x^2}\delta x.$$

(The negative sign appears because $x \to x - \delta x$, and hence the distance moved is $-\delta x$.) Thus the total work done in bringing up the charge from infinity to a distance r from q equals

$$\int_{\infty}^{r} -\frac{q}{x^2}dx = \frac{q}{r}.$$

If the charge is $-q$, the potential at a point distant r from it is $-q/r$. In this case work is done by the field on the unit, positive, test charge, and hence we speak of negative potential.

Example

ABC is an equilateral triangle of side 2 cm., and D is the midpoint of BC (Fig. 6). Charges of 5 and -3 e.s.u. are placed at B and C respectively. Find

(*a*) the strengths of the electric field at D and A,

(*b*) the electric potentials at D and A,

(*c*) the work done in taking a charge of 4 e.s.u. from D to A.

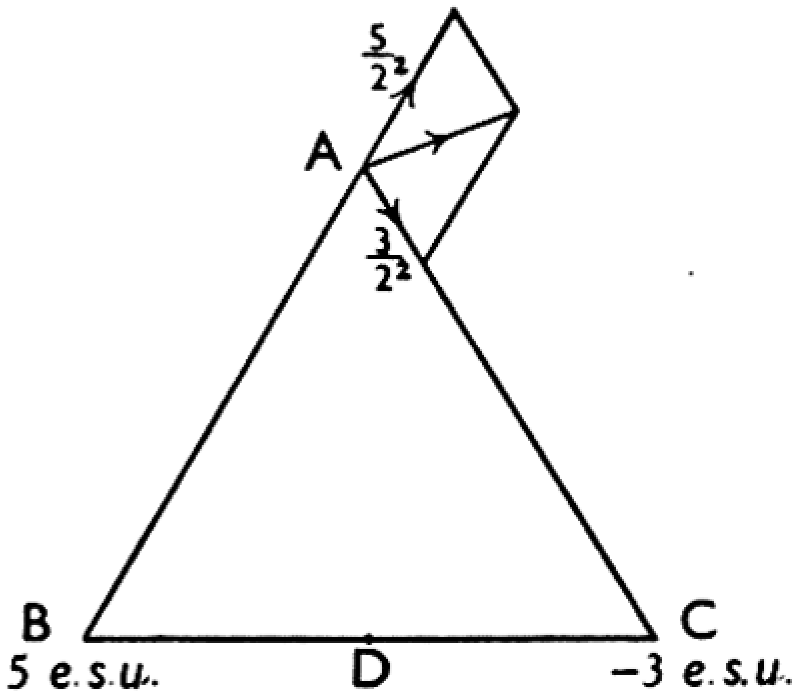


Fig. 6

(*a*) Strength of electric field at D

= force on unit positive charge at D in direction DC
= 8 dynes per e.s.u. of charge.

Strength of electric field at A due to charge at B
$= 5/2^2$ e.s.u. along BA produced.

Strength of electric field at A due to charge at C
$= 3/2^2$ e.s.u. along AC.

∴ Resultant, R, of the two fields is represented in magnitude and direction by the diagonal of a parallelogram as in Fig. 6 with sides in the ratio 5/4 : 3/4. It can be shown by trigonometry that R is 1·09 e.s.u. at an angle 23° 25′ with AC.

(*b*) Potential at $D = 5/1 - 3/1 = 2$ e.s.u. (or statvolts),
Potential at $A = 5/2 - 3/2 = 1$ e.s.u.

(*c*) Work done in taking a charge of 4 e.s.u. from D to A
$= 4 \times$ difference of potential between A and D
$= 4(2-1)$
$= 4$ ergs.

Potential gradient

Suppose the potentials at two points A and B, x cm. apart, in a uniform field of strength E, are V_A and V_B e.s.u., the former being the greater.

The work required to take a unit positive charge from B to A is (from the definition of potential) $V_A - V_B$. Incidentally, if the charge moves from A to B, an equal amount of work is done by the field.

Now the force due to the field on the unit charge (from the definition of field strength) $= E$.

∴ Work done on the unit charge in moving distance $x = Ex$.

$$\therefore\ Ex = V_A - V_B,$$

$$\therefore\ E = \frac{V_A - V_B}{x}.$$

This means that *the strength of an electric field is equal to the potential gradient*. It follows, of course, that the potential gradient of a uniform field is constant.

In a non-uniform field, where the potential gradient varies, the expression must be written

$$\mathbf{E} = -\frac{\mathbf{dV}}{\mathbf{dx}}.$$

The negative sign indicates that when the distance increases by δx in the direction of the field, the potential increases by $-\delta V$. The positive direction of the field is in the direction of decreasing potential, and a positive charge tends to move from higher to lower potential.

The strength of an electric field may thus be measured in terms of the potential gradient, i.e. in statvolts per cm.

It is worth noting that the work done in taking a charge from a point A to a point B in a field is independent of the path followed, whether a straight line or a curve. If this were not so it would be possible to push a charge along one path and to let it run back along another giving a larger amount of work, thus violating the principle of the conservation of energy. It is assumed, of course, that there are no frictional forces retarding the motion.

The work done by the field in moving a unit positive charge along a certain path is called the *electromotive force* along that path. This term occurs frequently in connection with current electricity.

Equipotential surfaces

Surfaces on which all points are at the same potential are called equipotential surfaces. Thus the equipotential surfaces in a uniform field are a set of parallel planes, equally spaced, the normals to which are lines of force. In the case of a point charge the equipotential surfaces are concentric spheres (Fig. 7).

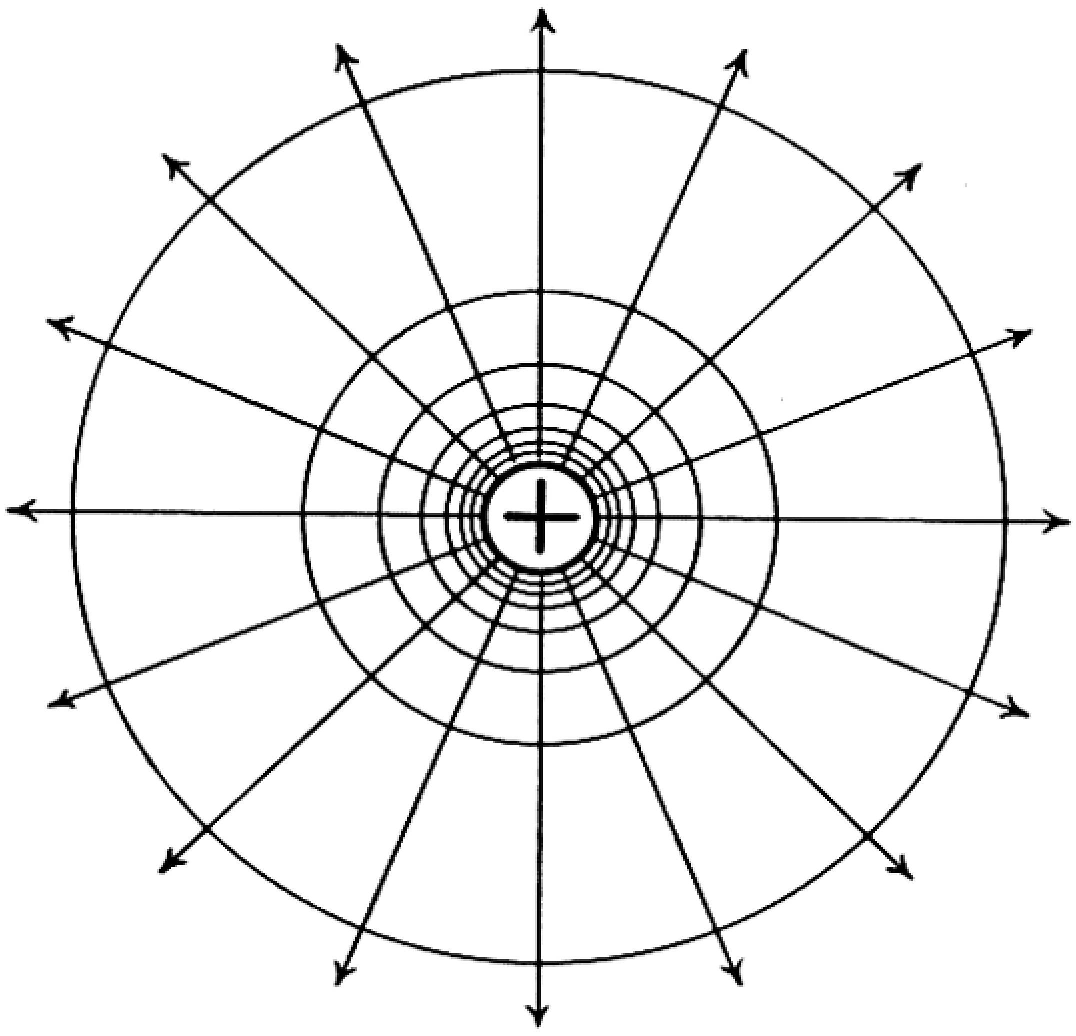


Fig. 7

It is instructive to note the analogy between electric and gravitational fields using Fig. 7. The circles or equipotential lines correspond to contours or lines of equal height above sea-level, while the lines of electric force correspond to lines of greatest gravitational slope. We can ascend a hillside by walking at right angles to the contours up the line of greatest slope or we can zigzag taking more gentle slopes. Similarly, the potential gradient at an angle to a line of electric force is less than along the line of force. Again, the force along the vertical in a gravitational field is $W = mg$; along a line of force in an electric field the force is $F = qE$.

The surface of a conductor must be an equipotential surface; if there were a difference of potential between two points of a conductor, charge would flow until the difference disappeared. Hence there can be no component of an electric field parallel to the surface of a conductor, and all lines of electric force must everywhere be perpendicular to the surface. No line of electric force can begin and end on the same conductor, since there must be a fall of potential down a line of force.

Capacitance

When a charge is given to a conductor its potential rises, just as when heat is given to a body the temperature rises. To produce the same rise in potential it is necessary to supply a larger charge to a large body than to a small body. We say that the large body has a greater electrical *capacitance* or capacity than the small body.

We can define the *capacitance of a conductor* as *the charge required to raise its potential by unity.*

Thus $$C = \frac{Q}{V},$$

where C = capacitance,
Q = charge,
V = potential.

If one statcoulomb raises the potential of a conductor by 1 statvolt, its capacitance is 1 statfarad.

The capacitance of a sphere

The lines of force due to a charged sphere appear to radiate from the centre, and the field at all points outside the sphere is the same as that of an equal point charge at the centre. The charge is, in fact, owing to self-repulsion, concentrated on the surface, and in the absence of neighbouring conductors it is uniformly distributed.

Suppose the sphere has a radius a and is given a charge q.

Potential of surface of sphere (or point just outside it)

$$= \text{potential due to point charge } q \text{ at distance } a = \frac{q}{a}.$$

$$\therefore \text{ Capacitance of sphere} = \frac{Q}{V} = \frac{q}{q/a} = a.$$

Thus the capacitance of a sphere is equal to its radius. On the electrostatic system the dimensions of capacitance are those of length, and a capacitance of 1 statfarad is sometimes given as 1 cm.

Potential due to a charged spherical conductor

For a charged spherical conductor the potential varies with the distance from the centre in the manner represented in Fig. 8. The height of the graph above a point on the diameter produced represents the potential at that point. Note that the potential is uniform throughout the sphere and falls off in the space surrounding it.

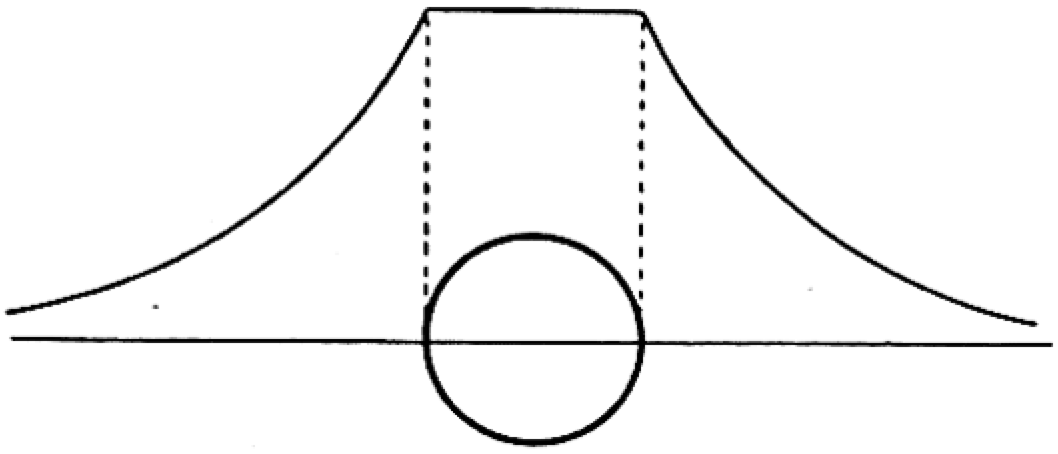


Fig. 8

The potential of the earth

The earth is so large a conductor that any small charges it may have on its surface will not raise its potential appreciably, and we can assume that its potential is constant. Its potential is taken as zero, that is, the same as a region remote from all electrical charges. Similarly, mean sea-level is taken as the zero for the heights of mountains and the depths of seas.

The potential of a conductor may be regarded as positive if, when connected to the earth, positive electricity flows from the body to the earth, or alternatively, negative electricity flows from the earth to the body. The conductor has a negative potential if the flow of electricity is in the opposite direction.

The charge resides on the surface

The charge on a conductor resides on its outside surface owing to self-repulsion. This can be demonstrated by charging an insulated hollow vessel, such as a deep metal can, and testing samples of the charge on various parts of its inside and outside surfaces with a gold-leaf electroscope by means of a proof plane (consisting of a metal disc on an insulating handle). The proof plane will be found to carry away no charge when touched deep inside the can. Faraday used a charged, conducting, linen butterfly net (Fig. 9); on turning the net inside out with an insulating silk thread, he found that the charge changed surface so that it resided once more on the outside.

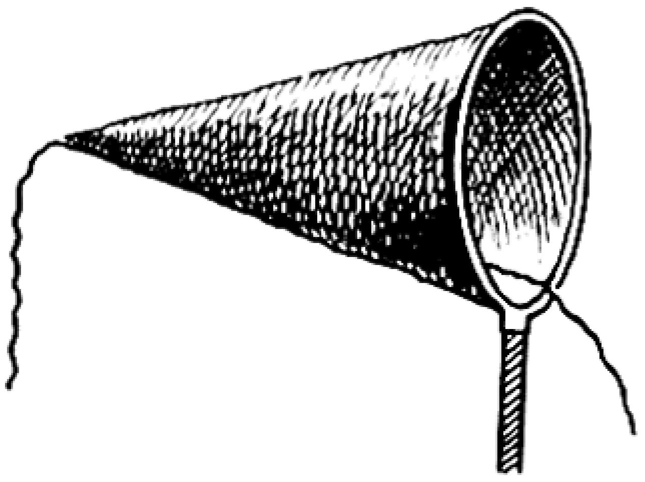


Fig. 9

Absence of field inside a hollow charged conductor

The space inside a hollow charged conductor is everywhere at the same potential as the conductor and hence there is no field there. Cavendish, and later Maxwell, verified this in the case of a hollow sphere, using the apparatus shown in Fig. 10. A hollow metal sphere has inside it another metal sphere on insulating supports, and also a trapdoor which, when closed, causes contact to be made between the two spheres. The outer sphere is charged with the trapdoor closed, and then the trapdoor is opened and the outer sphere discharged. On testing with a sensitive electroscope, the inner sphere is also found to be uncharged. Thus there can have been no electric field inside the charged outer sphere.

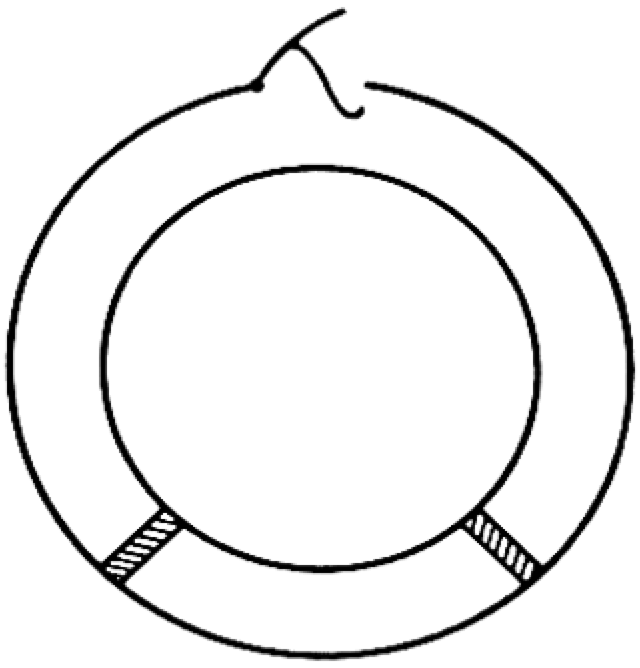


Fig. 10

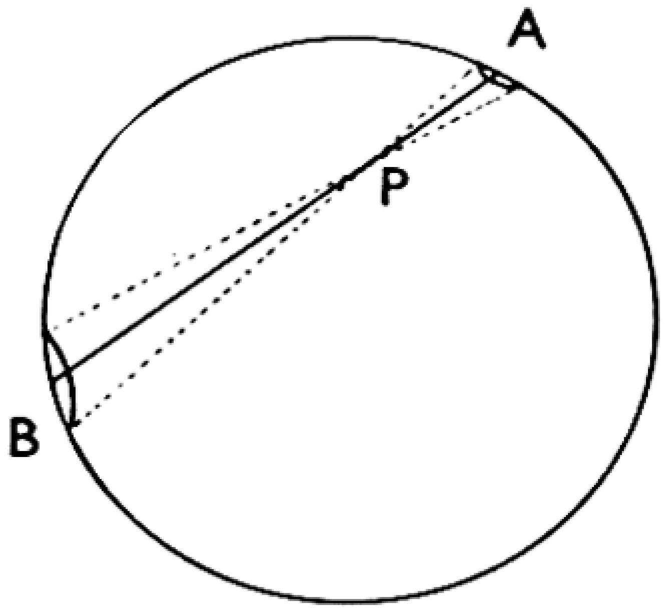


Fig. 11

Proof of the inverse square law

The above experiment is the basis of the most accurate proof of the inverse square law, usually ascribed to Cavendish, but derived earlier by Priestley.

Consider any point P inside a hollow, spherical conductor, the surface density (charge per unit area) being σ (see Fig. 11).

Draw any straight line APB, the points A and B being on the surface of the sphere, and describe a tiny area on the surface of the sphere around the point A. From points on the boundary of this area straight lines can be drawn through P which will demarcate an area round B, and we shall have two narrow cones as in Fig. 11. The areas at A and B are equally inclined to AB and hence

$$\frac{\text{Area at } A}{\text{Area at } B}=\frac{AP^2}{BP^2},$$

i.e.

$$\frac{\text{Area at } A}{AP^2}=\frac{\text{Area at } B}{BP^2}.$$

If the inverse square law is true,

$$\text{Field at } P \text{ due to charge on area at } A = \frac{(\text{Area at } A)\times\sigma}{AP^2} \text{ along } AP.$$

$$\text{Field at } P \text{ due to charge on area at } B = \frac{(\text{Area at } B)\times\sigma}{BP^2} \text{ along } BP.$$

These are equal and hence annul one another; but they would not do so if the force depended on, say, the inverse cube of the distance.

The whole surface of the sphere can be divided into pairs of tiny elements of area whose charges annul each other's effects at P.

By assuming the truth of the inverse square law we have thus deduced that there cannot be an electric field at any point inside a hollow charged sphere. The experimental fact that there is no field confirms the truth of the inverse square law.

Cavendish estimated that his experiments confirmed an nth power law, where $n=2\pm0{\cdot}02$; Maxwell, using a more delicate instrument (a quadrant electrometer) to test the lack of charge of the inner sphere, proved n to be $2\pm0{\cdot}00005$.

Electrostatic screening

Since the potential inside a hollow charged conductor is everywhere the same as that of the conductor and there is no internal field, a space can be electrostatically screened by surrounding it with a conductor. A large earthed wire cage is used to protect instruments and experimenters in high-voltage research. Similarly, coils and valves in wireless sets are often screened by enclosing them in metal boxes.

The distribution of charge

The charge on an isolated, spherical conductor is uniformly distributed on its surface, but in the case of a conductor which has not a constant curvature the charge is more dense on the more curved parts. This may be demonstrated by taking samples of the charge from various parts of the surface with a proof plane and testing them with a gold-leaf electroscope. In Fig. 12 the variation in the distance of the dotted line from the surface of the conductor indicates the variation in the surface density of the charge.

Consider two spheres, of radii r and R, connected by a wire (Fig. 13) and charged to a common potential, V. Since the capacitance of a sphere is equal to its radius, the charges on the spheres, q and Q, are given by

$$r=\frac{q}{V};\quad R=\frac{Q}{V}$$

$$\text{Surface density on small sphere}=\frac{q}{4\pi r^2}=\frac{V}{4\pi r}.$$

$$\text{Surface density on large sphere}=\frac{Q}{4\pi R^2}=\frac{V}{4\pi R}.$$

Thus the surface densities of the charges are inversely proportional to the radii of the spheres, and the surface density on the small sphere is greater than that on the large.

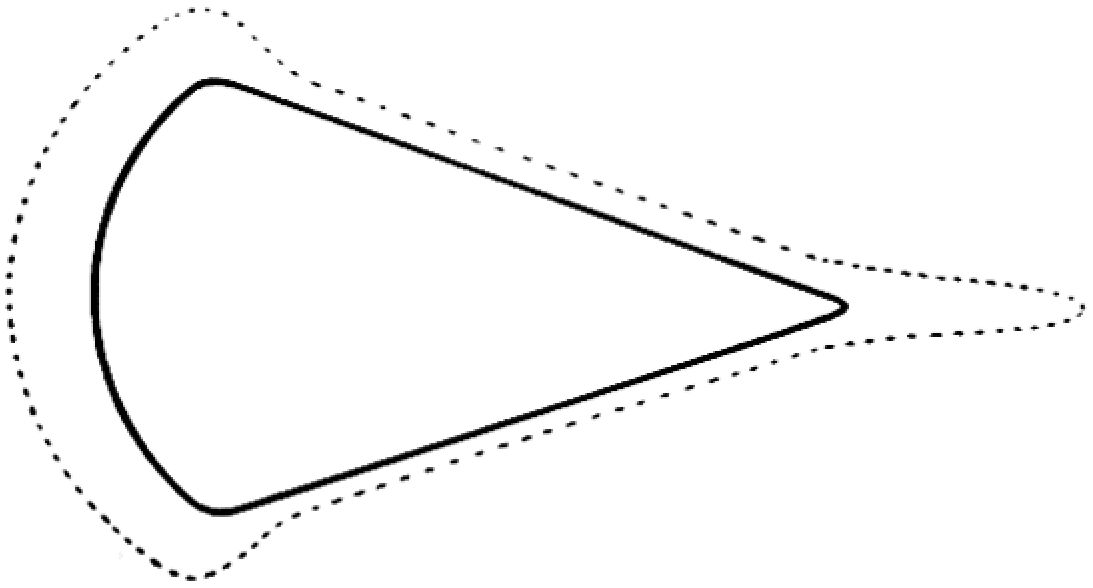


Fig. 12

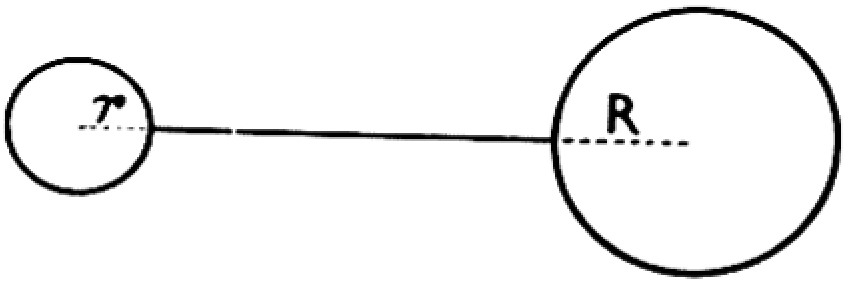


Fig. 13

The action of points

At the tip of a pointed conductor the curvature is very great, and hence there is a very high surface density of charge. This may be sufficiently great to charge the air and repel it in the form of an electric wind which can be demonstrated by its effect in blowing a candle flame (Fig. 14).

The air is in fact 'ionised'; the molecules are split into positively and negatively charged 'ions'. A positively charged point attracts the negative ions which tend to discharge it, and repels the positive ions to form the electric wind.

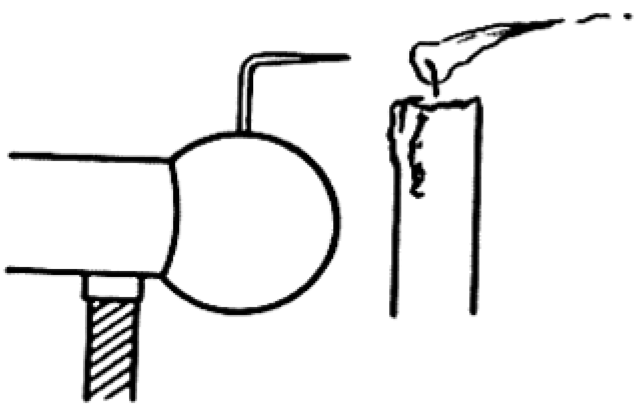


Fig. 14

Ionisation is often accompanied by a blue or violet glow known as a brush discharge which can be seen in the dark.

QUESTIONS

1. How has the inverse square law of force for electric charges been verified? (C. S.)

2. Two small gilded spherical pith balls, each of mass 0·004 g., are suspended from the same point by two threads each of length 10 cm. Assuming the balls to be equally charged and to remain in equilibrium with the balls 2 cm. apart, calculate the charge on each. ($g = 981$ cm.sec.$^{-2}$.)

3. Explain whether it is possible
   (*a*) for the potential at a point to be zero while the electric field at the point is not zero;
   (*b*) for the electric field at a point to be zero while the potential is not zero;
   (*c*) for charge to be distributed in such a way that both the potential and the electric field at a point are zero.

4. What is meant by (*a*) the *intensity of the electric field*, (*b*) the *electric potential*, at a point in air?

Point charges of electricity equal to $+100$, $+50$, -50, and $+100$ units are placed respectively at the corners of a square $ABCD$ of side 10 cm. Find (*a*) the electric intensity at O, the point of intersection of the diagonals, (*b*) the work done in taking a point charge of $+2$ units from O to E, the midpoint of AD. (O. & C.)

5. Explain the terms *electric intensity*, *potential*. Two electric charges of $+5$ units and -2 units respectively are 10 cm. apart. At what points in the line joining them will (*a*) the potential, (*b*) the electric intensity, be zero? Sketch the distribution of the lines of force and equipotential surfaces. (N.)

6. An isolated sphere 20 cm. diameter is charged to a positive potential of 50 e.s.u. What will be the charge on the sphere? How much work will be done by the field in taking a unit positive charge between two points A and B, A being 100 cm. from the sphere's centre and B being diametrically opposite to A and 50 cm. from the centre? (C.)

7. What initial velocity would an electron have to be given at infinity in order for it just to reach the surface of a sphere, radius 1 cm., with a charge -1 e.s.u. on it? (Electronic charge $= 4{\cdot}8 \times 10^{-10}$ e.s.u., electronic mass $= 9 \times 10^{-28}$ g.) (C. S.)

8. Draw on squared paper the potential distribution diagram due to an insulated metal sphere of radius 5 cm., charged with 100 e.s.u. of positive electricity.

Discuss the effect produced upon the potential distribution in the field if the potential of the sphere is kept constant and its radius is reduced.

Given that the potential gradient of 100 e.s.u. per cm. is sufficient to ionise the air and bring about discharge, explain by the aid of a diagram similar to the above the discharging action of points. (N.)

9. Define *intensity of the electric field at a point*, *potential gradient*. Obtain a relation between these quantities.

A spark passes in the air when the potential gradient at the surface of a charged conductor is 30,000 volts per cm. What must be the radius of an isolated metal sphere which can be charged to a potential of 3 million volts before sparking into the surrounding air? (300 volts $= 1$ c.g.s. unit of potential.) (C.)

10. Calculate the capacity of the earth in microfarads. The radius of the earth may be taken as $6{\cdot}4 \times 10^8$ cm. (1 microfarad $= 9 \times 10^5$ e.s.u.) (B.)

11. Two plane electrodes, 0·6 cm. apart, within an electron tube (or valve) are maintained at a difference of potential of 100 volts. Determine the field intensity in e.s.u. in the space between the electrodes.

An electron is a point charge of electricity of magnitude $4{\cdot}8 \times 10^{-10}$ e.s.u., and its mass is $8{\cdot}9 \times 10^{-28}$ g. How long would it take such a body, starting from rest at one electrode, to reach the other? (L.)

12. Two brass plates are arranged horizontally, one 2 cm. above the other, and the lower plate is earthed. The plates are charged to a difference of potential of 6000 volts. A drop of oil with an electrical charge of $4{\cdot}774 \times 10^{-10}$ e.s.u. is in equilibrium between the plates so that it neither rises nor falls. What is the mass of the drop? (1 e.s.u. of potential $= 300$ volts.) (O. & C.)

13. Show theoretically that there is no electric field inside a hollow, charged spherical conductor. How may this be tested experimentally? (C. S.)

14. Two spheres of radius 6 and 2 cm. at a considerable distance apart are connected by a wire. How will a charge of 120 e.s.u., given to the system, distribute itself, and what will be the surface densities on the two spheres?

15. What do you understand by the *potential* and *capacity* of a conductor?

A sphere of radius 10 cm. is charged to a potential of 10 e.s.u. One square millimetre of gold leaf spread on the surface of the sphere is removed to a point 20 cm. from the sphere. What work is done? (O. S.)

16. Water from a tap, maintained at a potential v, is allowed to drip in drops of radius a through a small hole in a hollow conducting sphere of radius R until it fills it.

Calculate the potential of the sphere and point out from what source the energy is derived. (O. S.)

17. Find an expression for the electric intensity at a point near an infinite plane charged with electricity.

Find the time of oscillation of a simple pendulum, which has a bob of mass m, charged with q units of positive electricity, when it swings with the bob just above a large horizontal plane charged with σ units of negative electricity per unit area. (C. S.)

CHAPTER 2

INDUCED CHARGES AND CONDENSERS

Electrostatic induction

When an uncharged conductor is placed in an electric field there is a rapid flow of charge in it until its potential is the same throughout; positive and negative *induced* charges appear at its ends. The phenomenon is known as electrostatic induction.

Fig. 15 illustrates the charges induced in a cylindrical conductor by a positively charged sphere; it shows also the potential distribution.

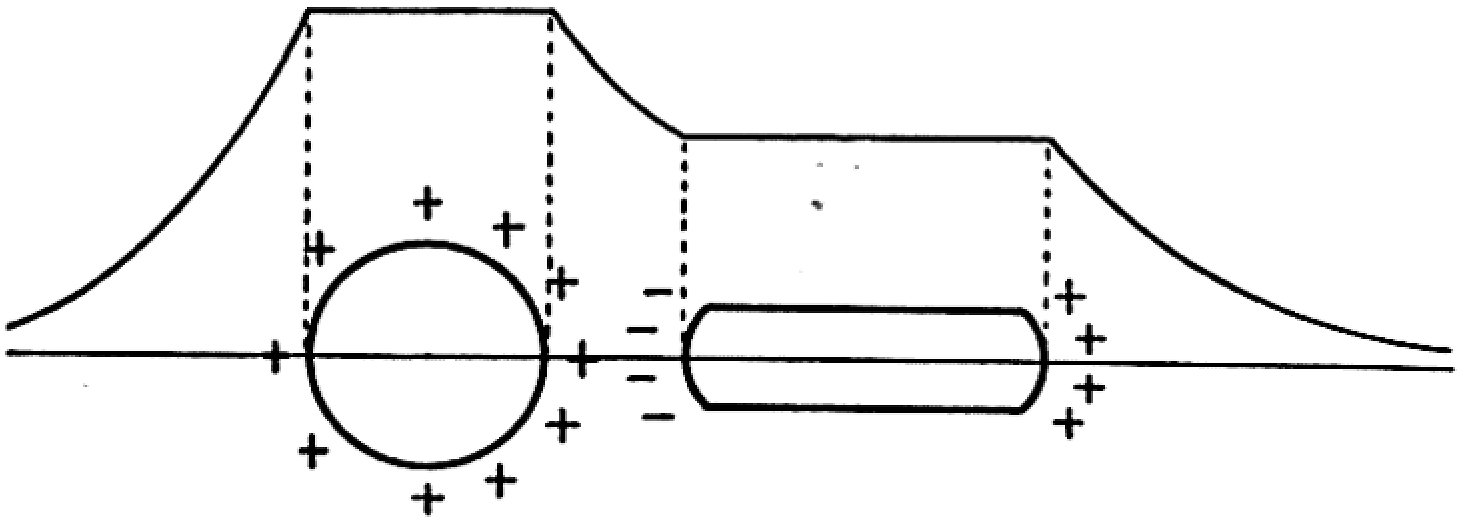


Fig. 15

The height of the graph above a point on the straight line through the centres of the sphere and cylinder represents the potential at that point. Note that the potential throughout the cylindrical conductor is the same and that therefore the field has disappeared inside it. This change in the field is, of course, due to the induced charges; the negative induced charge lowers the original potential at one end, and the positive induced charge raises it at the other.

The existence and sign of the induced charges may easily be verified by means of a proof plane. The proof plane is laid on one end of the cylindrical conductor and when removed carries away with it a sample of the induced charge which can be tested with a gold-leaf electroscope.

The induced charges are temporary, and disappear when the field is removed. If, however, the cylindrical conductor in Fig. 15 is touched to earth before the field is removed, it will be found to have a permanent negative charge. This can be explained by assuming that the induced positive charge was repelled to earth by the positive charge on the sphere, while the induced negative charge was held in position by attraction. Alternatively, we can say that the cylindrical conductor had a positive potential, and hence positive electricity flowed to earth until its potential was zero.

Electrostatic induction occurs only in conductors, since electricity cannot flow in insulators.

Theories of electricity

In electrification by friction equal and opposite charges appear on the rubber and the rubbed. Thus if an ebonite rod is rubbed with a piece of fur mounted on an insulating handle, and the two are presented together to a charged gold-leaf electroscope, the leaf does not move, since they neutralise each other's effects. When the ebonite and fur are brought up separately, however, the ebonite is seen to be negatively charged and the fur positively.

This fact, together with the phenomenon of electrostatic induction, suggests that all uncharged bodies contain equal quantities of positive and negative electricity.

In the eighteenth century it was suggested that there were two weightless, electric fluids, and that an electric current consisted of a flow of each in opposite directions. Benjamin Franklin proposed, for the sake of simplicity, that an electric current should be regarded as a flow of positive electricity only, a convention still retained. It is now known, however, that in a metal a current consists of a flow of negative electrons, whereas in a liquid it consists of a flow of both positively and negatively charged particles in opposite directions.

All matter is now believed to be composed of electrical charges. An atom consists of a small, massive nucleus of positive electricity with a number of negative charges, called electrons, surrounding it. An atom of copper has twenty-nine planetary electrons and a nucleus with an equivalent positive charge. The outermost electron of copper can easily break away from the atom. In all metals one or two of the outermost electrons can easily become free in this way. These free electrons form a kind of electron gas which moves slowly under the influence of an electric field and constitutes a current.

Metals viewed under the microscope are seen to be mosaics of microcrystals; in each of these tiny crystals the atoms are fixed by their mutual attractions in a simple, open-work arrangement known as a lattice, through which the free electrons move.

The process of induction, illustrated in Fig. 15, is therefore explained as a movement of electrons from right to left, giving the left-hand end of the cylindrical conductor an excess of electrons, and hence a negative charge, and the right-hand end a deficiency of electrons, and hence a positive charge.

When the cylindrical conductor is connected to earth, instead of positive electricity passing from the conductor to the earth, negative electrons pass up from the earth to the conductor.

The atoms of insulators do not readily release electrons and hence contain no free electrons to constitute a current. But when in the intimate contact with another substance necessitated by rubbing, they may either gain or lose electrons, and hence become charged in the region where they have been rubbed.

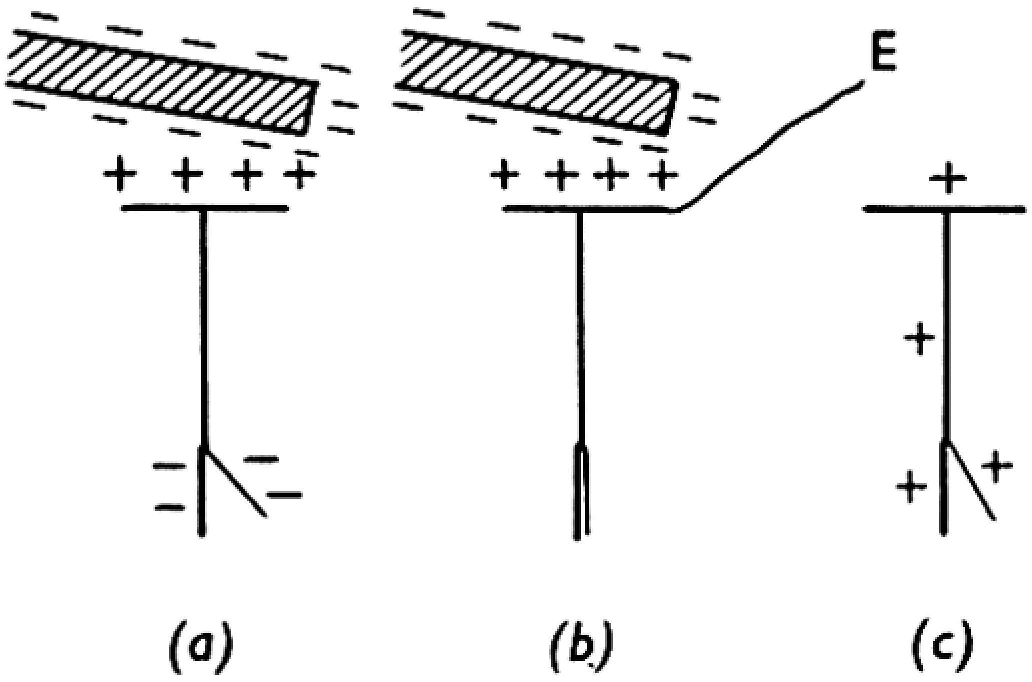


Fig. 16

Charging a gold-leaf electroscope by induction

Bring a rubbed ebonite rod near to the disc of a gold-leaf electroscope. Charges are induced in the electroscope as in Fig. 16 (*a*), and the leaf rises; electrons have been repelled from the disc to the lower end of the electroscope. Without moving the ebonite rod, earth the electroscope by momentarily touching the disc; electrons run from the electroscope to the earth and the leaf falls (Fig. 16 (*b*)). Remove the ebonite rod and the leaf rises (Fig. 16 (*c*)); electrons have run from the leaf to the disc, leaving the leaf positively charged and reducing the positive charge on the disc.

The electroscope measures potential rather than charge

In Fig. 16 (*a*) the electroscope has a negative potential owing to the presence of the negatively charged ebonite rod, but its resultant charge is zero; the leaf diverges. In Fig. 16 (*b*) the electroscope is at zero potential but it has a resultant positive charge; the leaf does not diverge. Thus the divergence of the leaf indicates the potential rather than the charge of the electroscope.

If a positively charged body is brought near to the electroscope of Fig. 16 (*c*), the positive potential of the electroscope increases and hence the leaf diverges farther. If a negatively charged body is brought near, the positive potential of the electroscope is reduced and the leaf falls. Thus the electroscope may be regarded as an electrostatic voltmeter.

Use of the electroscope to show that the potential is constant over the surface of a charged conductor

Charge a conductor by induction with an ebonite rod. Move a wire, wrapped round an insulating handle and connected to an electroscope (see Fig. 17), over the surface of the conductor. The divergence of the leaf of the electroscope is constant, showing that the potential over the surface of the conductor is constant.

In this experiment the conductor and the electroscope are permanently connected. In the experiment described on p. 13 the proof plane was disconnected from the conductor, and when taken away from the influence of the latter it acquired a potential determined by its capacitance and the magnitude of the sample of charge it carried away; it was this potential that the electroscope measured.

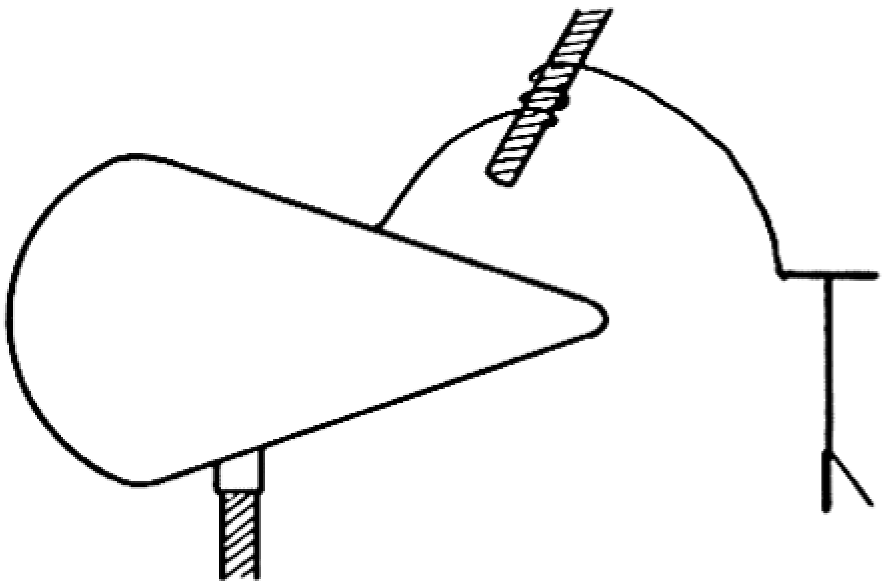
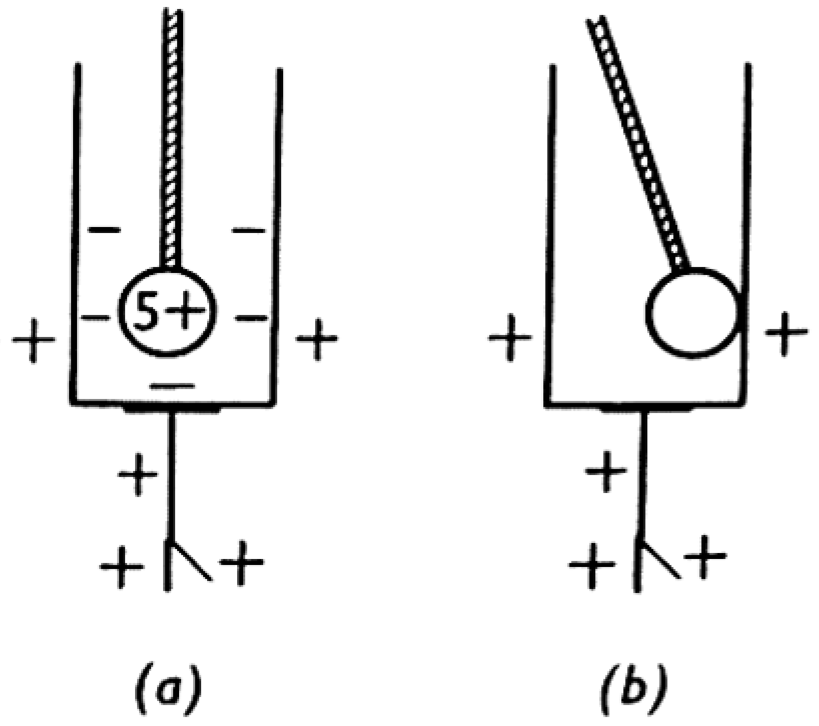


Fig. 17



(*a*) (*b*)

Fig. 18

Faraday's ice-pail experiment

Faraday performed famous experiments to show that the total induced charge is always equal to the inducing charge. He wrote that 'their value consists in their power to give a very precise and decided idea to the mind respecting certain principles of inductive electrical action, which I find are by many accepted with a degree of doubt or obscurity that takes away much of their importance'.

He used an ice-pail, replaced in Fig. 18 by a deep can, and the following is one of the experiments. Place the can on an electroscope and lower a positively charged sphere into the can. The leaf diverges. When the sphere is touched to the inside of the can, the divergence of the leaf does not change and the sphere, on removal, will be found to be totally discharged.

In Fig. 18 (*a*) five positive charges on the sphere are shown as inducing five negative charges on the inside of the can and these neutralise in (*b*). The five induced positive charges on the outside of the can remain when the sphere touches the inside and also when it is withdrawn.

The can must be deep so that all the lines of force from the charged sphere terminate on the can. The experiments show that lines of force which start on a positive charge always end on an *equal* negative charge. Faraday made this very clear by speaking of tubes of force, each starting on unit quantity of positive charge and ending on unit quantity of negative charge. Each tube may be regarded as made up of, or as bounded by, 4π lines.

The condenser

When an insulated metal plate A (Fig. 19) is given a positive charge and an earthed plate B is brought near to it, the positive potential of A is reduced, owing to the effect of the negative charge induced in B. If B is brought nearer, the potential of A is lowered still farther. This can be demonstrated by connecting A to a gold-leaf electroscope which measures its potential. The leaf of the electroscope falls as B is brought nearer to A.

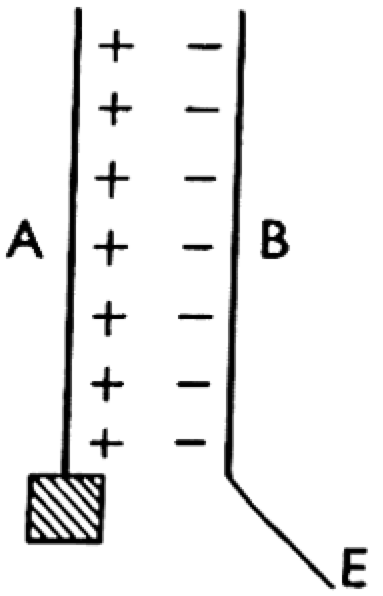


Fig. 19

The insulated plate's charge, Q, is unchanged, but its potential, V, is reduced by the presence of the earthed plate. Thus its capacitance, $C=Q/V$, has been increased. The arrangement of the two plates is called a condenser, and is a convenient means of storing a comparatively large charge without an excessively high potential. It is an essential component of a radio receiver.

Similarly, in Fig. 15 the potential of the sphere is lowered by the presence of the cylinder and hence its capacitance is increased. The capacitance of all bodies is influenced by the presence of nearby conductors.

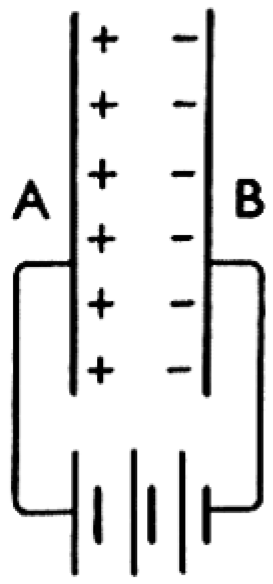


Fig. 20

Charging a condenser by a battery

A condenser may be charged by connecting its plates to the positive and negative terminals of a battery as in Fig. 20. We may regard the battery as pumping electrons from A to B, giving A a positive potential and B a negative.

Dielectrics

If a slab of an insulator, such as paraffin wax, is placed between the plates of a charged condenser, the potential difference between the plates falls and thus the capacitance increases. This can be demonstrated experimentally by connecting the positively charged plate of the condenser to a gold-leaf electroscope. When the slab of paraffin wax is inserted the leaf of the electroscope falls a certain distance, and rises again when the slab is withdrawn.

The ratio of the capacitance of the condenser with the insulator (called a dielectric) filling the space between the plates, to its capacitance with a vacuum between the plates is called the *dielectric constant* or *specific inductive capacity* of the dielectric, and is denoted by k.

Some values of k are:

Air	1·0006
Dry paper	2–2·5
Paraffin wax	2–2·3
Ebonite	2·7–2·9
Sulphur	3·6–4·3
Mica	5·7–7
Glass	5–10
Pure water	81

The fall in potential difference between the plates is due to 'polarisation' of the dielectric. Under the action of the electric field the molecules become distorted, the electrons being attracted towards the positive plate and the positive nuclei towards the negative plate, as shown diagrammatically in Fig. 21. The molecules may be regarded then as having positively and negatively charged ends, that is, as electric dipoles.

Positive and negative ends of adjacent molecules (Fig. 22) neutralise each other's effects, but the surface of the dielectric against the positive plate has a negative charge and this lowers the potential of the plate. Similarly, the positive charge at the other surface of the dielectric raises the potential of the negative plate.

The molecules of certain substances with a high dielectric constant, such as water, are permanent electric dipoles; that is to say, the 'centres of gravity' of the positive and negative charges do not coincide even when there is no electric field. Under the action of an electric field they align themselves rather like magnetic compasses in a magnetic field. This effect is superimposed on the normal effect and accounts for the high dielectric constant.

The analogy with magnetism is even more striking in the case of a substance known as carnuba wax. This becomes permanently polarised when melted and allowed to solidify in a strong electric field. The molecules are 'frozen' into alignment. If the wax so treated is cut into pieces, each piece will set itself in an electric field with its positive face in the direction of the field.

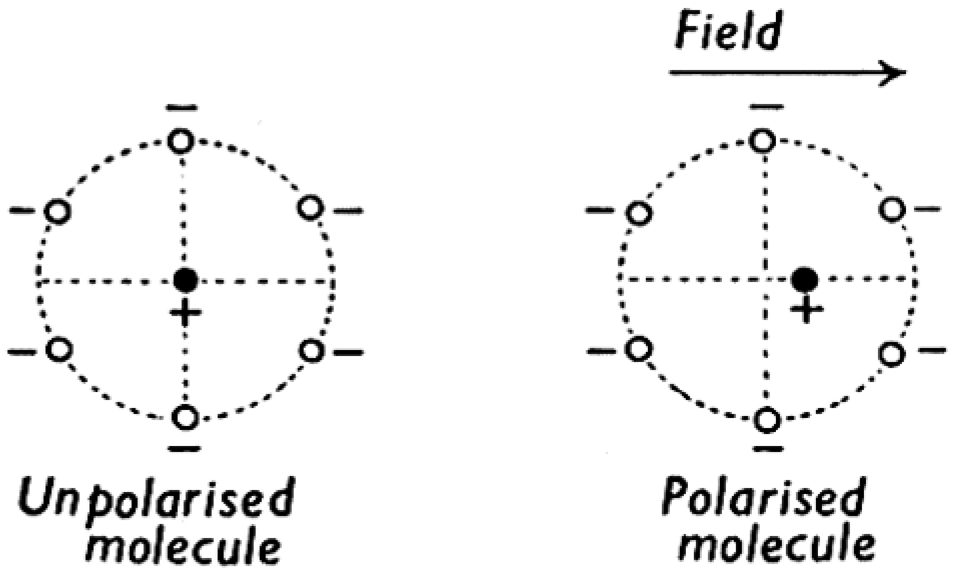


Fig. 21

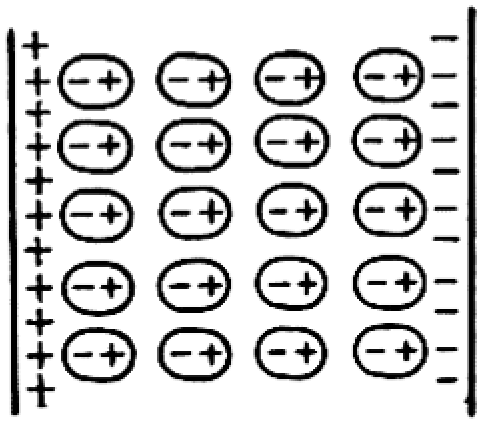


Fig. 22

Residual effect

When the plates of a charged condenser are connected, a spark passes immediately before contact is made and the plates are discharged. With a dielectric such as glass, several subsequent but much smaller sparks can be obtained without recharging. This is known as the residual effect, and may be explained by the dielectric retaining some of its polarisation for some time. As the polarisation slowly disappears, charges on the plates are released from the attraction of the dielectric and enable a small spark to be obtained.

An illuminating experiment can be performed with a condenser having a dielectric such as glass and removable plates mounted on insulating supports. After charging, say with a battery, the plates are removed from the glass and discharged. On replacing the plates against the glass a discharge can be obtained from the condenser. The charge must have remained on the glass when the plates were removed.

Piezo-electricity

An allied phenomenon is that of piezo-electricity. Certain crystals, notably quartz and Rochelle salt, when compressed or extended perpendicular to the axis of crystal symmetry become charged as shown in Fig. 23. Conversely, when quartz is placed in an electric field it expands or contracts according to the direction of the field.

The capacitance of a parallel plate condenser

The capacitance of a condenser depends only on the dimensions and the nature of the dielectric. Its value can be calculated in a few simple cases.

Consider a parallel plate condenser having plates of area A, at distance d apart, with a vacuum between them. Suppose one plate is given a charge Q raising its potential to V while the other plate is earthed.

The total number of lines of force from the charge Q is $4\pi Q$. These will all lie between the plates; provided that the area of the plates is large and their separation is small, the lines will be parallel so that the field is uniform. The strength of the field, E, is equal to the number of lines of force per sq.cm.; it is also equal to the potential gradient.

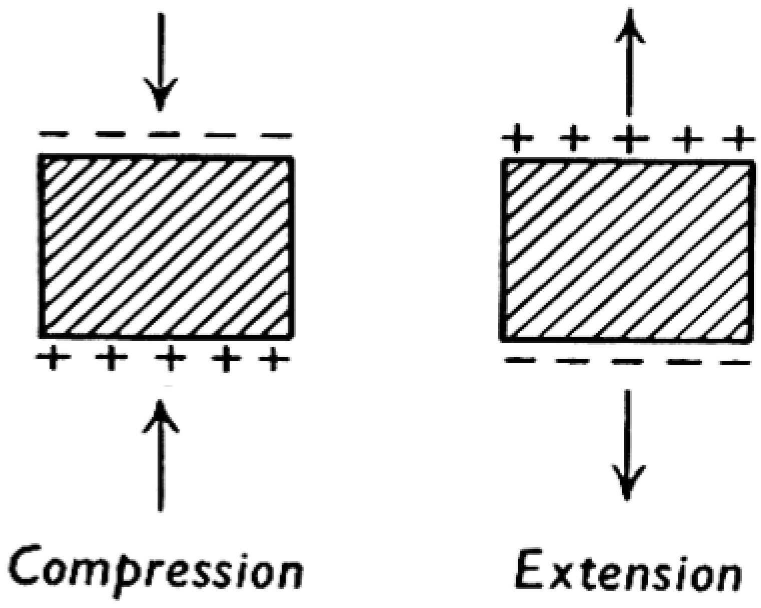


Fig. 23

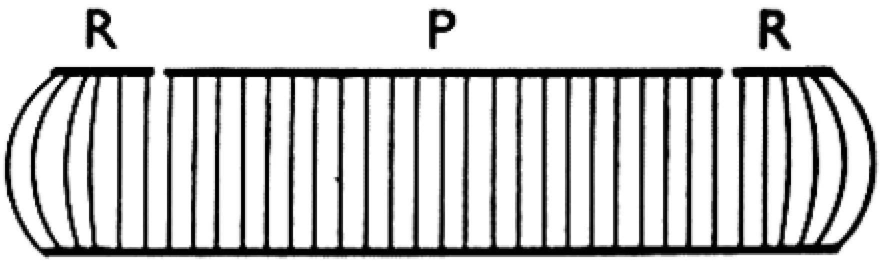


Fig. 24

$$\therefore\ E=\frac{4\pi Q}{A}=\frac{V}{d}.$$

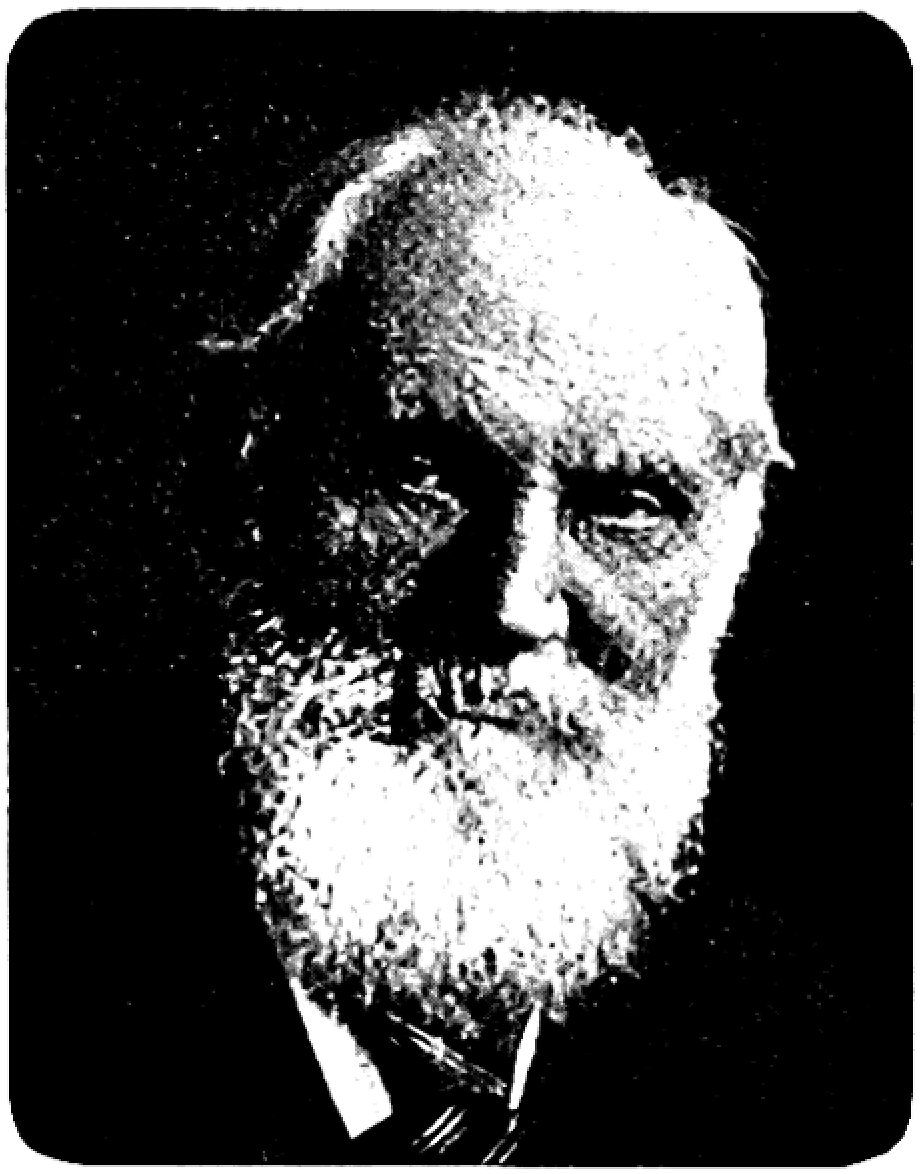
$$\therefore\ \text{Capacitance } C=\frac{Q}{V}=\frac{A}{4\pi d}.$$

If a dielectric, of dielectric constant k, replaces the vacuum between the plates

$$\mathrm{C}=\frac{\mathrm{kA}}{4\pi \mathrm{d}}.$$

The formula is not accurate if the lines of force at the edges of the plate are curved. The error can be eliminated by means of an experimental device known as a guard ring (Fig. 24). This consists of a ring *RR* surrounding the circular insulated plate *P* of the condenser and nearly touching it. *RR* is kept at the same potential as *P*. The earthed plate has an area equal to the combined areas of *P* and *RR*. It can be seen from Fig. 24 that the field between *P* and the earthed plate is uniform; the non-uniform field at the edges of *R* does not matter since *R* is not part of the condenser.

MICHAEL FARADAY



LORD KELVIN

MICHAEL FARADAY (1791–1867), *one of the greatest of experimental physicists, became first the laboratory assistant and then the successor to Sir Humphry Davy at the Royal Institution. His electrical researches began with the experiment, attempted earlier without success by Wollaston, in which a wire carrying a current rotated round a magnetic pole—the first electric motor. Ten years later, after repeated failures to convert magnetism into electricity, he discovered electromagnetic induction and made the first dynamo. His concepts of lines and tubes of force, and of the action of the dielectric, provided a new and revolutionary viewpoint which led Maxwell to formulate the electromagnetic theory of light. He discovered the laws of electrolysis and made numerous advances in electrostatics. He was a deeply religious man, wholly absorbed in his scientific investigations and indifferent to money. He refused a knighthood and the Presidency of the Royal Society.*

WILLIAM THOMSON, LORD KELVIN (1824–1907), *became Professor of Natural Philosophy at Glasgow university at the early age of 22, and held this chair for 53 years. For most of his life he was a busy man of affairs, as well as a professor, and much of his original scientific work was done in the railway train between Glasgow and London. Lord Rutherford said of him that he 'combined to an extraordinary degree the quality of great theoretical insight with the power to realize his ideas in a practical form'. Two of his finest theoretical achievements were the theory of electrical oscillations in a circuit containing resistance, inductance and capacitance, and the foundation (with Carnot and Clausius) of thermodynamics. His inventions included the quadrant electrometer, the direct-reading ammeter, improved forms of ship's compass and sounding apparatus, and the siphon recorder for use with the Atlantic cable.*

Energy of a parallel plate condenser

When the charged plates of a parallel plate condenser are moved towards each other the field between the plates is unchanged. The field is equal to the number of lines of force per sq.cm., and since the lines of force are parallel (assuming the presence of a guard ring), their number per unit area does not change when the plates are moved.

If one of the plates is earthed, the potential of the other plate falls as the plates are moved together. The potential gradient, V/d, however, remains unchanged since $E=V/d$, where E is the strength of the field, V is the potential of the insulated plate and d is the distance apart of the plates.

When the plates are about to touch, the potential of the insulated plate is almost zero, since the effect of its own charge $+Q$ is neutralised by the effect of the induced charge $-Q$ on the earthed plate.

Suppose the plates, originally at potentials of V and zero respectively, are allowed to move together until they touch,

$$\text{Average potential difference} = \tfrac{1}{2}V,$$

$$\therefore \text{ Work done in bringing the plates together } = Q \, . \, \tfrac{1}{2}V$$

(from the definition of potential difference). This work is derived from the energy of the charged condenser and could be utilised to lift a small weight attached to one of the plates. Since $C=Q/V$, the expression for the energy of the condenser can be written in several forms as follows:

$$\text{Energy of charged condenser} = \tfrac{1}{2}\mathbf{QV} = \tfrac{1}{2}\mathbf{CV}^2 = \frac{1}{2}\frac{\mathbf{Q}^2}{\mathbf{C}} .$$

Force of attraction between the charged plates of a parallel plate condenser

The plates of a parallel plate condenser attract each other because they are oppositely charged. Suppose the force is F dynes. This force will be constant when the plates are moved together because the electric field between the plates is constant.

When the plates, originally at a distance d apart, are allowed to move together so that they touch, the work done by the force of attraction between the plates is equal to the original energy of the condenser, i.e.

$$Fd = \tfrac{1}{2}QV,$$

$$\mathbf{F} = \tfrac{1}{2}\mathbf{Q}\frac{\mathbf{V}}{\mathbf{d}} .$$

It might, at first sight, be thought that the force should be $Q(V/d)$, since the field between the plates is V/d. But the charge on the insulated plate may be regarded as contributing half the value of the field and the induced charge on the earthed plate as contributing the other half; the force exerted on the charge on one of the plates is due to the field set up by the charge on the other.

The attracted disc or absolute electrometer

The force of attraction between the charged plates of a parallel plate condenser can be measured and may be utilised to determine potential. An instrument which can measure a quantity in terms of its fundamental units is called an absolute instrument. The gold-leaf electroscope must be calibrated by another instrument if it is to measure electrostatic units of potential or statvolts. The attracted disc electrometer, however, is an absolute instrument.

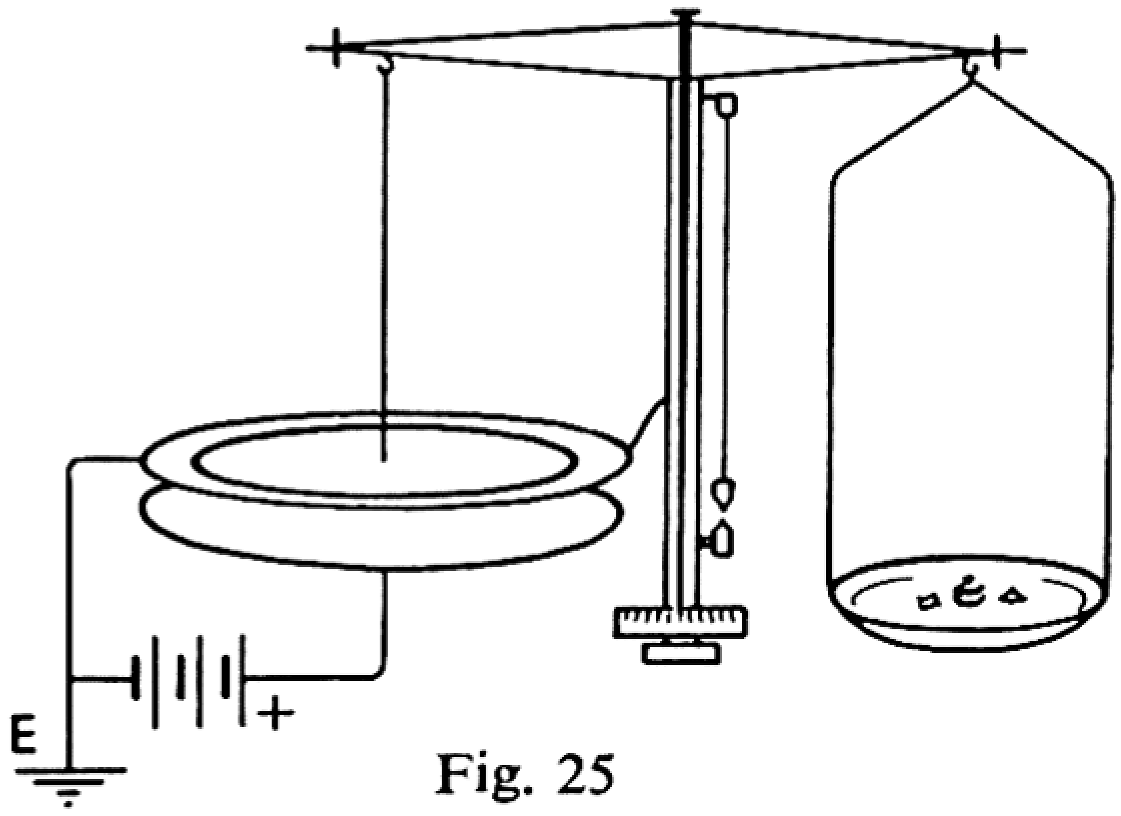


Fig. 25

It is a device for measuring the attraction between the parallel plates of an air condenser provided with a guard ring. Fig. 25 is a diagrammatic representation. The lower condenser plate is insulated and connected to the positive pole of a battery; the upper plate and guard ring are connected to the negative pole and to the earth via the frame of the balance. The attraction of the upper plate of the condenser by the lower plate, $\frac{1}{2}Q(V/d)$, is counteracted by a weight mg in the right-hand balance pan.

Thus

$$\tfrac{1}{2}Q\frac{V}{d} = mg.$$

With the usual nomenclature

$$Q = CV \qquad \text{(see p. 10)}$$

and

$$C = \frac{A}{4\pi d} \qquad \text{(see p. 25).}$$

$$\therefore \quad \frac{1}{2}\,\frac{VA}{4\pi d}\,\frac{V}{d} = mg,$$

$$V^2 = \frac{8\pi d^2}{A}\,mg.$$

Knowing d, A and mg, V can be calculated. In Fig. 25, V is the E.M.F. of the battery in statvolts.

Practical unit of capacitance

The formula $kA/4\pi d$, when A is in sq.cm. and d in cm., gives the capacitance of a parallel plate condenser in e.s.u. or statfarads or centimetres.

The practical unit of capacitance is the farad, which is the capacitance of a conductor requiring a charge of 1 coulomb to raise its potential by 1 volt. The farad is a large unit and the capacitances of wireless condensers are measured in microfarads, i.e. millionths of a farad. On p. 175 is given the proof that

$$1 \text{ farad} = 9 \times 10^{11} \text{ e.s.u.}$$

Practical forms of condensers

To possess a high capacitance a condenser must have plates of a large area as close together as possible and a dielectric with a high dielectric constant. A common form of wireless condenser consists of interleaved plates of tinfoil separated by thin sheets of mica or paraffin waxed paper (see Fig. 26). Sometimes a thin film of silver is deposited on both sides of thin strips of mica or ceramic base material.

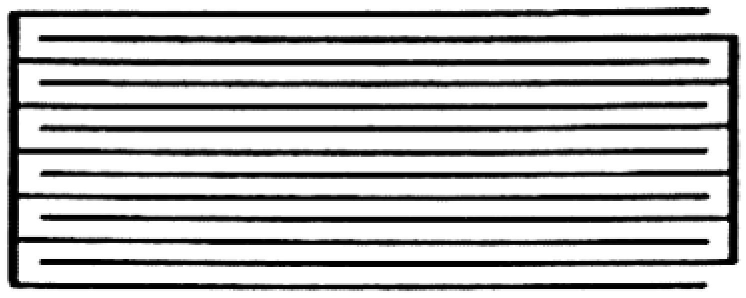


Fig. 26

Variable wireless condensers usually take the form of a set of metal vanes which can move between a set of fixed metal vanes, air being the dielectric (see Fig. 27). The moving plates of early wireless condensers were semicircular, in which case the capacitance is proportional to the angle of overlap. The ends of the tuning scale of the radio receivers were cramped because the wave-length is proportional to the square root of the capacitance. The shape of the moving vanes was therefore altered so that the capacitance increased as the square of the angle of overlap. These condensers are called square-law types.

The Leyden jar consists of a glass jar as the dielectric, with inner and outer coatings of tinfoil to act as the plates (see Fig. 28). The inner insulated plate is connected by a chain to an insulated knob for convenience of charge and discharge.

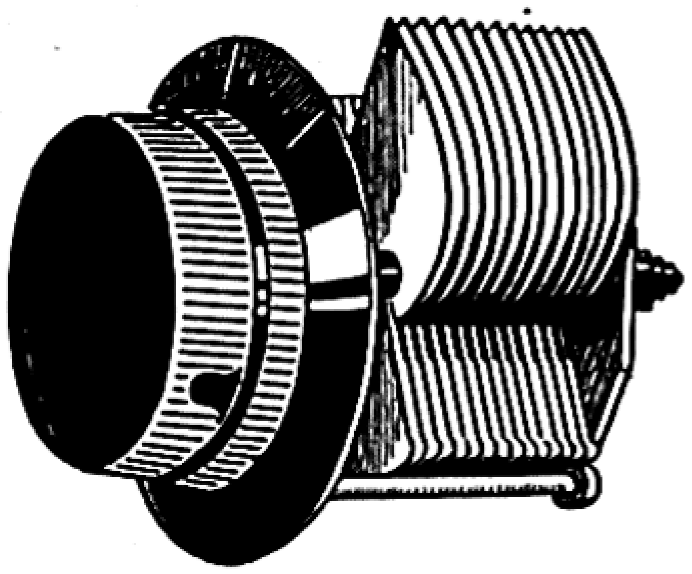
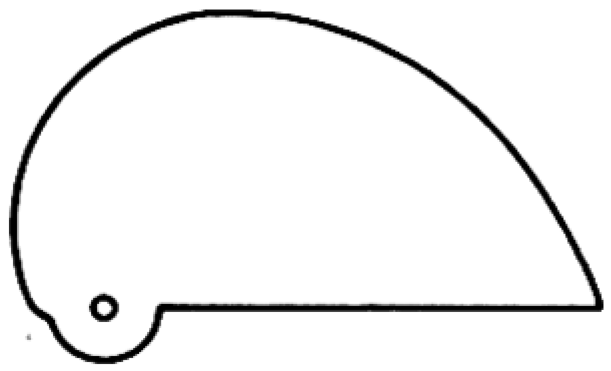


Fig. 27*a*



(*b*)

Fig. 27*b*. Shape of square law condenser plate.

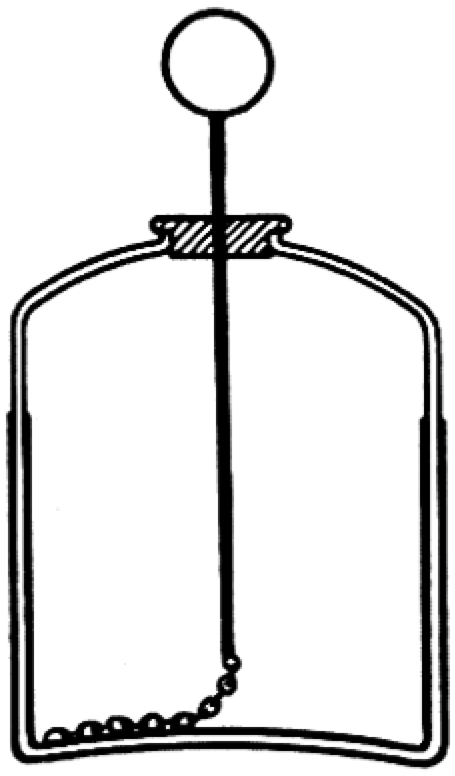


Fig. 28

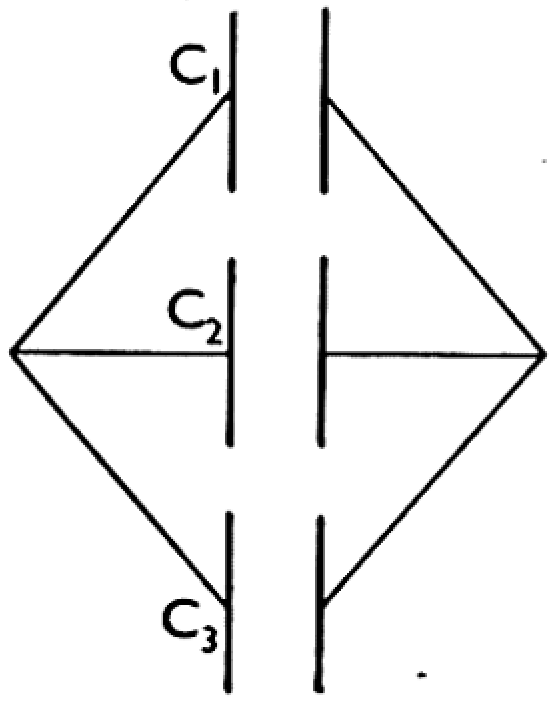


Fig. 29

Condensers in parallel

The condensers in Fig. 29 are said to be connected in parallel. It is clear that the three condensers are equivalent to a single condenser with plates of area equal to the sum of the areas of the plates of the separate condensers, and hence that

$$C = C_1 + C_2 + C_3.$$

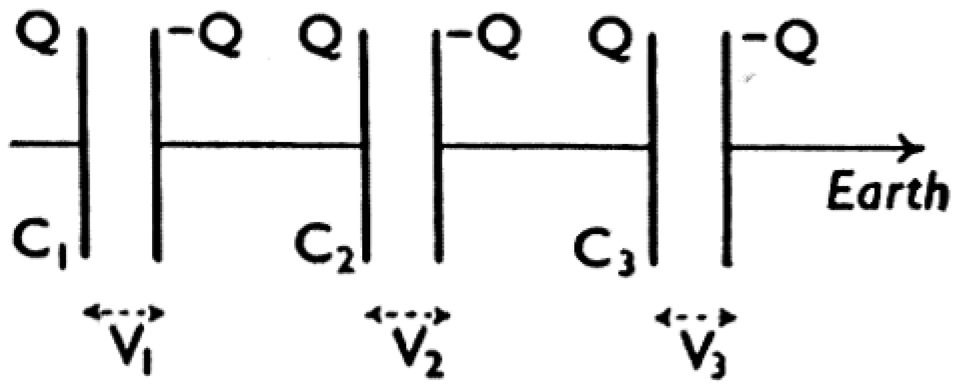


Fig. 30

Condensers in series or cascade

The condensers in Fig. 30 are said to be connected in series or cascade. When the extreme left-hand plate is given a charge Q, and the extreme right-hand plate is earthed, the outflow from each condenser is equal to the inflow to the next, and the plates acquire charges Q and $-Q$ as shown. The total P.D. V across all the condensers is equal to the sum of the P.D.'s across each.

Thus
$$V=V_1+V_2+V_3,$$

i.e.
$$\frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}.$$

$$\therefore\ \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}.$$

Energy of any charged conductor

By calculating the work done in charging the conductor it can be shown that the expression $\frac{1}{2}QV$ represents the energy of any charged conductor.

Suppose the conductor is charged by bringing up small charges from infinity. When it is at a potential v the work done in bringing up a charge δq, which is so minute that it does not appreciably change the potential, is $v\delta q$. Hence to charge the conductor with charge Q

$$\text{Work done}=\int_0^Q v\,dq.$$

But
$$v=q/C,$$

where C=capacitance of conductor.

$$\therefore\ \text{Work done}=\int_0^Q \frac{q}{C}\,dq$$

$$=\frac{Q^2}{2C}=\tfrac{1}{2}QV.$$

Law of force in a dielectric

When a dielectric replaces a vacuum between the charged plates of a condenser, the potential gradient, and hence the field strength, is reduced by the factor k, the dielectric constant. Now field strength is force per unit charge and thus the law of force in the dielectric must be

$$F=\frac{q_1q_2}{kr^2}.$$

This can be explained as the effect of an opposing field due to the polarisation charges on the surfaces of the dielectric adjacent to the plates (see Fig. 22).

Similarly, the field strength at a distance r from a point charge q, in a medium of dielectric constant k, is $\mathbf{q/kr^2}$ and the potential is $\mathbf{q/kr}$. These are the combined effects of the charge q and a polarisation charge 'induced' on the walls of the cavity of the dielectric containing q.

Lines of induction

The insertion of a slab of dielectric between the plates of a charged condenser reduces the field between the plates, but it does not affect the magnitudes of the charges on the plates. The induced charge is always equal to the inducing charge whatever the dielectric.

It is therefore convenient to introduce the concept of lines of electric induction in addition to that of lines of electric force. A unit charge has always 4π lines of induction whether in a vacuum or in a dielectric because it always induces the same charge; it has 4π lines of force in a vacuum but appears to have only $4\pi/k$ lines of force in a dielectric.

As already indicated, however, we prefer to regard a charge as having 4π lines of force in all media and to explain the reduction to $4\pi/k$ as due to the opposing field of the surface polarisation charges of the dielectric. On our view the lines of induction are the total lines of electric force due to the original charge and to the polarisation charge adjacent to it, plus those which run through the interior of the dielectric from each charged molecule of the dielectric to the next (Fig. 22). To prove this, suppose that the charge on the positive plate of the condenser in Fig. 22 is Q per sq.cm. and that the polarisation charge on the surface of the dielectric next to the plate is $-P$ per sq.cm. The number of lines of electric force due to these charges, which represents the electric field in the dielectric, is $4\pi(Q-P)$. If the dielectric is cut somewhere in a plane parallel to the plates, induced charges P and $-P$ per sq.cm. will appear on the walls of the cavity produced by the cut and $4\pi P$ lines of force will run across the cavity in addition to those due to the electric field.

$$\begin{aligned}\therefore\ \text{Induction} &= 4\pi(Q-P)+4\pi P\\ &= 4\pi Q.\end{aligned}$$

This accords with our statement that unit charge has 4π lines of induction in all media.

Maxwell called the number of lines of induction per sq.cm. the displacement D and thus, if E is the electric field, $D=kE$; if P is the polarisation charge of the dielectric, $D=E+4\pi P$. We shall make no use of these equations but mention them because of their analogues in magnetism (p. 108).

Maxwell thought of electric field as a stress in the aether and of induction as a strain akin to an elastic deformation of the aether. *D* and *E* were regarded as quantities with different physical natures and hence with different dimensions; but on the view we have adopted *D* and *E* are physically identical. We shall discuss this more fully in Chapter 9.

EXAMPLE 1

A conductor of capacitance 4 e.s.u. is charged to a potential of 90 e.s.u. It shares its charge with an uncharged conductor of capacitance 2 e.s.u. Find their common potential and the loss of energy.

$$\text{Charge} = CV = 4 \times 90 = 360 \text{ e.s.u.}$$

Let v e.s.u. = common potential.

No charge is lost on sharing.

$$\therefore\ (4+2)\,v = 360,$$

$$v = 60 \text{ e.s.u.}$$

$$\text{Initial energy} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}.4.90^2 = 16{,}200 \text{ ergs.}$$

$$\text{Final energy} = \tfrac{1}{2}.6.60^2 = 10{,}800 \text{ ergs.}$$

$$\therefore\ \text{Energy lost} = 16{,}200 - 10{,}800$$

$$= 5{,}400 \text{ ergs.}$$

The energy is lost in the form of a spark on connecting the conductors.

EXAMPLE 2

A parallel plate condenser has a capacitance of 100 e.s.u. Calculate the work done in separating its plates to double their original distance apart (*a*) if the plates are charged to a potential of 2 e.s.u. and insulated before being separated; (*b*) if the plates are permanently connected to a battery which maintains the P.D. at 2 e.s.u.

The capacitance *C* of the condenser changes from 100 to 50 e.s.u. when the distance between the plates is doubled.

In case (*a*) the charge *Q* on the plates remains constant at

$$100 \times 2 = 200 \text{ e.s.u.}$$

$$\text{Energy of condenser} = \frac{Q^2}{2C}.$$

$$\therefore\ \text{Gain in energy} = \frac{200^2}{2}\left(\frac{1}{50} - \frac{1}{100}\right)$$

$$= 200 \text{ ergs.}$$

$$\therefore\ \text{Work done in separating the plates} = 200 \text{ ergs.}$$

In case (*b*) the P.D. between the plates remains constant at 2 e.s.u.

$$\text{Energy of condenser} = \tfrac{1}{2}CV^2.$$

$$\therefore\ \text{Gain in energy} = \tfrac{1}{2}.2^2(50-100)$$

$$= -100 \text{ ergs.}$$

Hence despite the fact that work must be done to separate the unlike charges on the plates, the condenser loses energy. The work lost is used to charge up the battery.

$$\begin{aligned}\text{Charge flowing into battery} &= (C_1 - C_2)V \\ &= (100-50)2 = 100 \text{ e.s.u.} \\ \text{Work done in charging battery} &= Eit \text{ (see p. 128)} \\ &= 2 \times 100 = 200 \text{ ergs.}\end{aligned}$$

Some of this work comes from the loss in energy of the condenser.

$$\begin{aligned}\therefore\ \text{Work done in separating plates} &= 200 - 100 \\ &= 100 \text{ ergs.}\end{aligned}$$

EXAMPLE 3

Calculate the capacitance of a spherical condenser consisting of two concentric spheres, radii a and b, with a vacuum (or air) between the spheres (Fig. 31).

Suppose the inner sphere is given a charge $+q$ and the outer sphere is earthed. The outer sphere will have an induced charge $-q$.

The charges will be distributed on the surfaces of the spheres but will behave as though concentrated at the centres of the spheres.

The potentials due to the charge $+q$ will be q/a on the inner sphere and q/b on the outer sphere. The potential of the large sphere due to its charge $-q$ will be $-q/b$ and the potential throughout the space inside this sphere, due to $-q$, will be $-q/b$.

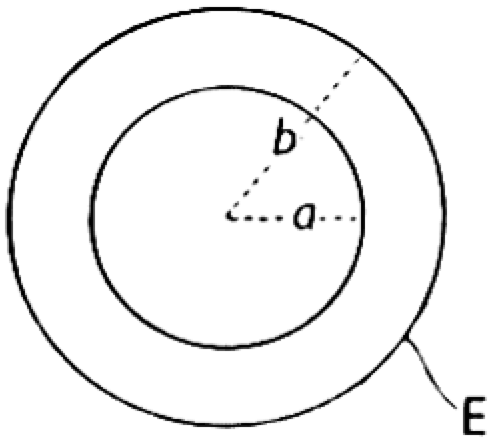


Fig. 31

$$\therefore\ \text{Potential of inner sphere} = q/a - q/b,$$

$$\text{Potential of outer sphere} = q/b - q/b = 0.$$

$$\begin{aligned}\therefore\ \text{Capacitance of condenser} &= \frac{\text{charge}}{\text{P.D.}} \\ &= \frac{q}{q/a - q/b} \\ &= \frac{\mathrm{ab}}{\mathrm{b-a}}.\end{aligned}$$

EXAMPLE 4

Calculate the capacitance of a cylindrical condenser consisting of two concentric cylinders, radii a and b and length l, with a vacuum (or air) between the cylinders.

Suppose the inner cylinder is given a charge $+q$ and the outer cylinder is earthed. There will be $4\pi q$ lines of force between the cylinders and they will diverge, showing that the field is not constant. At distance x from the axis of the cylinders (where $b > x > a$), the lines of force are spread over an area $2\pi xl$.

Field at distance x from axis, F = number of lines of force per sq. cm., i.e.

$$F = \frac{4\pi q}{2\pi xl} = \frac{2q}{xl}.$$

But $$F=-\frac{dV}{dx},$$

where V=potential at distance x from axis.

$$\therefore \int dV=\int -Fdx,$$

$$\int_0^v dV=\int_b^a -\frac{2q}{xl}dx,$$ where v=potential of inner cylinder,

$$v=\frac{2q}{l}\log_e\frac{b}{a}.$$

$$\therefore \text{ Capacitance of condenser}=\frac{q}{v}$$

$$=\frac{l}{2\log_e b/a}.$$

(This method could be used in Example 3 above.)

QUESTIONS

1. An isolated metal sphere is given a positive charge. Draw and explain diagrams showing the manner in which the potential in the neighbourhood varies from point to point (i) before, (ii) after an insulated conductor is brought near. (C.)

2. How would you demonstrate experimentally any variations which occur in the potential at points on, and in the neighbourhood of, the surface of a charged insulated spherical conductor?

What factors determine the potential of a charged conductor? Describe an experiment to illustrate the effect of one of these factors. (N.)

3. Describe and explain how you would charge a gold-leaf electroscope positively by induction. Draw diagrams showing the lines of force at three stages of the process.

Explain why the presence of a negative charge in a conductor should not be tested by means of a positively charged electroscope. (N.)

4. Describe the experiment known as Faraday's ice-pail, and show what important conclusions can be deduced from it. Do you consider it an *exact* proof of these conclusions?

Being supplied with an insulated metal sphere charged with 10 units of positive electricity, and a small and a large calorimeter each with an insulating stand and handle, how would you produce (*a*) a negative charge of 20 units, and (*b*) a charge of 20 positive units? (B.)

5. A gold-leaf electroscope is placed on a block of paraffin wax and the cap, or knob, is connected by a wire with the metal case of the instrument. A small electric charge is given to the cap. The connecting wire is then removed by means of insulated tongs. Finally the cap is touched with the finger. Describe and explain the sequence of events. (D.)

6. It is required to construct a condenser of capacity 0·05 microfarad consisting of two plane metal sheets separated by a sheet of mica of thickness 0·003 cm. and of dielectric constant 6. What must be the area of each of the metal sheets? (D.)

7. Calculate the capacitance in microfarads of a parallel-plate condenser whose plates have an area of 400 sq.cm. and are separated by mica ($k=6$) of thickness 0·001 cm. If the condenser is connected to the poles of a 4-volt battery, calculate the charge on each plate.

8. A parallel-plate air condenser is charged to a potential difference of 300 volts and then connected in parallel with another condenser of equal dimensions with ebonite as dielectric. The potential of the combination is found to be 75 volts. Calculate the dielectric constant of ebonite. (N.)

9. Obtain an expression for the energy of a charged condenser. A condenser of capacity 10 e.s.u. is charged to a potential of 100 e.s.u. and connected to the plates of an uncharged condenser of capacity 40 e.s.u. Calculate the energy of the condensers (*a*) before, (*b*) after being connected and account for any difference in the two cases. (D.)

10. Calculate the work done in charging a sphere of radius 5 cm. with a charge of 100 e.s.u.

11. A condenser consists of two circular, parallel plates each of radius 10 cm., distant 1 cm. apart, with a slab of glass, of dielectric constant 5, filling the space between them. One of the plates is given a charge of +250 e.s.u. while the other plate is earthed. Calculate the potential of the insulated plate, and also its electrical energy.

The slab of glass is removed, the plates remaining the same distance apart. What are now the potential and the electrical energy of the insulated plate? Where did the extra energy come from?

12. Three insulated parallel plates A, B, C, of equal area, are equidistant. The middle plate B is given a positive charge and A and C are momentarily earthed. A and C are connected to two gold leaves insulated from each other and hanging side by side. A slab of shellac is interposed between A and B and the leaves attract each other. Explain carefully with a diagram showing the charges. Which of the leaves has a potential positive with respect to the other?

13. Give an account of the action of an electrostatic condenser, and show that the energy of such a condenser is $\frac{1}{2}CV^2$, where C is its capacity and V the potential difference between its electrodes.

To what potential must a condenser of capacity 40 microfarads be charged to have the same energy as a fully charged 2-volt, 10-ampere-hour accumulator? (O. & C.)

14. Show that the capacity of a sphere is equal to its radius.

Two equal spherical raindrops, charged to the same potential, meet and coalesce. Compare their total electrical energies before and after collision. How do you reconcile your result with the principle of the conservation of energy? (O. S.)

15. Find the electrical capacity of a condenser consisting of a sphere and a concentric spherical shell, of radii 9 and 10 cm. respectively, separated by air.

What would be the potential of the sphere if it were given a charge of 40 e.s.u. while the outer shell was (*a*) insulated, (*b*) earth-connected? (L.)

16. A conducting sphere of radius 5 cm. has a positive charge of 100 e.s.u. and is surrounded by a concentric spherical conductor, the radii of whose surfaces are 8 and 10 cm. respectively.

Calculate the capacity of the inner sphere and the energy of the charge upon it when the surrounding shell is (*a*) insulated, (*b*) earthed. (L.)

17. Derive from first principles the capacitance of a parallel-plate condenser which has a slab of dielectric, of dielectric constant k and thickness t, between the plates. Assume that the rest of the space between the plates is filled with air, that the plates have an area A and that they are a distance d apart.

18. Find an expression for the combined capacity of three condensers C_1, C_2 and C_3 in series (cascade).

If $C_1=20$, $C_2=30$ and $C_3=15$ e.s.u. and the insulated plate of C_1 be at a potential of 45 e.s.u., one plate of C_3 being earthed, what is the difference of the potential between the plates of C_2, the three condensers being in series? (L.)

19. A condenser consists of three parallel plates, the two outer of which are fixed and connected together, while the centre one is capable of sliding between them. If the condenser is charged and insulated, show by considerations of energy that there is a force tending to pull the movable plate into the space between the fixed plates. Interpret this result, using the conception of lines of force. (C. S.)

20. What is the capacity in farads per kilometre of a cable consisting of a copper core of sectional area 2 sq.mm. which is surrounded by a rubber insulation 5 mm. thick, and encased on the outside by a lead covering? (Dielectric constant for rubber=3.) (O. S.)

21. A condenser of capacity 1 microfarad provided with terminals A and B is connected in series with a condenser of capacity 4 microfarads provided with terminals C and D. Terminals B and C are connected together and A and D are connected to a 1000-volt battery. The condensers are now disconnected from the battery and from one another, and connection is made between the terminals A and C, and between B and D. Discuss the storage of energy in the condensers and calculate and explain the loss in electrical energy when the condensers are connected in the second way. (C. S.)

22. Give an account of the attracted disc electrometer, pointing out the advantages which it may have over other forms of electrometer.

Calculate the force of attraction between the lower and upper discs when a potential of 1000 volts is applied between them, given that they are 0·5 cm. apart and of area 10 sq.cm. (C. S.)

23. The moving plate of an attracted disc electrometer is of radius $r=10$ cm.; it is separated from the fixed plate by an air space $d=1$ mm. Find the voltage V necessary to produce an attraction $F=11{\cdot}49$ g. wt.

What is the maximum possible error in V if r can be measured to 0·02 mm., d to 0·005 mm., and F to 0·002 g.? Examine whether this error is increased by working with an air space of 2 mm., and find the air space that would give the least error in measuring this voltage V. (Modified O. S.)

24. Explain how you would calibrate a gold-leaf electroscope so thati s potential could be deduced from the deflection of the leaves.

The electroscope, when calibrated, is charged and the deflection noted. An uncharged insulated metal sphere of known radius is then connected to the cap and the deflection again observed. Show how the capacity of the electroscope can be deduced from these observations. How would you use the electroscope to compare the capacities of two Leyden jars? (B.)

25. A condenser consists of two parallel plates of 10 cm. radius separated by an air gap of 1 mm. Calculate the work done in separating the plates to a distance of 1 cm. when (*a*) the plates are connected to a 100-volt battery, (*b*) the plates are charged to a potential of 100 volts and the plates are insulated before being separated. (1 e.s.u. of potential = 300 volts.) (C. S.)

26. Prove, by the principle of virtual work, that there is an outward pressure $2\pi\sigma^2$ acting on the surface of a conducting sphere whose density of electrification is σ.

The pressure of the air inside an electrically charged soap bubble, of radius 2 cm., is the same as that outside. If the surface tension of the soap solution is 25 dynes per cm., what is the electric potential of the bubble?

27. The leaf of an electroscope of capacity 10 e.s.u. is connected to one of two equal parallel metal discs of 10 cm. radius and 2 cm. apart, the case being connected to the other. The case is then earthed while the leaf is charged to a high potential. On ionising the air between the plates by means of X-rays the potential of the leaf is found to fall at the rate of 100 volts per minute. Find the ionisation current in amperes. (Neglect edge effects when calculating the capacity of the parallel-plate condenser.) (N.)

28. A condenser of capacity 10 microfarads is charged so that the potential difference between its plates is 100 volts. It is then discharged through a resistance of 5×10^6 ohms.

Find corresponding values of the remaining charge q and current flowing i, plot a graph of q (x axis) and $1/i$ and hence or otherwise find the time required for the charge on the condenser to fall to 10^{-4} coulombs. (N.)

29. Derive an expression for the electrostatic capacity of two concentric spheres, of radii a and b respectively, separated by a medium of dielectric constant k, when the outer sphere is earthed. Hence find the capacity (neglecting 'edge effects') of two parallel plane plates, each of area A, when they are separated by a medium of dielectric constant k and thickness t.

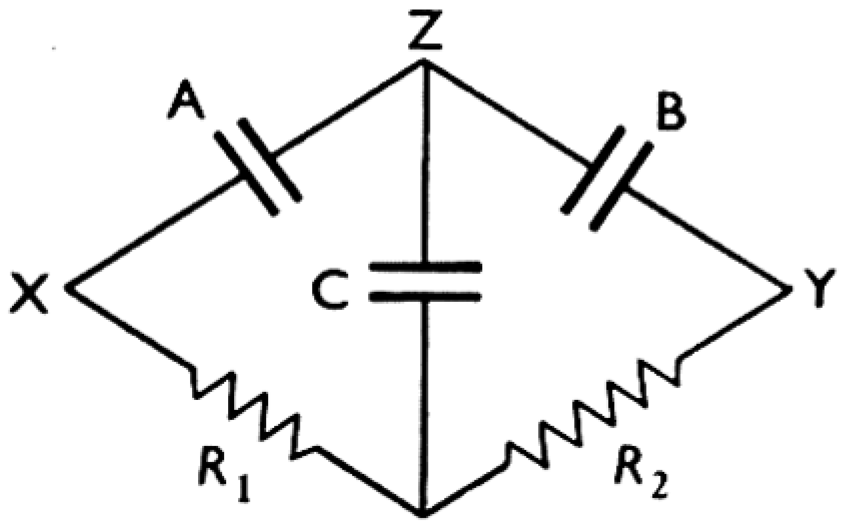


Fig. 32

In Fig. 32 the three condensers are initially uncharged. The point Y is then raised to a positive potential of 4 volts with respect to X. Calculate the potential of Z with respect to X when a steady state has once more been reached. What are the final charges on the three condensers? $A = 1$ microfarad, $B = 2$ microfarads, $C = 3$ microfarads, $R_1 = 1$ ohm, $R_2 = 3$ ohms. (C. S.)

30. Two condensers, each consisting of a pair of plates, area A sq.cm., separated by d cm., are connected in series. One condenser is air spaced, the other is filled with a sheet of material of dielectric constant ϵ. Find an expression for the capacity of the series combination.

The condensers, so combined, are connected to a battery of potential difference V volts, by means of copper wires. The sheet of dielectric filling the second condenser is withdrawn at a uniform rate, the process of withdrawal occupying a time of t seconds. What current flows through the copper wires in the process? What would be the current if a *very large* resistance, R ohms, were included in series with one of the connecting wires? (One e.s.u. of capacity may be taken as 10^{-12} farad.) (C. S.)

CHAPTER 3

FURTHER ELECTROSTATICS

ELECTROSTATIC MEASUREMENTS

The quadrant electrometer

An instrument for measuring potential difference, more sensitive and accurate than the gold-leaf electroscope, known as the quadrant electrometer, was designed by Lord Kelvin. In its modern form (Figs. 33 *a* and 33 *b*) it consists of four brass quadrants on insulating pillars in the middle of which a light paddle or needle, made of paper coated with aluminium, is suspended from a fine quartz fibre.

Opposite quadrants are joined by wires, and the two pairs are connected to the points whose P.D. v is to be determined. The paddle is maintained at a comparatively high voltage V (say at 100 volts if the P.D. to be measured is of the order of 2 or 4 volts), through the suspending fibre which is coated with some conducting material.

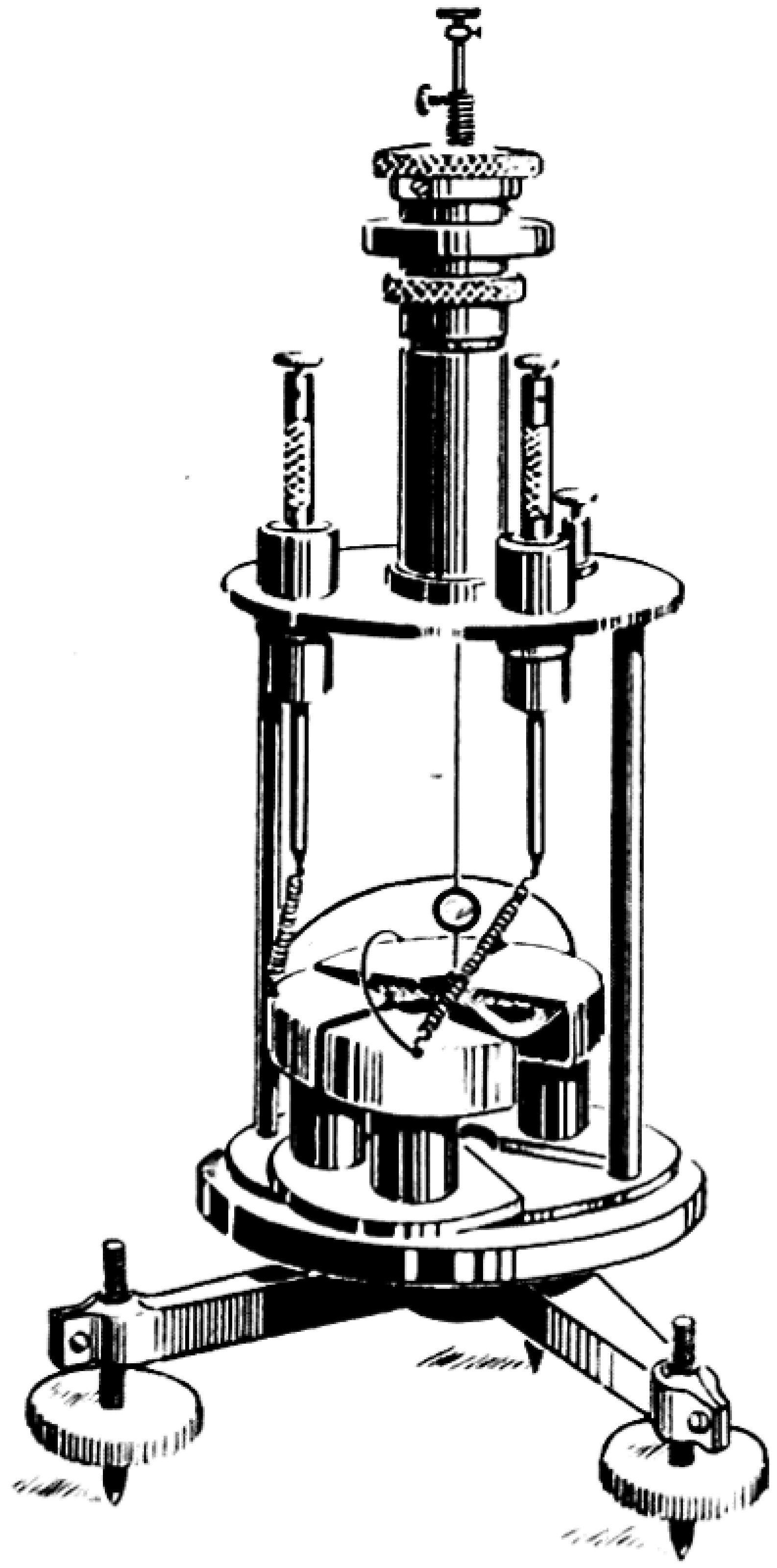


Fig. 33*a*. Quadrant electrometer with quadrants opened to show the interior.

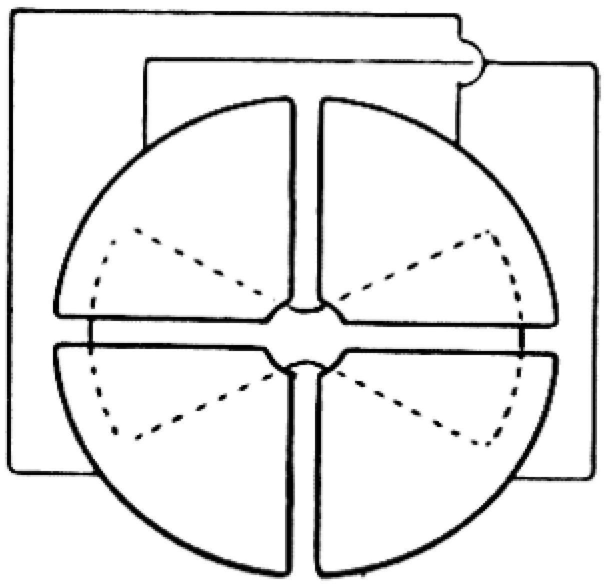


Fig. 33*b*. Quadrants and paddle.

If the paddle is at a positive potential it will move from the quadrants at the higher potential to those at the lower potential, as a ball rolls downhill, until the twist of the supporting fibre produces a sufficiently large opposing couple. The deflection θ is measured by means of a beam of light reflected from a mirror attached to the suspension, on to a scale. It can be shown that $\theta \propto v$ so long as V is large compared with v.

For measuring alternating potentials, the paddle may be connected to one pair of quadrants (instead of maintaining it at a high fixed potential); then $\theta \propto v^2$, and the deflection is in the same direction whatever the direction of the P.D.

Measurement of a very small current by a quadrant electrometer

Considerable use has been made of the quadrant electrometer in measuring small electric currents through gases, known as ionisation currents. Suppose the air between the two plates A and B in Fig. 34 is irradiated with X-rays. Positively and negatively charged molecules of air, called ions, are formed by the detachment of electrons from some of the air molecules and by the attachment of electrons to other molecules. The positive ions will move in the direction of the electric field set up by the battery, from A to B, while the negative ions will move in the opposite direction. Thus one pair of quadrants of the electrometer will receive a gradually increasing positive charge.

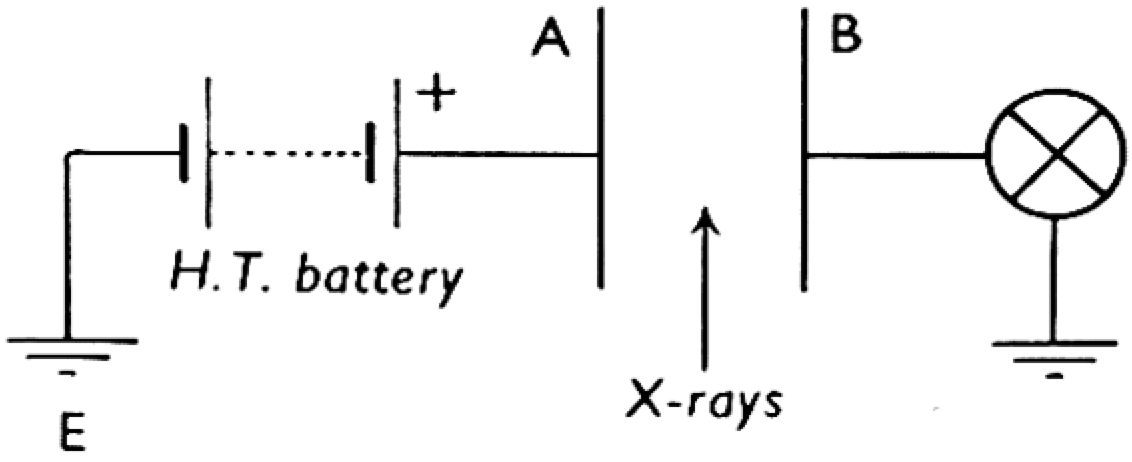


Fig. 34

Suppose that a charge Q given to the quadrants raises their potential by V and that their capacitance is C. Then

$$Q = CV.$$

The deflection, θ, of the paddle, if the latter is at a high potential with respect to the quadrants, is proportional to V.

$$\therefore \ \theta = aV, \quad \text{where } a \text{ is a constant.}$$

$$\therefore \ Q = CV = \frac{C}{a}\theta.$$

The current, i, flowing into the electrometer, is given by

$$i = \frac{dQ}{dt} = \frac{C}{a}\frac{d\theta}{dt}.$$

Thus the current is proportional to the rate of change of the deflection of the paddle; knowing C and a its value can be obtained.

The electrostatic voltmeter

The electrostatic voltmeter shown in Fig. 35 works on the same principle as the quadrant electrometer, except that one pair of quadrants only is provided and the difference of potential is applied between the needle and the quadrants. The controlling force is provided by an adjustable weight at the bottom of the lower vane. The instrument can be used for measuring alternating potential differences.

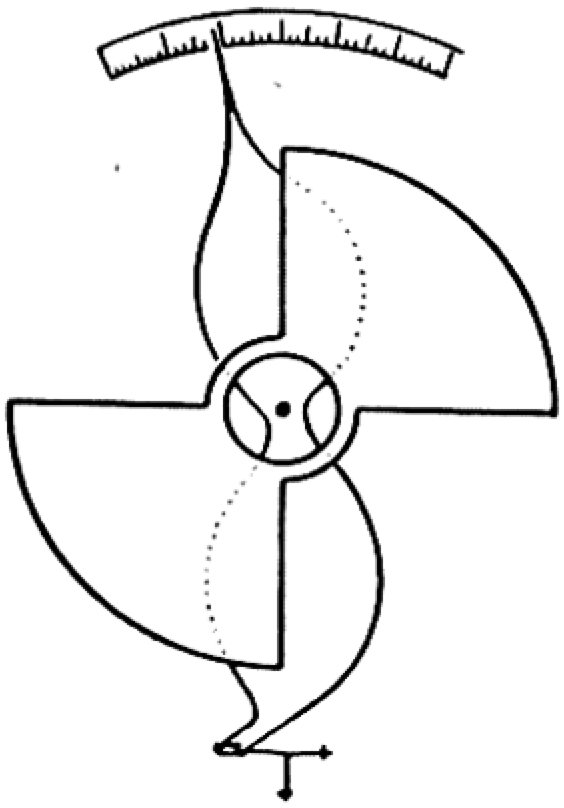


Fig. 35

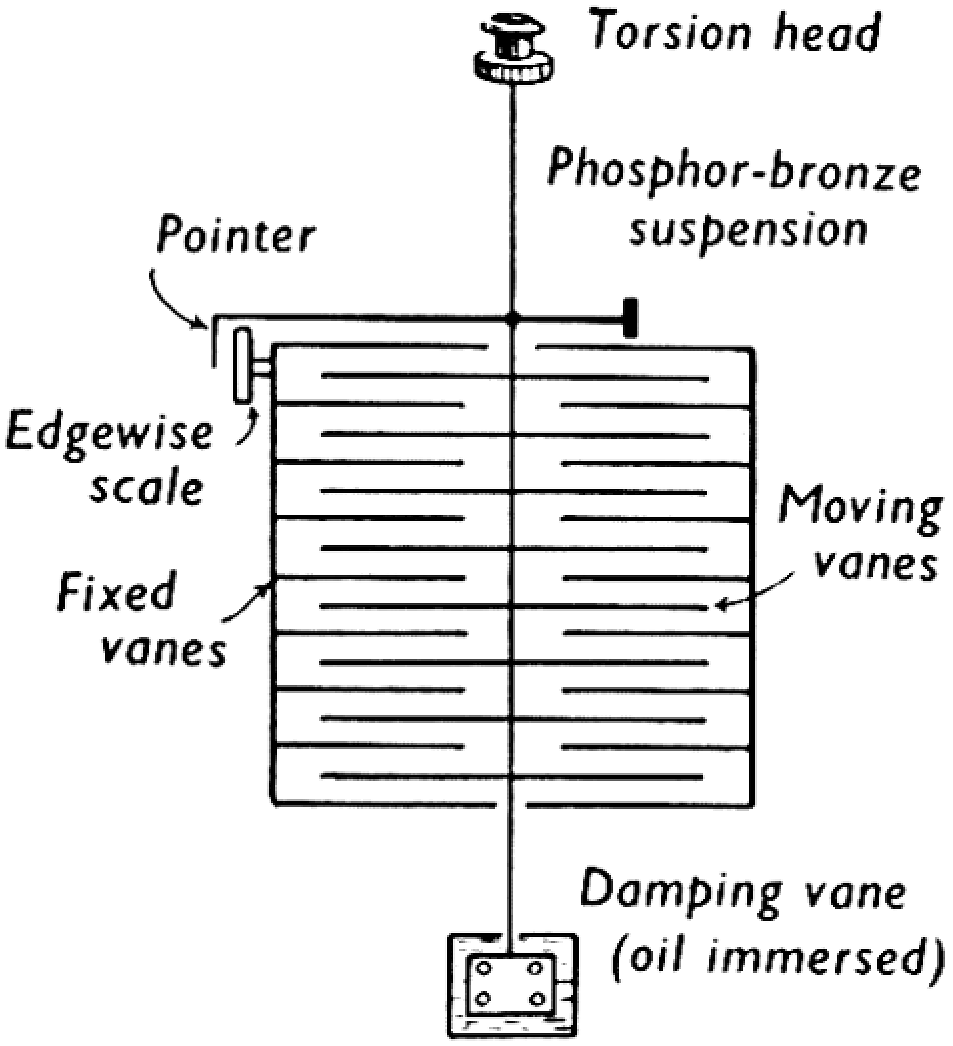


Fig. 36

A more sensitive type, with a number of vanes rigidly attached together, moving in an equal number of pairs of quadrants, is shown in Fig. 36.

Comparison of capacitances

To compare the capacitances C_1 and C_2 of two condensers, the former is given a charge, Q, and the potential difference, V, between its plates is measured with a quadrant electrometer. The charge is then shared with the other condenser and the common potential difference V' measured,

$$Q=C_1V=(C_1+C_2)V'.$$

$$\therefore \frac{C_1}{C_2}=\frac{V'}{V-V'}.$$

The circuit is shown in Fig. 37. The switch S_1 is closed and the deflection of the electrometer found; then S_1 is opened and S_2 closed and the deflection again determined. Great care is needed with the insulation of S_1 and S_2.

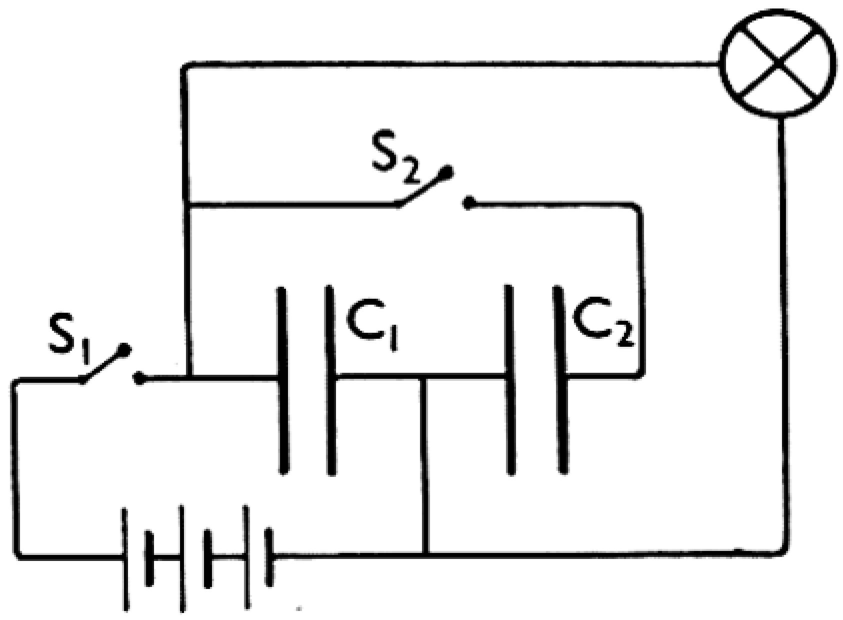


Fig. 37

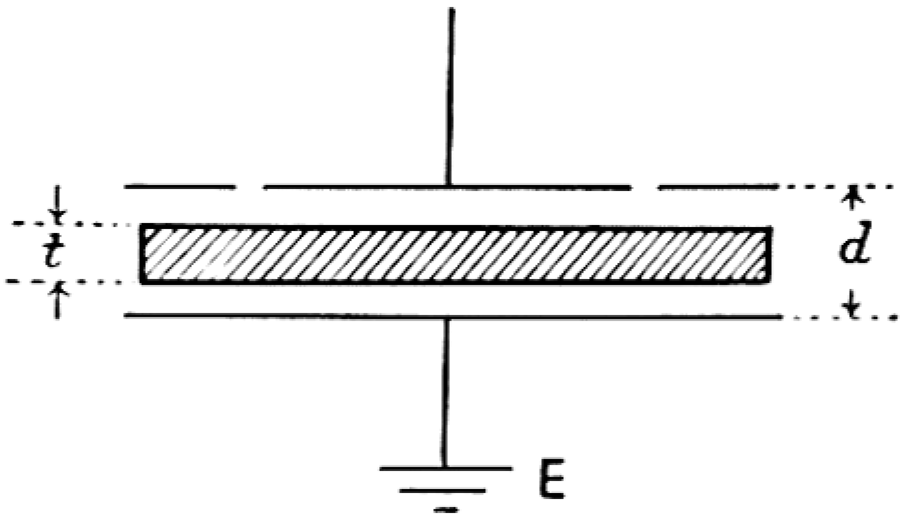


Fig. 38

Measurement of dielectric constant

The following method of measuring dielectric constant is due to Boltzmann.

A slab of the dielectric of uniform thickness is placed between the parallel plates of an air condenser equipped with a guard ring (Fig. 38). The condenser is connected to an electrometer and charged. The slab of dielectric is now removed and the earthed plate moved nearer to the insulated plate by means of a micrometer screw until the deflection of the electrometer is the same as before.

It can be proved by the method described on p. 25 that the capacitance of the condenser, with the slab of dielectric between the plates, is

$$C=\frac{A}{4\pi(d-t+t/k)},$$

where A is the area of the insulated plate, d the separation between the plates, t the thickness of the dielectric and k the dielectric constant. The student should verify this.

Suppose that, after removal of the slab, the distance between the plates must be changed to d' for the electrometer to read the same as before, i.e. for the capacitance of the condenser to be the same,

$$C=\frac{A}{4\pi d'},$$

$$\therefore\ d'=d-t+t/k.$$

$$\therefore\ k=\frac{t}{t-(d-d')}.$$

The only measurements required are the two distances, t and $(d-d')$.

Comparison of capacitances using a ballistic galvanometer

One of the condensers C_1 is charged by a battery (Fig. 39) to a potential V, and then connected to a ballistic galvanometer whose throw, θ_1, is proportional to the charge, Q_1, of the condenser,

$$Q_1 = C_1 V.$$

The experiment is repeated, replacing the first condenser by the second. Using the same nomenclature,

$$Q_2 = C_2 V.$$

$$\therefore \frac{C_1}{C_2} = \frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}.$$

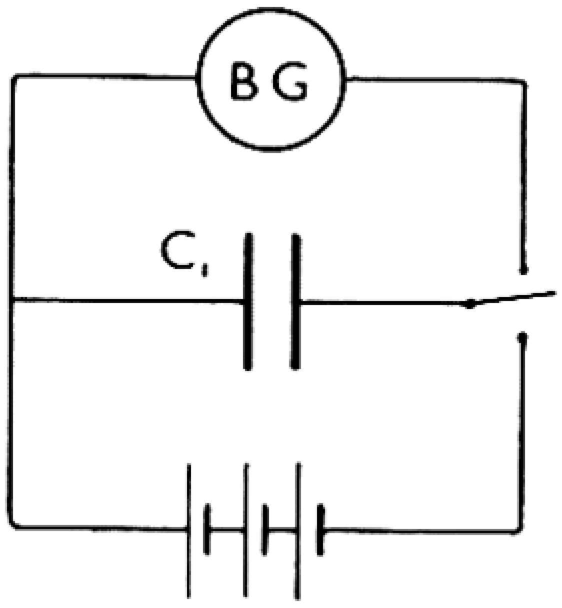


Fig. 39

ELECTROSTATIC MACHINES

The earliest machines for generating electric charges were friction machines. Otto von Guericke cast a globe of sulphur and caused it to rotate while rubbing it with a dry hand. Later the bare hand was covered with a leather glove, and then replaced by a pad.

Modern machines generate charges by induction instead of by friction.

The electrophorus

The electrophorus, invented by Volta, consists of a disc of some insulator such as ebonite which, on rubbing with fur or flannel, acquires a negative charge (see Fig. 40). A brass disc with an insulating handle, called a cover, is placed on the charged ebonite and touched to earth. It becomes charged positively by induction. On raising the cover away from the influence of the negative charge it acquires a considerable

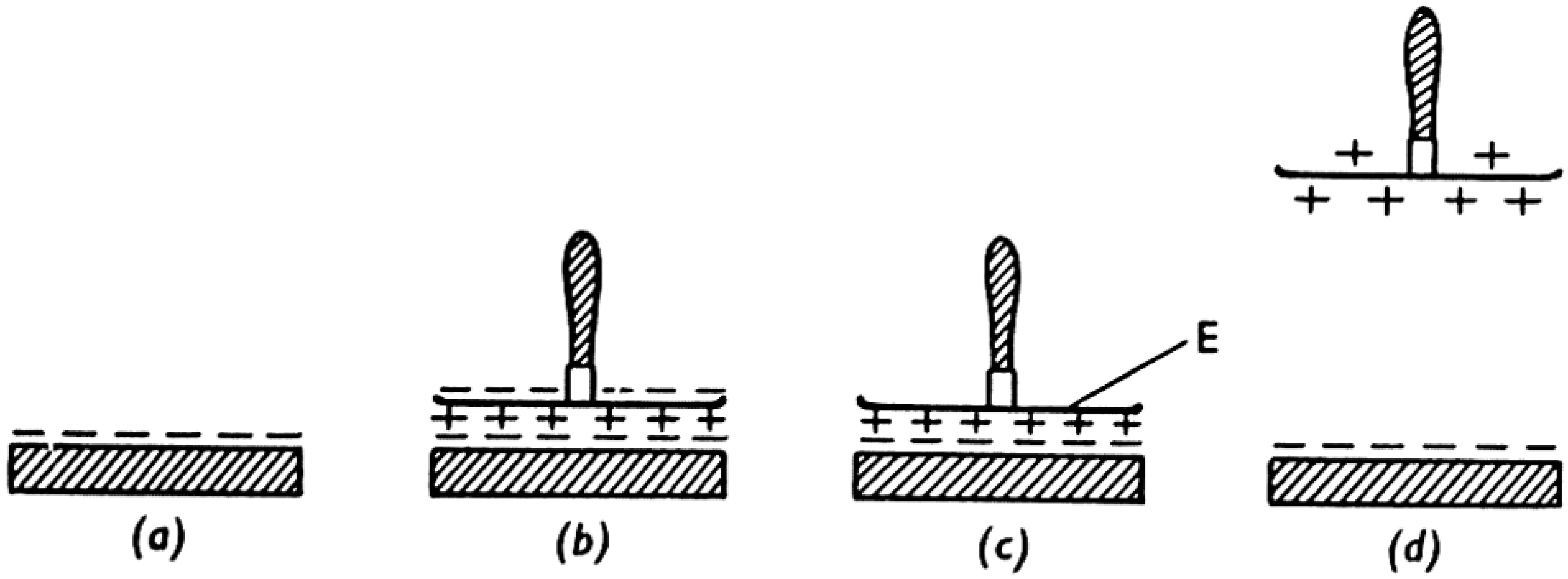


Fig. 40

positive potential and will provide a spark. The energy of the spark is derived from the mechanical work done in raising the cover against the attraction of the unlike charges. Numerous charges may be obtained by repeating the process without any further rubbing of the ebonite. The latter is usually mounted on a metal disc called a sole; a positive charge is induced on the sole which tends to prevent the charge on the ebonite from leaking away.

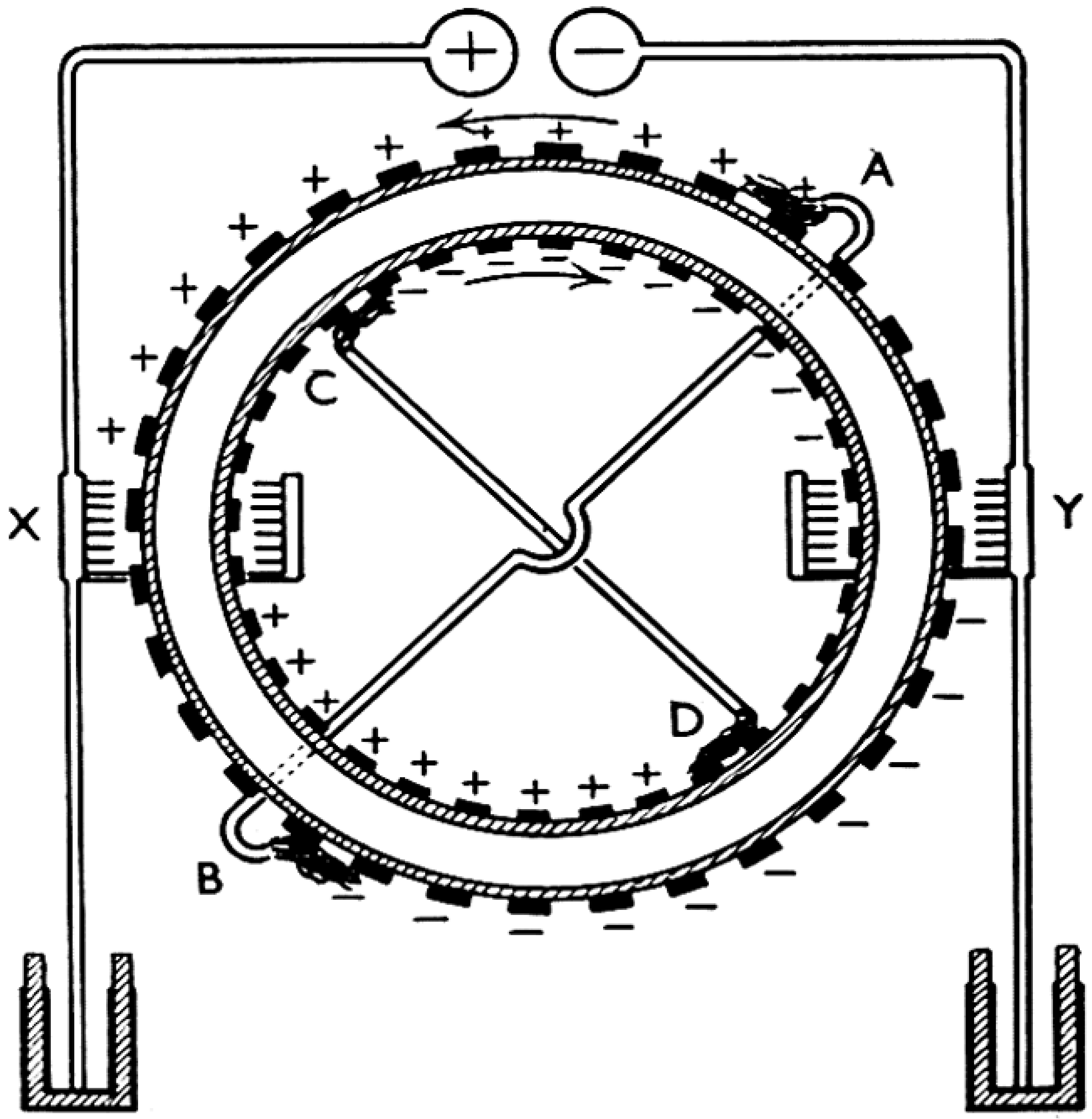


Fig. 41

The Wimshurst machine

The Wimshurst machine consists of two discs of glass coated with shellac or of ebonite, to which are stuck numerous sectors of metal foil (see Fig. 41), and which are geared to rotate in opposite directions. In Fig. 41 these discs are represented by circles and the sectors by arcs.

At the ends of the two metal rods *AB* and *CD* are wire brushes which touch the metal foil sectors as they rotate. *X* and *Y* are collecting combs, attached to the inner coatings of Leyden jars, and also to metal knobs between which sparks pass when the machine is working.

Suppose a rubbed ebonite rod is held near to *A*; it induces a positive charge on the foil sector in contact with *A* and a negative charge on the sector in contact with *B*. These charges are separated as the disc rotates, and carried away by the metal foil sectors. When the positively charged sector from A is opposite C it induces a negative charge on the sector at C and a positive charge on the sector at D. These also are separated by the rotation of the disc, and as a result of a continuous series of processes of this kind all sectors approaching X are positively charged and all approaching Y are negatively charged.

The sectors passing X induce negative charges on the points of the comb and positive charges on the knob and Leyden jar connected to the comb. The negative charges on the points stream on to the sectors as an electric wind (see p. 14) and neutralise their charges. A similar process occurs at Y.

If the Leyden jars are disconnected from the knobs a stream of feeble sparks passes; when the jars are connected, fewer but more vigorous sparks pass, since the jars store up the charge until the potential difference is sufficient for a spark to pass.

The energy of the sparks is provided by the work done in revolving the discs against the attraction of the unlike charges.

The identity of static and current electricity

The static electricity produced by rubbing or by a Wimshurst machine is identical in nature with the current electricity produced by an electric cell or dynamo. The apparent difference in their effects is due to the fact that static electricity is usually at a very high potential but small in quantity, whereas current electricity is usually generated at a low potential difference but in comparatively large quantity. The E.M.F. of a simple cell is 1 volt, whereas the P.D. required to produce a spark 1 cm. long between the knobs of a Wimshurst machine is of the order of 30,000 volts. The shock from a Wimshurst machine is not fatal because the quantity of electricity it generates is small; a shock from a dynamo or cells at a P.D. higher than 250 volts is extremely dangerous, since the quantity of electricity passing depends almost solely on the resistance of the human body; also the supply is continuous.

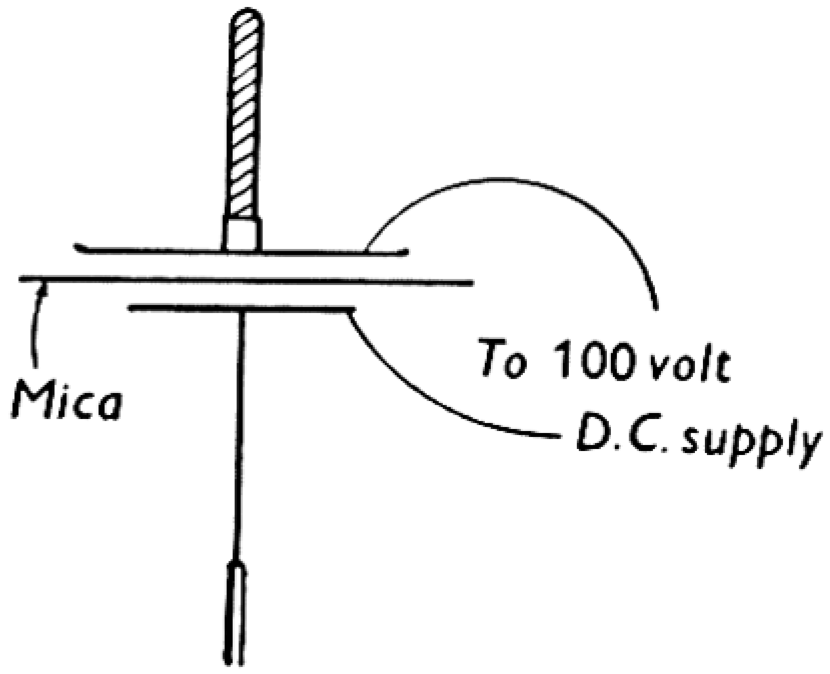


Fig. 42

A battery or direct current generator giving a P.D. of a few volts will cause a divergence of the leaf of an electroscope if the latter is combined with a condenser (see Fig. 42). The P.D. is applied between the top of the electroscope and a metal disc with an insulating handle, the two being separated by a thin sheet of mica or waxed paper. When the leads to the generator are disconnected and the disc is then raised, the leaf of the electroscope will diverge. The disc is oppositely charged to the electroscope, and when it is removed the potential of the electroscope rises considerably.

ATMOSPHERIC ELECTRICITY

Lightning

The similarity between an electric spark and a lightning flash was noticed as soon as electrostatic generating machines were invented.

Fig. 43. Franklin's kite experiment. Being also an American statesman and one of the signatories to the Declaration of Independence, Franklin was said to have 'snatched the lightning from heaven and the sceptre from tyrants'.

In 1752 Benjamin Franklin demonstrated, by utilising the action of points, that thunderclouds are charged with electricity (Fig. 43). Suppose a pointed conductor is held near to a negatively charged ebonite rod (Fig. 44). A positive charge is induced on the point and streams away as an electric wind which tends to neutralise the charge on the ebonite, while a negative charge is induced on the lower end of the pointed conductor.

Franklin attached a pointed conductor to a kite which he flew in a thunderstorm, and when the string became wet and a conductor, he was able to draw a spark from a key attached to the lower end of the string. The psychological effect of this famous experiment was much greater than its theoretical importance; it gave to a capricious and terrifying phenomenon a material explanation and related it to scientific law. Moreover, Franklin showed how the striking of buildings by a lightning discharge, previously ascribed to divine wrath, could be circumvented. A pointed conductor, efficiently earthed and attached to the highest point of a building, gives off an electric wind tending to neutralise a charged cloud above it; if, despite this action, the discharge still occurs, the conductor conveys it safely to earth.

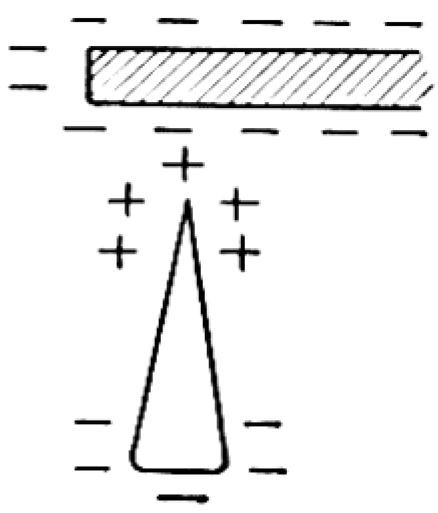


Fig. 44

The charge on a thundercloud

The distribution of the charge on a thundercloud was, until recently, deduced from observations made at the earth's surface. Between 1934 and 1936 Sir George Simpson and his co-workers at Kew Observatory sent up sounding balloons (Fig. 45) utilising, like Franklin, the action of points; the main object was to ascertain which part of a thundercloud is charged positively and which part negatively.

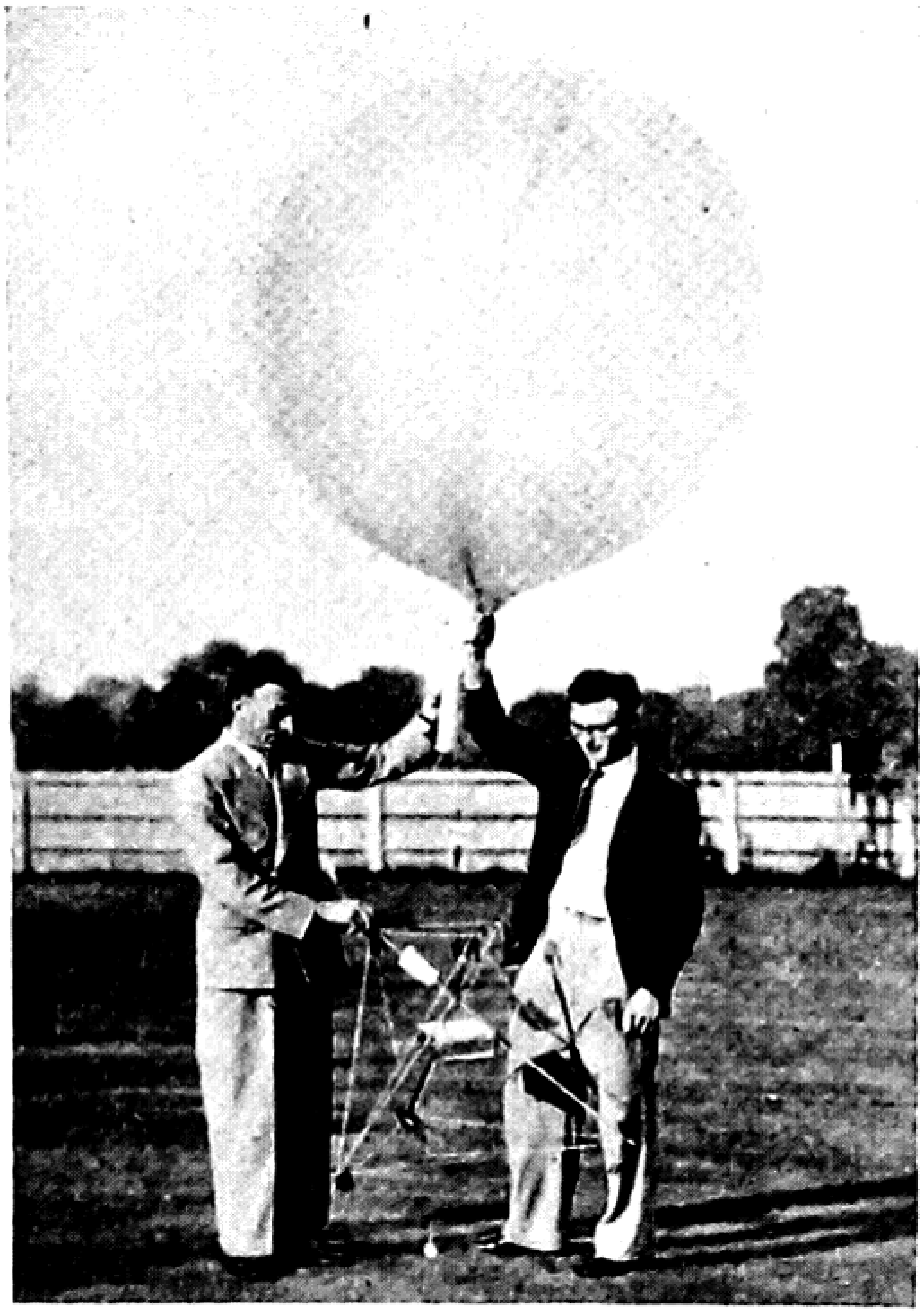


Fig. 45. Balloon and recording apparatus ready for launching.

Below the balloon were attached two copper wires *AB* and *CD* (Fig. 46) of total length 20 metres, making electrical contact with each other through a piece of pole-finding paper *BC* impregnated with starch and potassium iodide solution.

Suppose that the balloon was flying between two clouds, the upper one being charged positively and the lower one negatively. A negative electric wind streamed from the point *A* and a positive electric wind from *D* causing a positive current to flow through the wire from *A* to *D*.

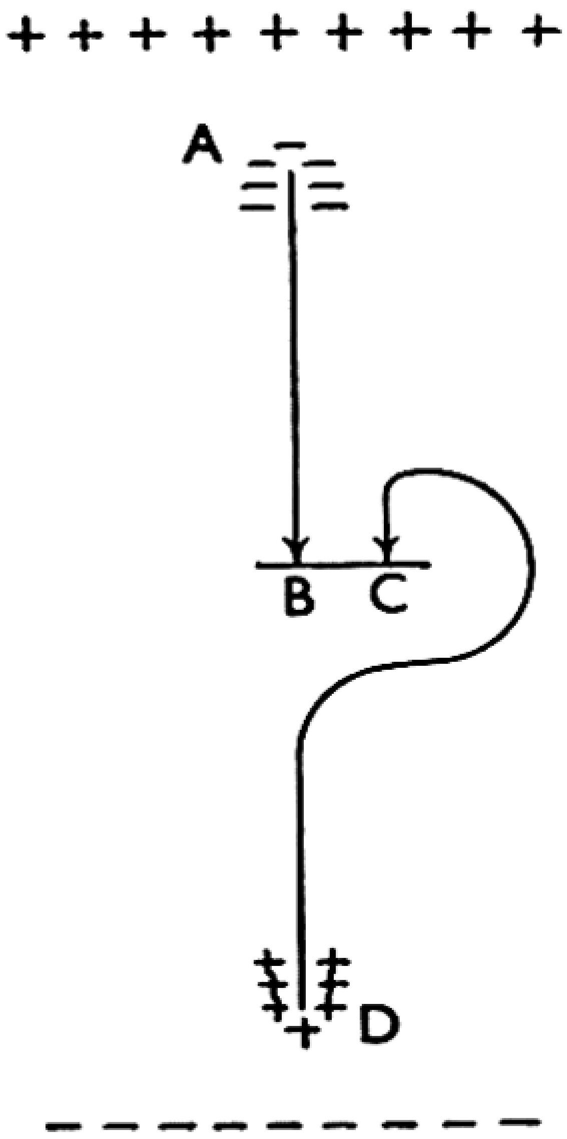


Fig. 46

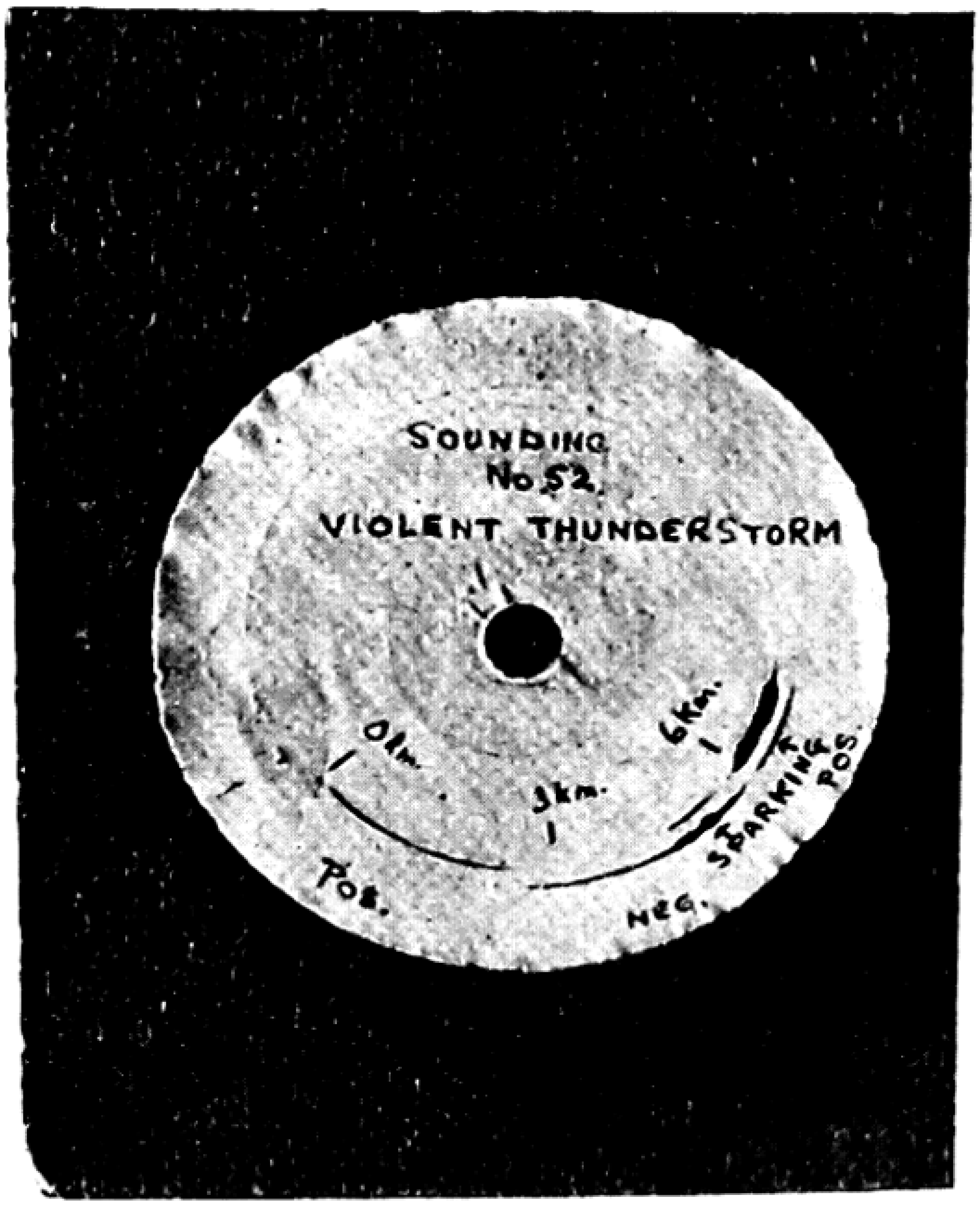


Fig. 47. Record taken during violent thunderstorm

Electrolysis took place in the solution impregnating the pole-finding paper, and at the positive electrode *B* a brown stain appeared. If the potential gradient in the cloud was in the opposite direction the brown stain appeared at *C* instead of *B*.

The pole-finding paper rested on a small disc which was made to rotate slowly by clockwork, and the whole was carried in a bamboo framework. A typical record is shown in Fig. 47. Inside the framework there was also a self-recording aneroid barometer to register the height, and a hair hygrometer to register the relative humidity.

At a predetermined height the expanding aneroid box released a catch causing the bamboo framework with its contained instruments to become detached from the balloon and to float gently to earth under a parachute.

Fig. 48 represents the distribution of charge in a typical thundercloud obtained from the results of these experiments. The continuous lines with arrow heads represent streams of air, their distance apart being proportional to the wind velocity. Thunderstorms are always associated with vertical convection currents of air and occur in this country most frequently in the early summer months when the ground becomes hot while the upper atmosphere is still cool.

It will be seen from Fig. 48 that the lower part and main body of the cloud is charged negatively and the upper part positively. There is a local concentration of positive charge at the base of the cloud just in front of the region of heaviest rainfall. This heavy rain is positively charged and the succeeding rain is negatively charged.

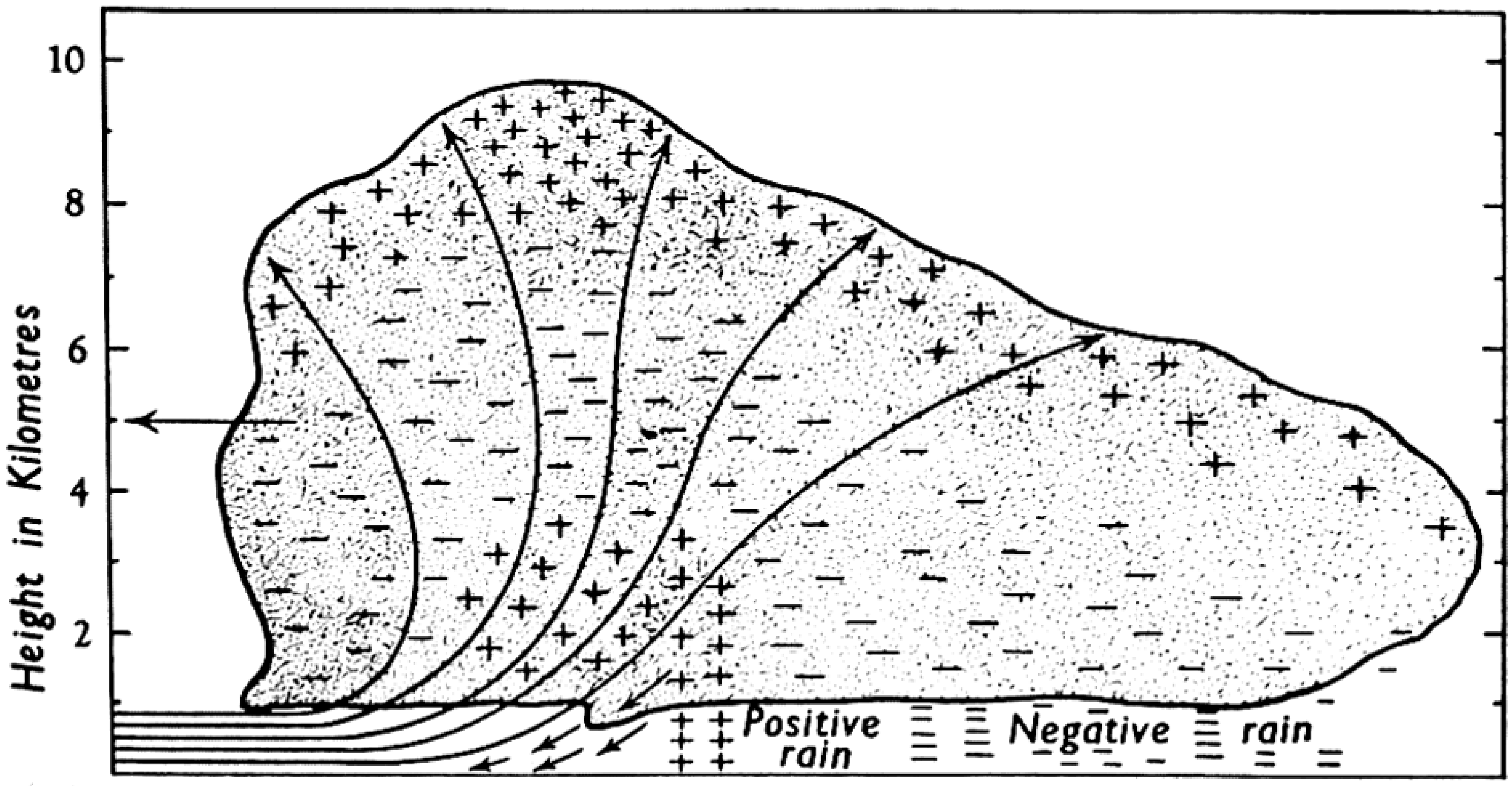


Fig. 48

There are two theories to account for the separation of the electric charges in a thundercloud, one due to Simpson and the other to C. T. R. Wilson.

Simpson bases his theory on the experimental fact that when drops of water are broken up in an air current the drops become positively charged and the air negatively. This explains satisfactorily the positively charged heavy rain in Fig. 48 and the negative charge in the middle and lower part of the cloud. As the moist air currents sweep upwards, drops form, break up and then grow again or coalesce. The heavy positively charged drops begin to fall and the negatively charged air current sweeps through the cloud charging it negatively.

How can the positive charge on the top of the cloud be accounted for? C. T. R. Wilson's theory was designed to explain this and also the remarkably short time—less than a minute—required to renew the charges on the top and bottom of a cloud after they have been neutralised by a lightning flash. In a thundercloud the air is ionised, that is, the molecules are positively and negatively charged, particularly just after the passage of a discharge. Assuming that there is some positive charge on the top of the cloud and some negative charge on the bottom, a falling drop will have an induced negative charge on its upper side and an induced positive charge on its lower side. As it falls its lower positive side will attract the negative ions through which it passes but repel the positive ions (see Fig. 49) and thus will leave the positive ions at the top of the cloud and carry the negative ions to the bottom.

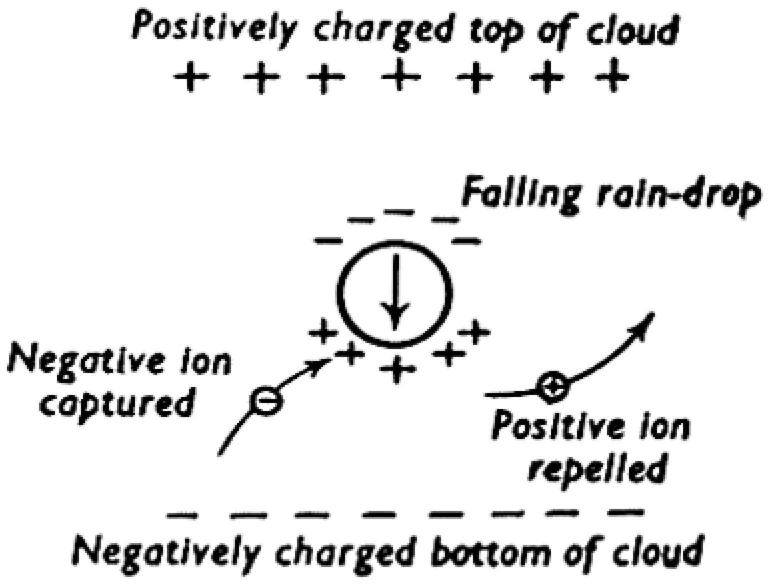

Positively charged top of cloud
Falling rain-drop
Negative ion captured
Positive ion repelled
Negatively charged bottom of cloud

Fig. 49

Simpson criticises this theory on the grounds that the top of a thunder-cloud is at a temperature well below freezing-point, usually below −10° C., and hence the condensed moisture there must be in the form of ice crystals. These ice crystals are non-conductors and cannot acquire the induced charges necessary for Wilson's theory. He suggests that the ice crystals become charged by the air currents, as happens during blizzards in polar regions. The air becomes positively charged on its way to the upper part of the cloud, while the ice crystals acquire a negative charge and descend slowly to the bottom of the cloud.

Under the diverse meteorological conditions possible in thunder-clouds it is probable that several charging mechanisms are at work.

The earth's electric field

On a fine day the earth's surface has a negative charge and the lower atmosphere has a positive charge. There is therefore a vertical electric field which varies from about 300 volts per metre at Kew to 126 volts per metre over the oceans and 60 volts per metre at Davos, Switzerland.

If E is the value of the field, dv/dx the potential gradient, and σ the charge per unit area of the earth's surface,

$$E = -\frac{dv}{dx} = 4\pi\sigma \quad \text{(pp. 8 and 5).}$$

Thus E can be found either by determining the potential gradient or by laying an insulated plate over a hole flush with the ground and measuring the charge on it.

Using the first method, a hollow copper sphere about 30 cm. in diameter is mounted on a rod 5 metres long hinged at its lower end (Fig. 50). When the rod is moved from a horizontal to a vertical position the copper sphere acquires an induced negative charge, $-Q$, sufficient to neutralise the potential of the earth's field, V. The copper sphere is insulated from the rod but attached by a wire to a delicate charge-measuring instrument, called a capillary electrometer, which registers the induced positive charge Q, passing to earth. Then if r is the radius or capacitance of the sphere, $V=Q/r$. When the rod is moved back from the vertical to the horizontal position the charge $-Q$ passes to earth through the electrometer, and the first reading may be checked.

It is of interest to note that when a man stands erect there is a small induced negative charge on the top of his head; otherwise his head would be at a potential several hundred volts higher than his feet.

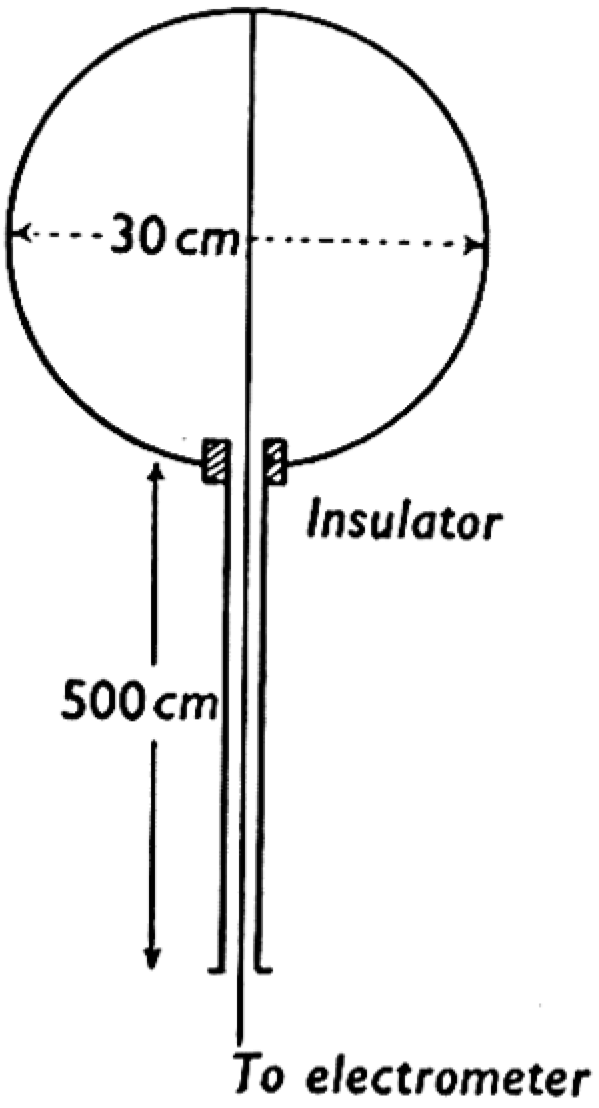


Fig. 50

The fine-weather current

The air is not a perfect insulator, but has a slight conductivity due to ionisation by radioactive materials in the earth and by cosmic rays. There is therefore a downward current of 2×10^{-16} ampere per sq.cm.—small, but totalling about 1000 amperes for the whole earth's surface. This current is sufficient to neutralise the charges on the earth and the atmosphere in a few minutes, and the problem arises as to how the charges are maintained.

The earth's electric field varies during the day. The graph in Fig. 51 was not understood until it was realised that a graph of atmospheric pollution has almost exactly the same shape. The maxima at 8 a.m. and 9 p.m. correspond to the times when people stoke up their fires for breakfast and at night. When summer time was introduced the maxima shifted.

Over the ocean there is no atmospheric pollution, and the daily variation is shown in Fig. 52. It is found that the variations over the Atlantic and the Pacific differ when referred to local times, but correspond when referred to a universal time.

Various explanations of the maintenance of the earth's charge have been put forward, but the theory of C. T. R. Wilson, that thunderstorms are responsible, is now almost universally accepted. At any moment there are some 1800 thunderstorms occurring over the earth's surface. Assuming that the quantity of electricity brought to the earth by each flash is 20 coulombs (mainly negative), and that three flashes occur per minute, the current passing is $\frac{1800 \times 20 \times 3}{60} = 1800$ amperes. This is easily sufficient to maintain the earth's charge.

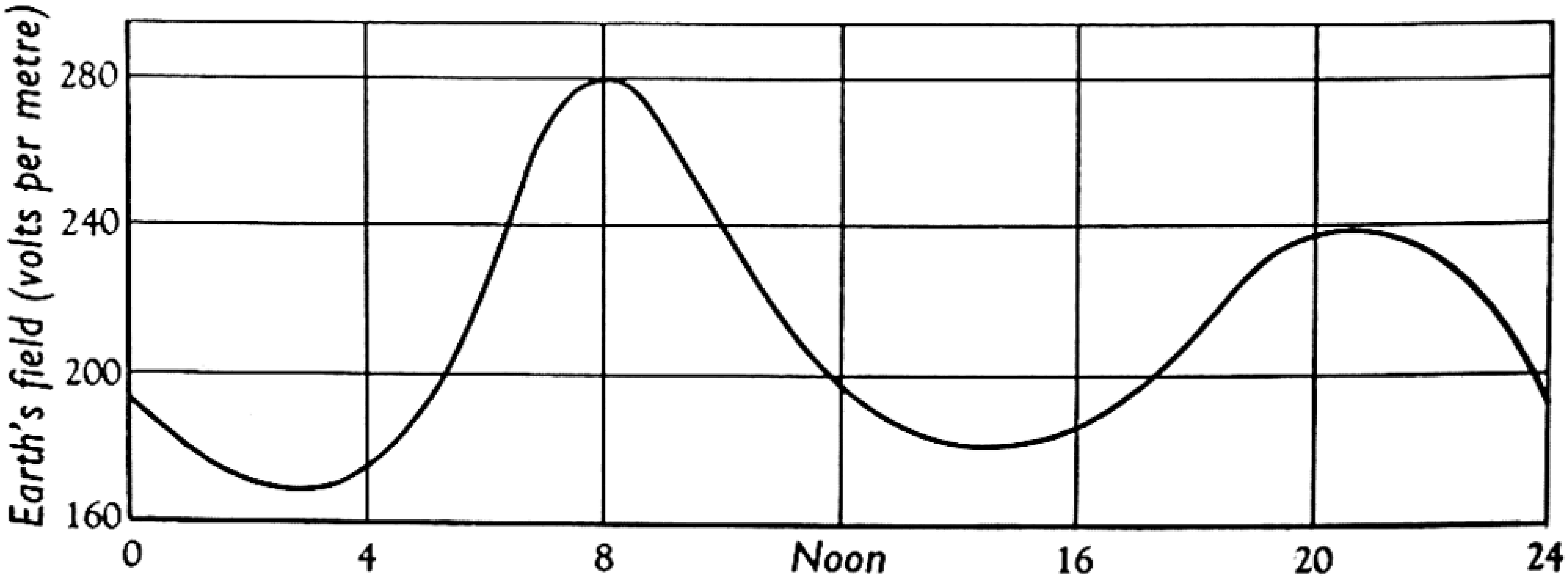


Fig. 51. Hourly variation of the earth's electric field at Kew. The graph summarises a long series of readings.

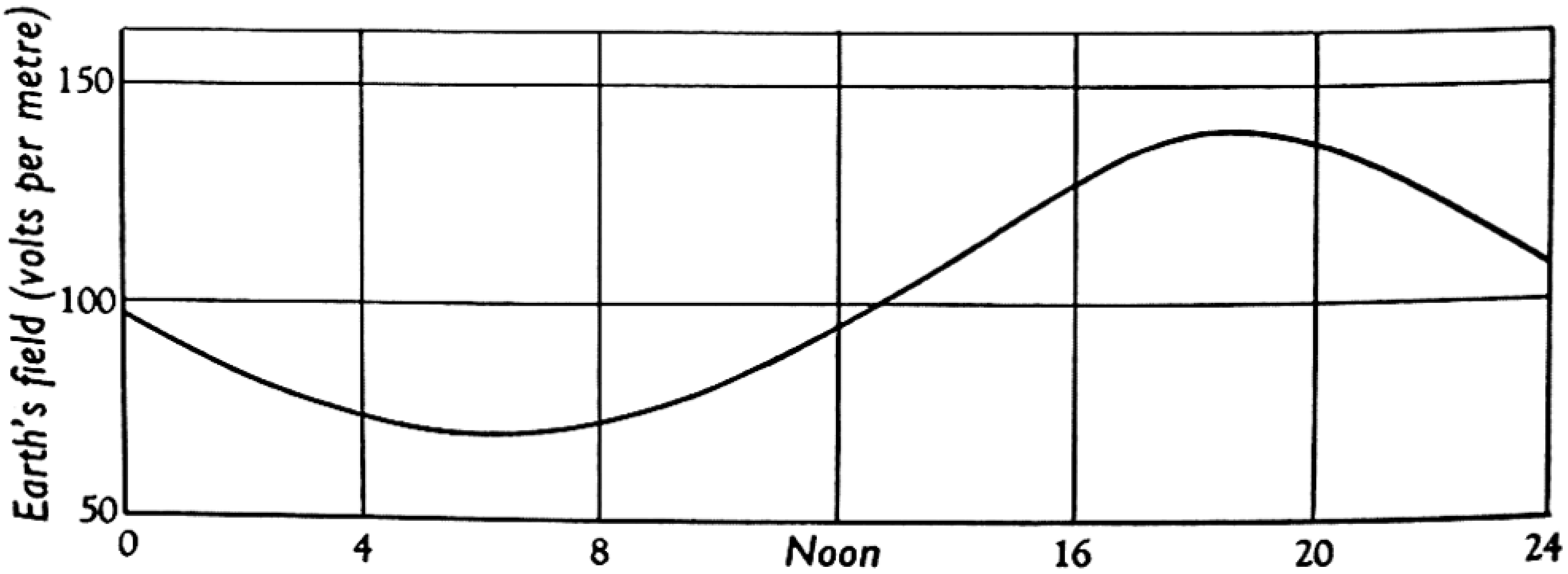


Fig. 52. Universal daily variation of the earth's electric field over the sea.

The daily variation can also be accounted for. There are three very thundery areas in the world—Dutch East India, South Africa and South America. Thunderstorms are most frequent about 4.0 p.m. local time, and in the areas mentioned this corresponds to between about 04.00 and 20.00 hours Greenwich Mean Time, which is the time interval in Fig. 52 during which the earth's negative charge is increasing.

Many lightning flashes occur inside clouds and do not pass to the ground. Thus 1800 amperes is a considerable overestimate. It has been discovered, however, that a great deal of negative charge reaches the earth from a thundercloud, which is charged negatively underneath, by the action of points. Dr Schonland insulated a thorn bush in South Africa and found that the current passing into it was very considerable.

QUESTIONS

1. Describe and explain the action of the quadrant electrometer. How would you use a quadrant electrometer (*a*) to compare the electromotive forces of two cells, (*b*) to measure an electric current? (L.)

2. Describe some form of electrostatic voltmeter, and show how it may be used to measure (*a*) steady, (*b*) alternating, potential differences. (O. & C.)

3. How would you show (*a*) that there are two kinds of electric charge, 'positive' and 'negative', and (*b*) that a charge from a voltaic cell can be made to give a deflection on a gold-leaf electroscope?

Describe how, given an electrometer and accessory apparatus, you would determine the sign and compare the magnitude of the charges on two small insulated bodies. You can assume that the charge on a glass rod rubbed with silk is positive. (O. & C.)

4. The plates of a parallel-plate condenser are 3 cm. apart. They are connected to the quadrants of an electrometer and are given a charge which causes a deflection of 5·0 divisions on the scale. When a slab of dielectric 1 cm. thick is inserted between the plates the deflection falls to 4·5 divisions. Determine the specific inductive capacity of the material. (The needle of the electrometer is kept throughout at a constant high potential.) (C.)

5. A parallel-plate air condenser has its plates 2 cm. apart. If a slab of glass 2 cm. thick is placed between the plates it is found necessary to increase their distance apart by 1·6 cm. in order to restore the capacity to its original value.

Explain this, giving the underlying cause of the effect produced by the dielectric. Calculate the dielectric constant of the glass. What proportion of the total energy of the electric field will be in the glass in the final arrangement of the condenser? (N.)

6. Two condensers *A* and *B* are charged by means of a certain battery when they are connected (*a*) in series, (*b*) in parallel. On discharging them through a ballistic galvanometer the kicks are respectively 4 and 18 divisions. Determine the ratio of their capacities. (C.)

7. Describe an electrophorus and, with the aid of diagrams, explain how it is used to give positive charges to an insulated conductor. Why is the plate of the electrophorus at a high potential when lifted from the base although it has previously been earthed?

Why is it possible to obtain a succession of charges from an electrophorus? What is the source of the energy associated with these charges? (N.)

8. Give an account of the phenomena of electrostatic induction and describe some electrostatic machine in which they are applied.

An electrophorus, the upper plate of which has a capacity of 10 e.s.u., is used to charge a condenser of large capacity across which is a leak of resistance 2 e.s.u. If the plate is at a potential of 10 e.s.u. when removed from the charged surface, and if all its charge is communicated to the condenser, what will be the ultimate average potential of the latter (in e.s.u. and in volts) if the electrophorus cycle is completed in 2 seconds? (O. & C.)

9. Describe the action of some form of rotary electrostatic influence machine. Contrast the output of a machine of this kind with that of a voltaic cell.

A condenser of capacity 0·00015 microfarad is connected across the discharge knobs of a Wimshurst machine. When the potential reaches 40,000 volts the condenser is completely discharged by a spark. If the sparks occur at intervals of 1·75 sec., estimate the power of the machine in watts. (N.)

10. If the spark balls of a Wimshurst machine are adjusted to a discharge at 60,000 volts, and a condenser of 0·01 microfarad is connected across the gap, it is found that a spark passes every 5 seconds. Deduce the average current in milliamperes. It can be assumed that when a spark passes the potential difference is reduced to zero. (O. S.)

11. A quadrant electrometer of 50 cm. capacity is set up for use in the ordinary way (i.e. heterostatically) and one pair of quadrants is given a charge. They are then connected to an uncharged parallel-plate condenser and the deflection of the 'needle' is observed to decrease to one-quarter of its initial value. The distance between the plates of the condenser is then increased by 0·5 cm., and the deflection increases to half its first value. Find the capacity of the condenser and the separation of the plates in the first instance. (N.)

12. Describe briefly the use of a quadrant electrometer.

One end of a very high resistance is maintained at a potential of 10 volts above that of the earth, and the other end is joined to the quadrants A and C of a quadrant electrometer. Initially all four quadrants A, B, C, D are earthed. When A and C are disconnected from the earth the needle is deflected at the rate of 48 divisions per minute. When the experiment is repeated with a condenser of capacity 100 e.s.u. connected between the quadrants A, C and the earth, the rate of deflection is 16 divisions per minute. Calculate the value of the high resistance in ohms. (Sensitivity of electrometer $=100$ divisions per volt. 1 microfarad $=9\times10^5$ e.s.u.) (O. & C.)

13. If there is an electric intensity of 300 volts per metre at the surface of the earth, what is the charge in coulombs per square kilometre on the earth's surface? (O. & C.)

14. Write a brief account of the earth's electric field.

CHAPTER 4

MAGNETISM

THE properties of the lodestone (Fe_3O_4)—its power to attract iron and to set itself north and south—were the starting point of the study of magnetism. The elementary phenomena were described in two books of the same name, *de Magnete*, published respectively in 1269 by Peter Peregrinus and in 1600 by Sir William Gilbert. Gilbert, who spent £5000 on his researches, was a great original genius. Before the work of Bacon and Newton, he began the collection and checking of facts by experiment, and arrived at laws and generalisations by induction from the facts. He was, in fact, a pioneer of the scientific method in an age which had not emerged from medieval credulity about natural phenomena—an age which believed that lodestone in small doses could preserve youth, cure headaches, or if pulverised and embedded in plaster, draw an arrow from the human body, losing its power only when smeared with garlic.

In 1785 Coulomb applied Newtonian mechanics to the phenomena and made magnetism a quantitative science. Magnetic theory is analogous to electrostatic theory—an inverse square law, parallel definitions of magnetic pole strength and electric charge, and of the strength of magnetic and electric fields.

The inverse square law

The magnetism of a magnet is concentrated in regions at its ends called its poles. By using long, ball-ended magnets Coulomb was able to experiment on virtually isolated poles concentrated at points, the centres of the balls. He suspended a magnet from a fine wire, and estimated from the twist of the wire (Fig. 53) the force exerted on one of its poles by a pole of another magnet. In this way he discovered the inverse square law: *the force between two magnetic poles varies inversely as the square of their distance apart.*

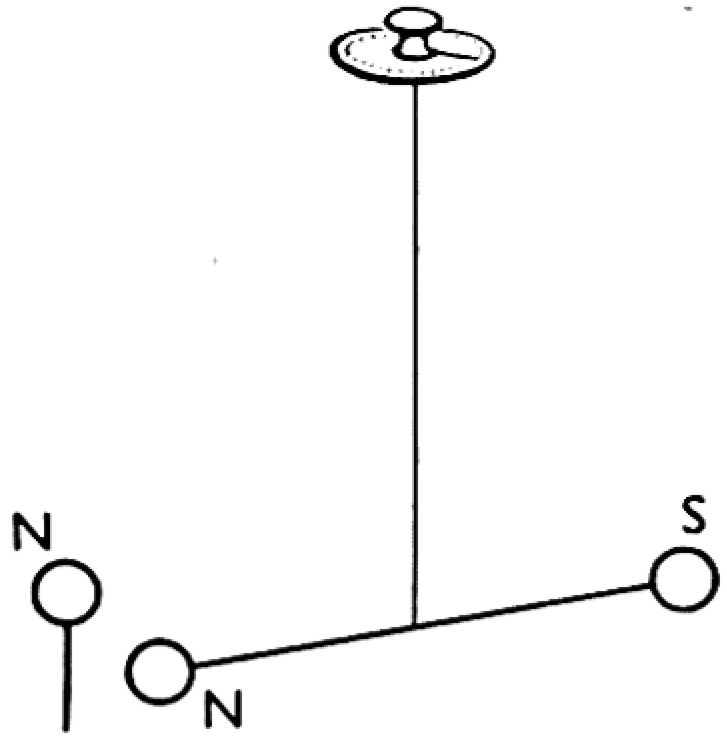


Fig. 53

Definition of unit pole

In 1832 Gauss devised a unit in which the strength of a magnetic pole may be measured. *Two poles are of unit strength if, when placed* 1 *cm. apart in a vacuum, they exert a force on each other of* 1 *dyne.* This force is an attraction if the poles are unlike (i.e. N. and S.) and a repulsion if they are like; it is not appreciably changed if there is air between the poles instead of a vacuum.

If one of the poles is replaced by a pole of double strength, the force is 2 dynes, and if the other pole is also replaced by one of strength 3 c.g.s. units, the force will be $2 \times 3 = 6$ dynes.

Thus the force F dynes, between two poles of strengths m_1 and m_2 c.g.s. units at a distance r cm. apart in a vacuum or in air, is given by

$$\mathbf{F} = \frac{\mathbf{m_1 m_2}}{\mathbf{r^2}}.$$

Definition of magnetic field strength

The space surrounding a magnet in which its influence can be detected is called a magnetic field. *The strength or intensity of a magnetic field at a point is the force it exerts on a unit pole placed at the point.* A field which exerts a force of 1 dyne on a pole of strength 1 c.g.s. unit is said to have a strength of 1 *oersted.*

Thus a field of strength H oersteds exerts on a pole of strength m c.g.s. units, a force of mH dynes.

Magnetic field due to a single pole

The strength of the magnetic field due to a pole of strength m c.g.s. units at a distance r cm. from it is, by definition, the force exerted on a unit pole, and hence is

$$\frac{m \times 1}{r^2} = \frac{m}{r^2} \text{ oersted.}$$

Field strength and lines of magnetic force

A magnetic field of strength 1 oersted is represented by 1 line of magnetic force per sq.cm., and a field of H oersted by H lines per sq.cm.

According to this convention *a unit pole must have* 4π *lines of force,* and a pole of strength m c.g.s. units $4\pi m$ lines of force. Consider a sphere of radius r cm. described round a pole of strength m c.g.s. units as centre. The strength of the magnetic field at all points on the sphere due to the pole is m/r^2 oersted. Since the surface area of the sphere is $4\pi r^2$ sq.cm., the total number of lines of force must be $m/r^2 \times 4\pi r^2 = 4\pi m$.

Magnetic field due to a bar magnet

(*a*) *End-on or Gauss A position.* The magnetism of a bar magnet may be regarded as concentrated at two points near its ends called its poles. The position at which the strength of its magnetic field may be calculated most simply is on its axis produced.

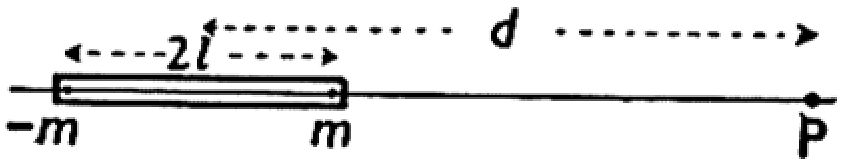


Fig. 54

Consider a point *P* (Fig. 54) at a distance *d* from the centre of a bar magnet whose pole strength is *m* and magnetic length (the distance between the two poles) 2*l*.

Strength of field at *P* due to the magnet

$$= \text{Force exerted by the magnet on a unit pole imagined at } P$$

$$= \frac{m\times 1}{(d-l)^2} - \frac{m\times 1}{(d+l)^2}$$

$$= \frac{m(d^2+2dl+l^2-d^2+2dl-l^2)}{(d-l)^2\,(d+l)^2}$$

$$= \frac{4mdl}{(d^2-l^2)^2} = \frac{2Md}{(d^2-l^2)^2},$$

where $M=2lm=$ the *moment* of the magnet.

The value and significance of this new concept, moment of a magnet, which is the *product of the magnetic length and pole strength*, will be seen on p. 63.

If l is small compared with d, l^2 can be ignored in comparison with d^2. Therefore d^2 can be written instead of d^2-l^2.

Thus if $d=20$ cm. and $l=2$ cm.,

$d^2-l^2=400-4=396, \quad d^2=400.$

Hence for a short bar magnet at a considerable distance along its axis,

$$\text{Strength of magnetic field} = \frac{2M}{d^3}.$$

(*b*) *Broadside or Gauss B position.* Another position at which the strength of its magnetic field may be calculated simply is at a point on the perpendicular bisector of a bar magnet, known as the broadside position.

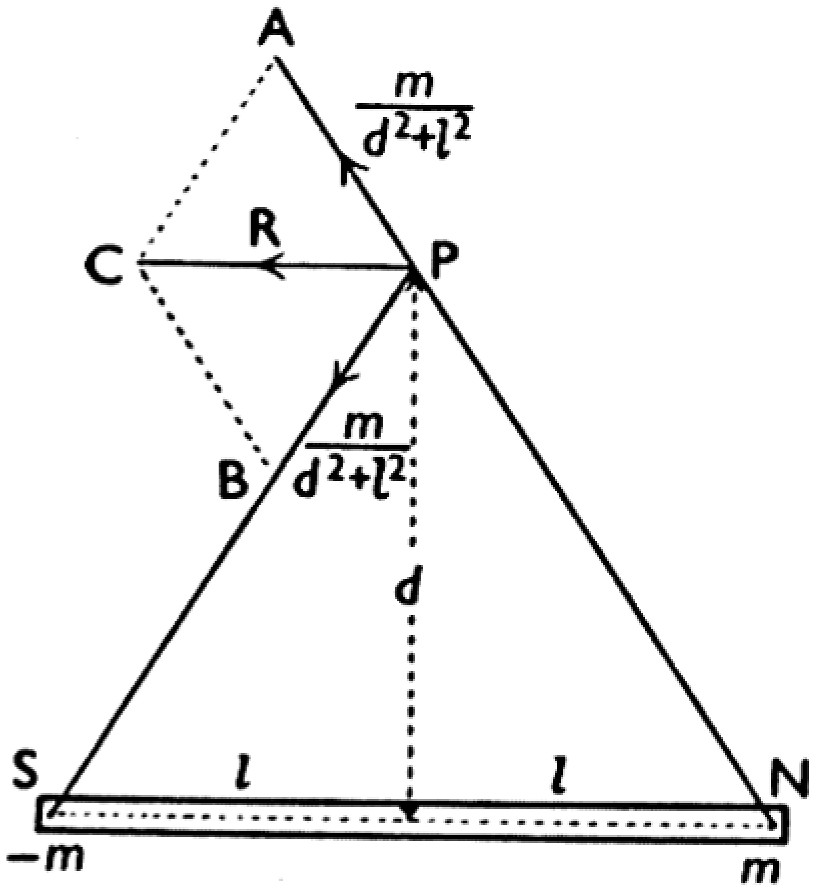


Fig. 55

Consider a point *P* (Fig. 55) distant *d* from the centre of the magnet. The forces on a unit pole at *P* due to the two poles of the magnet are

$m/(d^2+l^2)$ along PA and PB. The resultant may be obtained by the parallelogram of forces. If PA and PB represent the two forces $m/(d^2+l^2)$, then PC represents their resultant, R.

The triangles APC and PSN are similar.

$$\therefore\ \frac{CP}{AP}=\frac{SN}{PN},$$

i.e.

$$\frac{R}{m/(d^2+l^2)}=\frac{2l}{\sqrt{(d^2+l^2)}},$$

$$R=\frac{2lm}{(d^2+l^2)^{\frac{3}{2}}}=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}.$$

$$\therefore\ \text{Strength of field at } P=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}.$$

If l is small compared with d,

$$\text{Strength of field due to a short bar magnet}=\mathbf{\frac{M}{d^3}}.$$

Gauss's proof of the inverse square law

The strength of the magnetic field of a short bar magnet is $2M/d^3$ in the end-on position and M/d^3 in the broadside position, assuming the truth of the inverse square law.

Gauss demonstrated by experiment that the field in the end-on or Gauss A position is double that in the broadside or Gauss B position at equal distances from the centre of a short magnet; he thus proved the truth of the inverse square law. His indirect method was considerably more accurate than the direct method of Coulomb.

Gauss used a deflection magnetometer (described later in this chapter) to compare the two magnetic fields.

Magnetic field at any point due to a bar magnet

In order to find the strength of the field due to a bar magnet at any point P, the parallelogram construction is necessary.

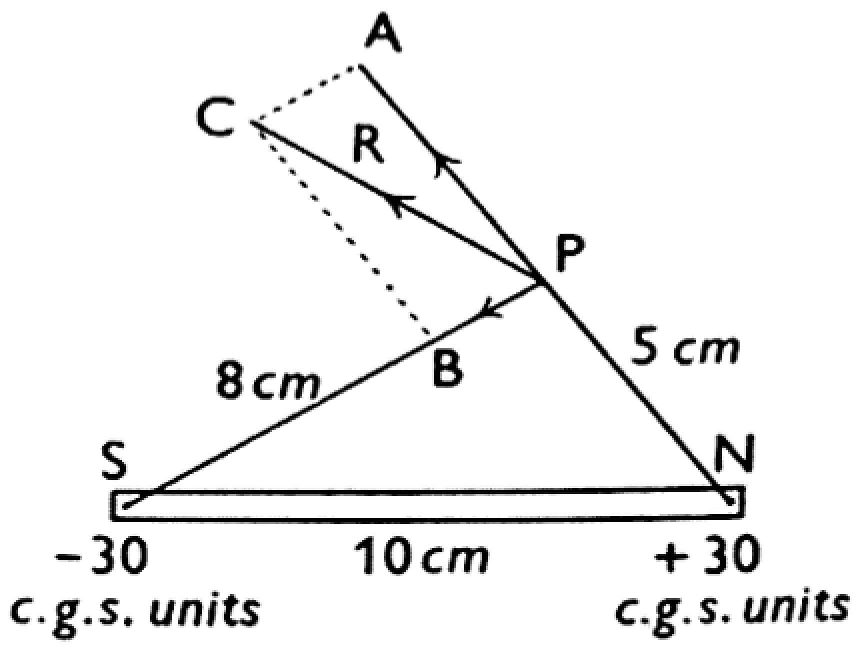


Fig. 56

In Fig. 56 suppose $PN=5$ cm., $PS=8$ cm., $NS=10$ cm., and the pole strength of the magnet is 30 c.g.s. units. To find the strength of the field at P we must find the resultant force exerted by the poles of the magnet on a unit pole imagined at P.

$$\text{Force along } NP = \frac{30 \times 1}{5^2} = 6/5 \text{ dynes.}$$

$$\text{Force along } PS = \frac{30 \times 1}{8^2} = 15/32 \text{ dynes.}$$

Thus the parallelogram $PACB$ must be constructed so that PA and PB are proportional to 6/5 and 15/32 dynes respectively. Then the length of PC will be proportional to the resultant force, and hence to the resultant field at P due to the magnet. The value is 1·35 oersted.

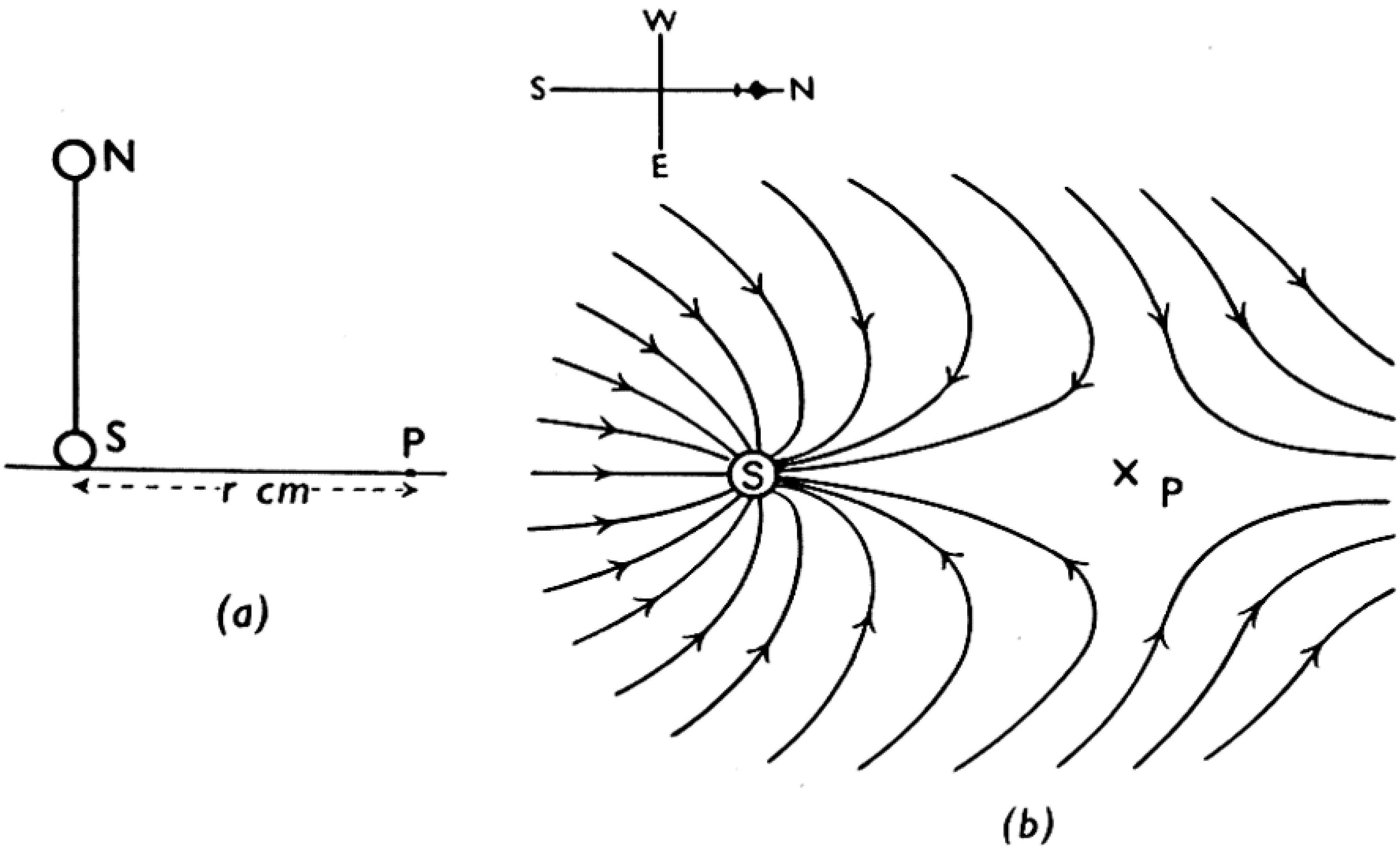


Fig. 57. A vertical magnet with one pole resting on a table. The magnetic field due to the lower pole and the earth is shown in (*b*).

Determination of pole strength and magnetic moment by neutral points

(*a*) *Single pole.* The lines of magnetic force due to a single, spherical pole should theoretically be radial, similar to Fig. 3. If the field is plotted with a small compass, however, the combined field of the pole and the earth (Fig. 57) is obtained. (The earth's field may be represented by parallel lines of magnetic force running magnetic N. and S.)

The point P in Fig. 57 is called a neutral point; here the fields due to the pole and the earth are equal and opposite. By measuring the distance of this point from the centre of the pole, r cm., it is possible to calculate the strength of the pole, m c.g.s. units, if the strength of the horizontal component of the earth's field, H_e oersted, is known, since

$$H_e = \frac{m}{r^2}.$$

(*b*) *Bar magnet lying magnetic N. and S. with S. pole pointing N.* In this case the neutral points are in the positions shown in Fig. 58; m or M can be calculated from the formula

$$H_e = \frac{2Md}{(d^2-l^2)^2},$$

where d is the distance of a neutral point from the centre of the magnet.

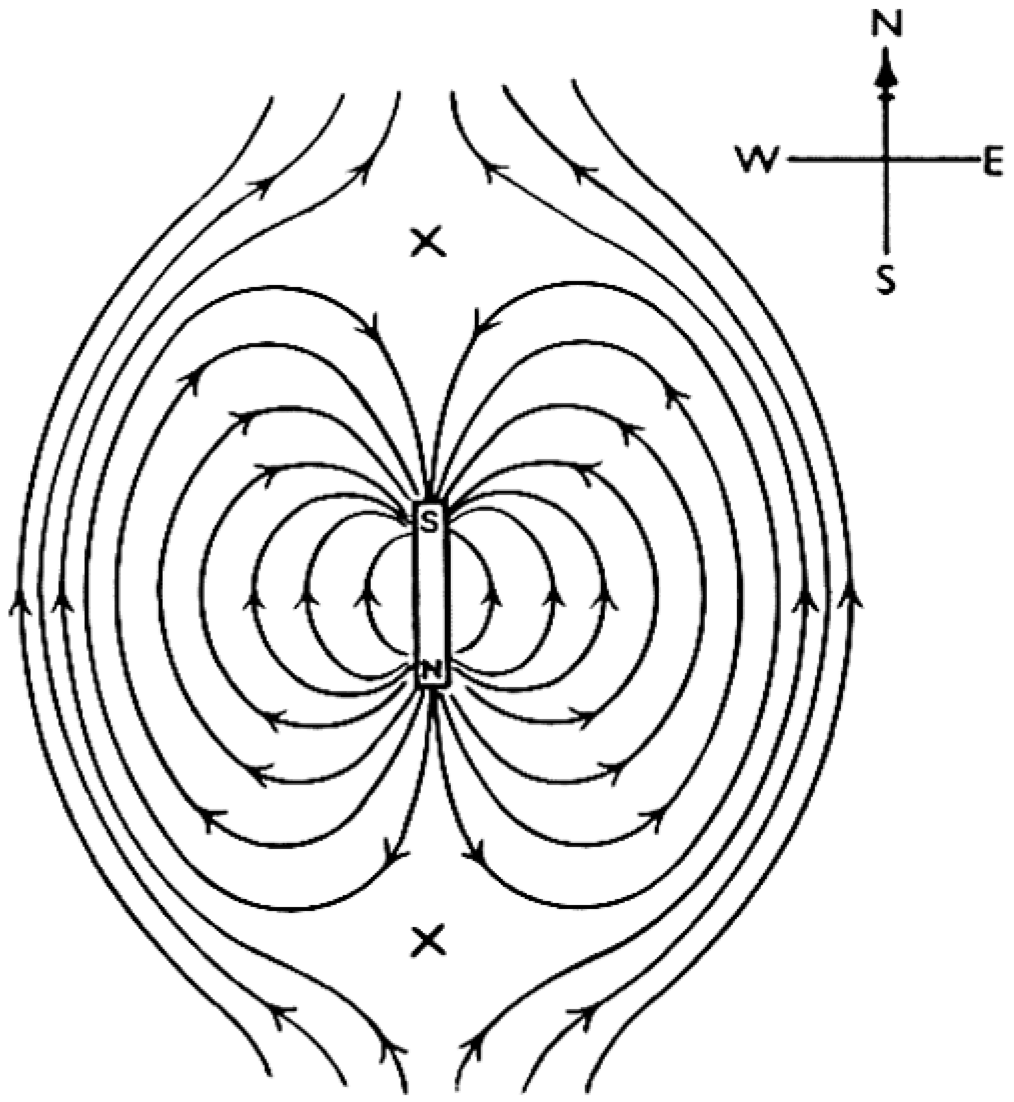


Fig. 58

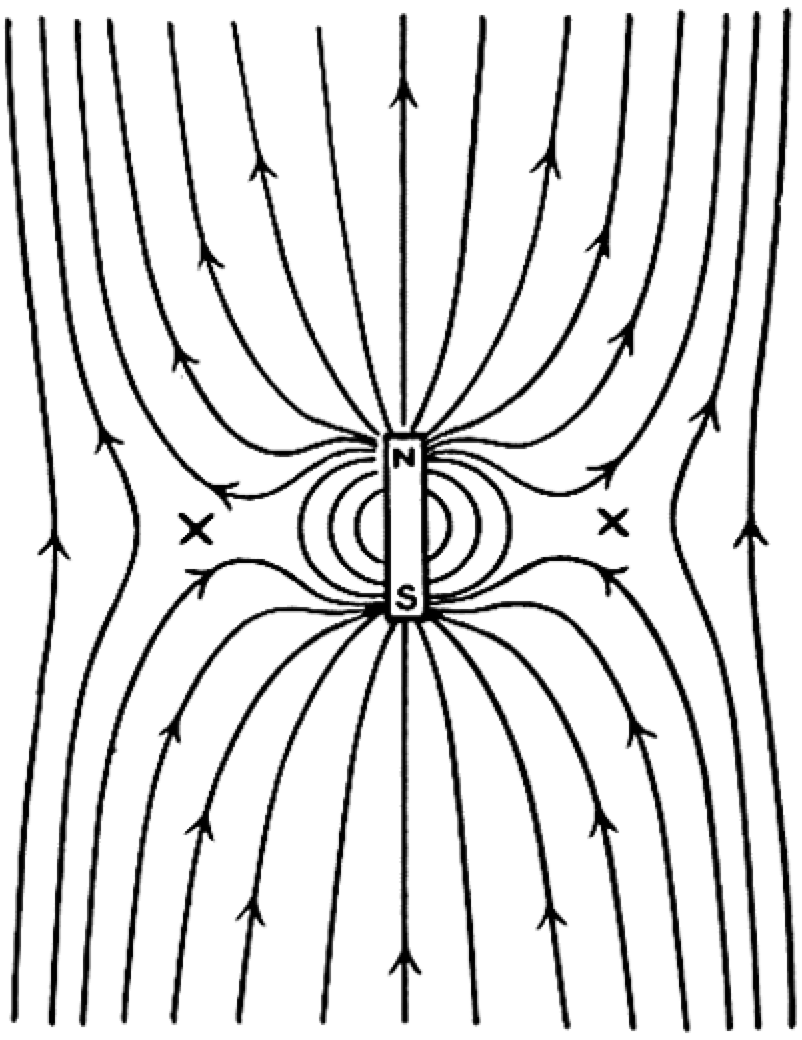


Fig. 59

(*c*) *Bar magnet lying magnetic N. and S. with N. pole pointing N.* In this case the neutral points are in the positions shown in Fig. 59; m or M can be calculated from the formula

$$H_e = \frac{M}{(d^2+l^2)^{\frac{3}{2}}},$$

where d is the distance of a neutral point from the centre of the magnet.

The deflection magnetometer

The deflection magnetometer is an instrument for comparing the strengths of two magnetic fields. It consists of a short pivoted magnet to which is attached a long aluminium pointer moving over a scale marked in degrees (Fig. 60).

The two fields to be compared must be at right angles. One of them is usually the horizontal component of the earth's magnetic field, H_e; suppose the other is H. The resultant of the two fields is the diagonal of the rectangle in Fig. 61 and the short pivoted magnet will set itself in this direction when the two fields are applied.

Clearly

$$\mathbf{H = H_e \tan \theta.}$$

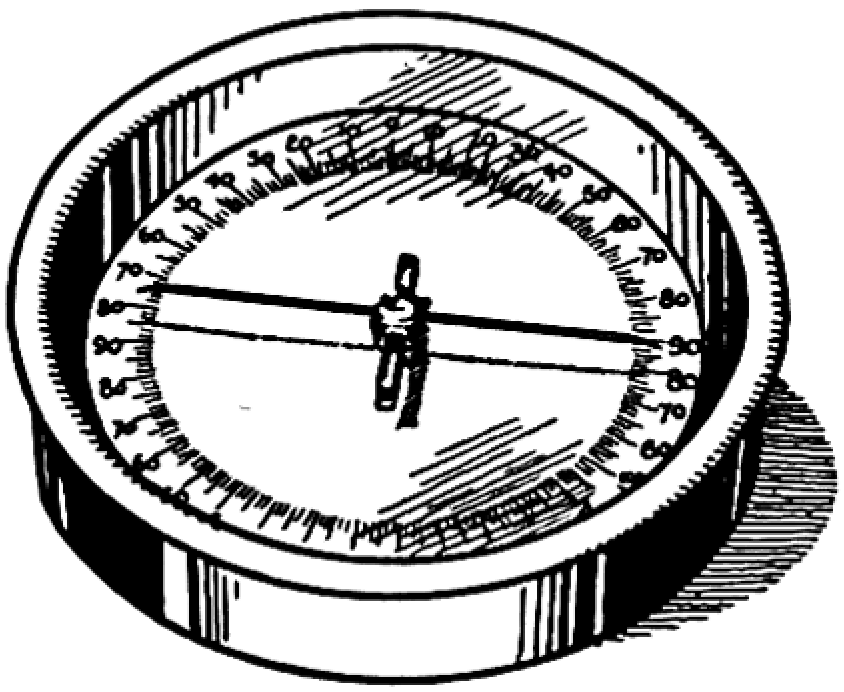


Fig. 60

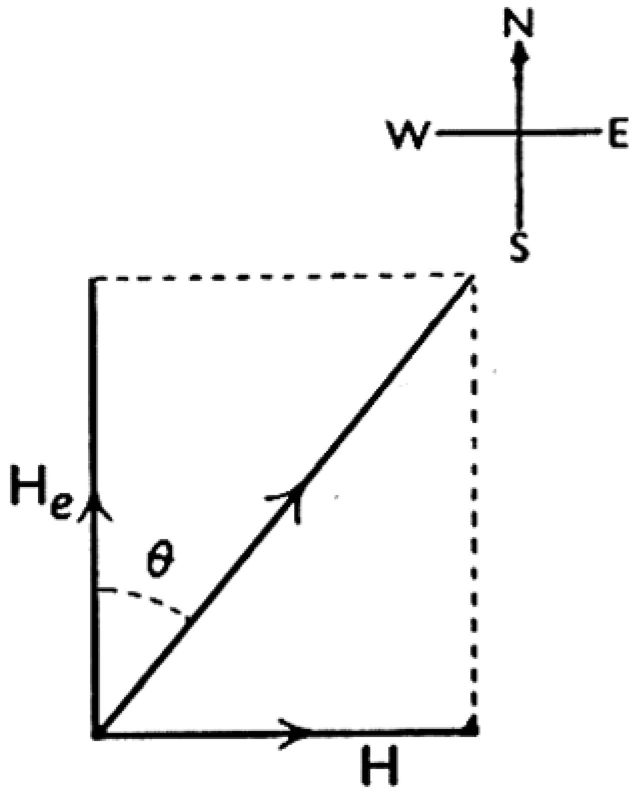


Fig. 61

Use of the deflection magnetometer

(*a*) The deflection magnetometer may be used to find the strength of the magnetic field due to a bar magnet in the end-on position and hence to determine its magnetic moment. The magnetometer is fixed in the middle of a metre rule (Fig. 62) which must first be set magnetic E. and W. The magnet is placed as shown on the ruler when its field at the magnetometer pivot will be at right angles to the earth's field. The readings of both ends of the pointer are taken. The magnet is now reversed so that its S. pole is nearer to the magnetometer and the two readings of the pointer taken. The magnet is placed on the other side of the magnetometer at the same distance and four readings of the pointer are again taken. In this way errors such as those due to imperfect setting of the instrument are corrected. If θ is the mean deflection, then the field H due to the magnet at the centre of the magnetometer is given by

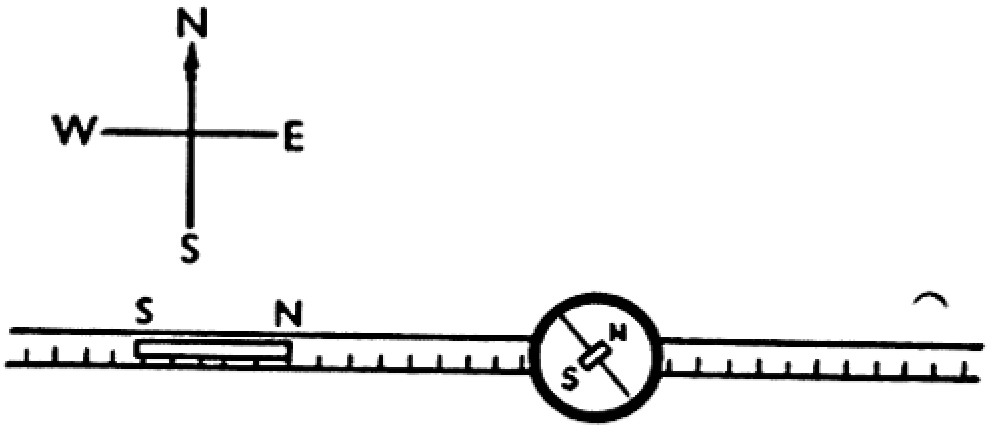


Fig. 62

$$H = H_e \tan \theta.$$

The moment of the magnet may be calculated from the equation

$$H = \frac{2Md}{(d^2 - l^2)^2}.$$

(*b*) The deflection magnetometer may be used with the magnet in the broadside position (see Fig. 63), but the metre scale must be set N. and S. in this case since the field due to the magnet is parallel to its axis. Again eight readings of the pointer are taken for a single distance d

$$H = H_e \tan\theta = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}.$$

(*c*) To compare the moments of two magnets, M_1 and M_2, the two magnets may be placed in turn at equal distances d from the centre of the magnetometer and the mean deflections, θ_1 and θ_2, found.

Suppose the end-on position is used and the magnets are short compared with d:

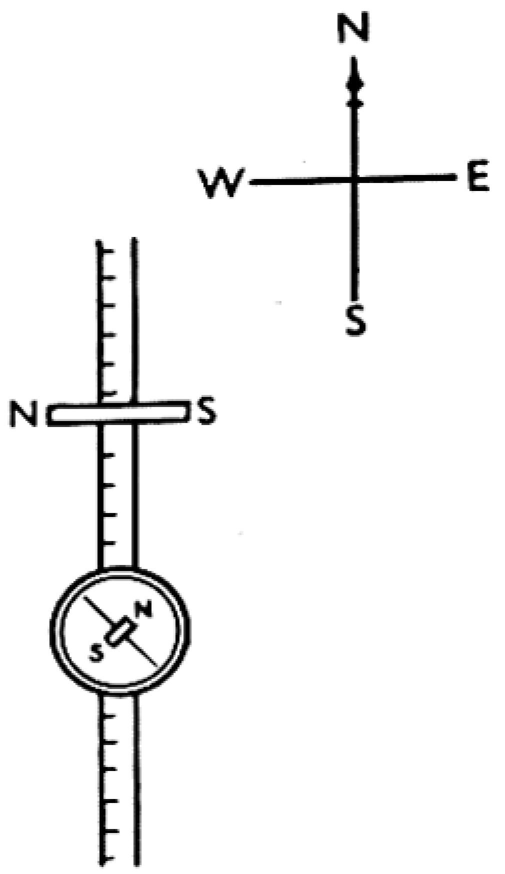


Fig. 63

$$H_1 = \frac{2M_1}{d^3} = H_e \tan\theta_1,$$

$$H_2 = \frac{2M_2}{d^3} = H_e \tan\theta_2.$$

$$\therefore \frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}.$$

Alternatively, a null method may be employed. The two magnets are placed on opposite sides of the magnetometer at such distances d_1 and d_2 that the deflection is zero. Their magnetic fields at the centre of the magnetometer must then be equal. Again assuming that the magnets are short and that they are used in the end-on position,

$$\frac{2M_1}{d_1^3} = \frac{2M_2}{d_2^3}.$$

$$\therefore \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}.$$

The couple exerted on a magnet by a magnetic field

Suppose a magnet, pivoted at its centre, of pole strength m c.g.s. units and magnetic length $2l$ cm., is at an angle θ to a magnetic field of strength H oersted (Fig. 64). Equal and opposite forces, mH dynes, will be exerted on the poles, constituting a couple tending to turn the magnet until it lies in the direction of the field. Taking moments about the centre of the magnet

$$\text{Moment of couple} = 2mHl\sin\theta = \mathbf{MH\sin\theta}.$$

If $H=1$ and $\theta=90°$, moment of couple$=M$. Thus the *magnetic moment of a magnet may be defined as the moment of the couple required to keep it at right angles to a magnetic field of unit strength.*

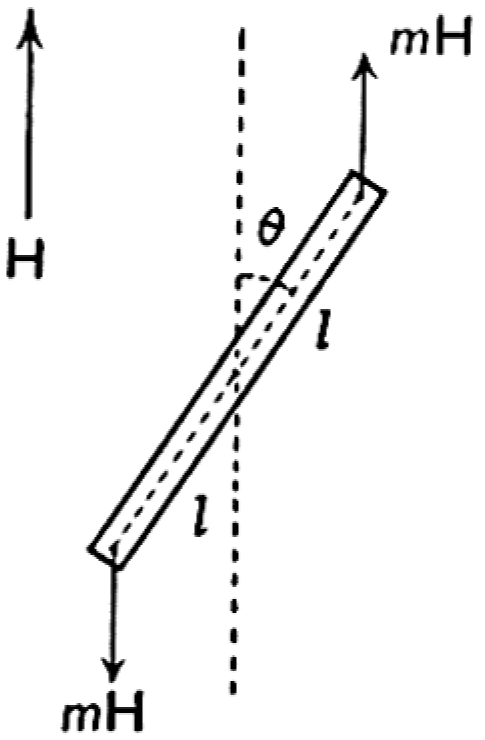


Fig. 64

The expression $MH\sin\theta$ does not involve the very uncertain magnetic length $2l$, which is always less than the geometrical length, and the magnetic moment can therefore be determined more accurately than the pole strength.

Another advantage of magnetic moment is that it can be resolved into components like a force. Thus the magnet in Fig. 64 is equivalent to two magnets of moments $M\sin\theta$ and $M\cos\theta$ lying perpendicular and parallel to H respectively. This sometimes simplifies the solution of problems (see p. 79).

The vibration of a magnet in a magnetic field

A freely suspended magnet lying in a magnetic field, when turned slightly from the direction of the field and released, begins to vibrate like a pendulum oscillating in a gravitational field.

As we have just seen, when the magnet is at an angle θ with the field

$$\text{Moment of restoring couple} = MH\sin\theta$$
$$= MH\theta \quad \text{if } \theta \text{ is small.}$$

Since the restoring couple is proportional to the angular displacement, θ, the magnet is executing simple harmonic motion, but it should be noted that this is true only for small vibrations.

The periodic time of simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{\text{Moment of inertia of body about its axis of rotation}}{\text{Restoring couple per unit angular displacement}}}.$$

Thus
$$\mathbf{T = 2\pi\sqrt{\frac{I}{MH}}},$$

where I is the moment of inertia. In the case of a rectangular bar magnet of mass m g., length (geometrical) $2a$ cm. and breadth $2b$ cm., oscillating about an axis as in Fig. 65,

$$I - m\frac{a^2+b^2}{3} \text{ c.g.s. units.}$$

Determination of the horizontal component of the earth's magnetic field

To determine the strength of the horizontal component of the earth's magnetic field, H_e, a magnet must be employed and two experiments are necessary, since there are two unknowns H_e and the moment of the magnet M.

The magnet is suspended by means of a long thin copper wire from the ceiling, the object of the long wire being to reduce torsion to a minimum. The periodic time, T, for small vibrations is determined. The moment of inertia I may be calculated from the formula given above.

A deflection magnetometer is then placed with its centre exactly below the copper wire suspension—that is, in the place where the value of H_e is required. The mean deflection, θ, for a suitable distance, d, of the magnet is determined using, say, the end-on position. The value of H_e may be found from the two equations

$$T=2\pi\sqrt{\frac{I}{MH_e}},$$

$$H_e \tan\theta=\frac{2Md}{(d^2-l^2)^2}.$$

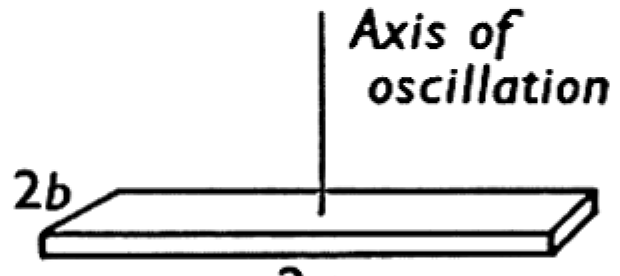


Fig. 65

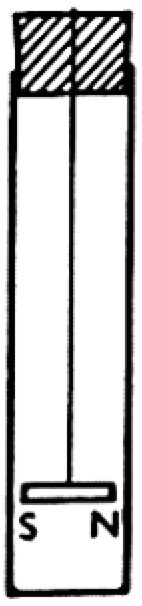


Fig. 66

The vibration magnetometer

The vibration magnetometer is used to compare the strengths of magnetic fields. It consists of a small magnet usually suspended by a thread of unspun silk in a glass vessel to protect it from draughts (Fig. 66).

The times of vibration, T_1 and T_2, of the small magnet are found in the two fields, H_1 and H_2. Then

$$T_1=2\pi\sqrt{\frac{I}{MH_1}} \quad \text{and} \quad T_2=2\pi\sqrt{\frac{I}{MH_2}}.$$

$$\therefore\ \frac{H_1}{H_2}=\frac{T_2^2}{T_1^2}.$$

This is a convenient method of determining how the earth's field varies at different points in a laboratory.

Suppose the fields of two magnets are to be compared by a vibration magnetometer. Allowance must be made for the effect of the earth's magnetic field by finding the time of vibration in the earth's field alone, T_e. Suppose that when the magnets are placed in turn in such positions with respect to the magnetometer that their magnetic fields reinforce the earth's field, the times of vibration are T_1 and T_2 respectively. Let the fields due to the magnets alone be H_1 and H_2 respectively:

$$H_1+H_e \propto \frac{1}{T_1^2},$$

$$H_2+H_e \propto \frac{1}{T_2^2},$$

$$H_e \propto \frac{1}{T_e^2},$$

$$\therefore \frac{H_1}{H_2}=\frac{\dfrac{1}{T_1^2}-\dfrac{1}{T_e^2}}{\dfrac{1}{T_2^2}-\dfrac{1}{T_e^2}}.$$

The earth's magnetic field

A magnetic needle, when suspended freely so that it can turn in both a horizontal and a vertical plane, sets itself, in England at the present time, in a vertical plane 10° W. of N. with its N. pole dipping 67° below the horizontal; it follows that the magnetic field due to the earth must lie in this direction.

The following terms are in common use (see Fig. 67):

The geographic meridian is the vertical plane in a direction geographic N. and S., i.e. which passes through the earth's geographic poles.

The magnetic meridian is the vertical plane in which a magnet sets itself at a particular place.

The angle of declination (or variation of the compass) is the angle between the magnetic and geographic meridians.

The angle of dip (or inclination) is the angle between the horizontal and the magnetic axis of a magnet free to swing in the magnetic meridian about a horizontal axis. It is the angle between the directions of the earth's magnetic field and the horizontal.

The magnetic field of the earth, often called the *total intensity*, *I*, is resolved for convenience into a *horizontal component*, *H*, and a *vertical component*, *V*.

The quantities declination, dip, I, H and V are known as the *magnetic elements*. The earth's magnetic field at a particular place may be specified by the declination and any two of the other magnetic elements because I, H, V and θ are mutually related; e.g. $V/H=\tan\theta$.

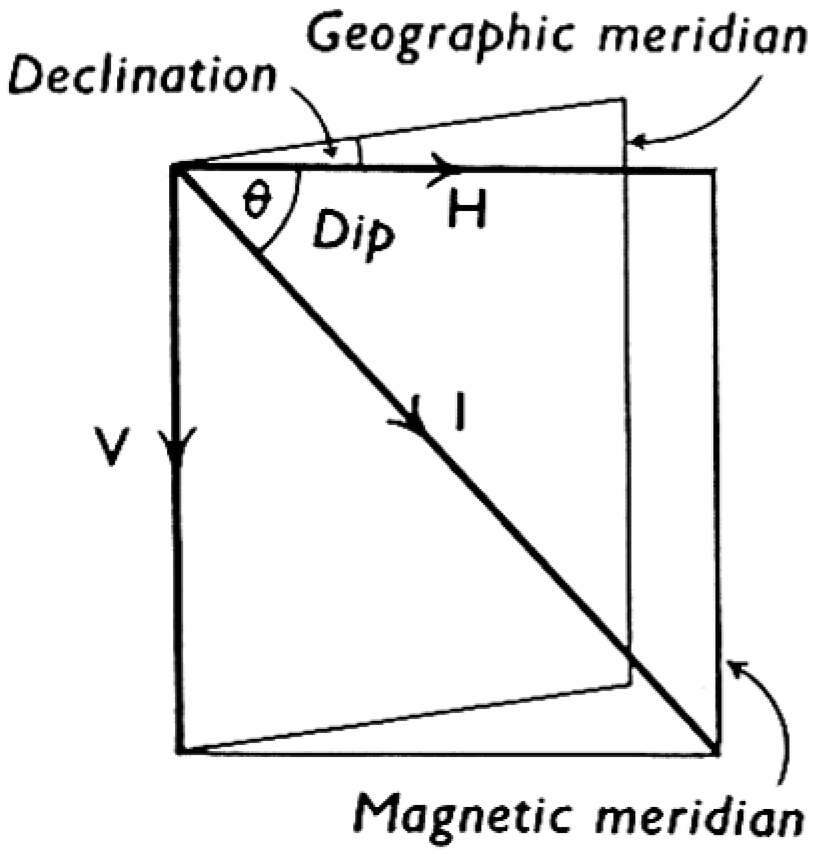


Fig. 67

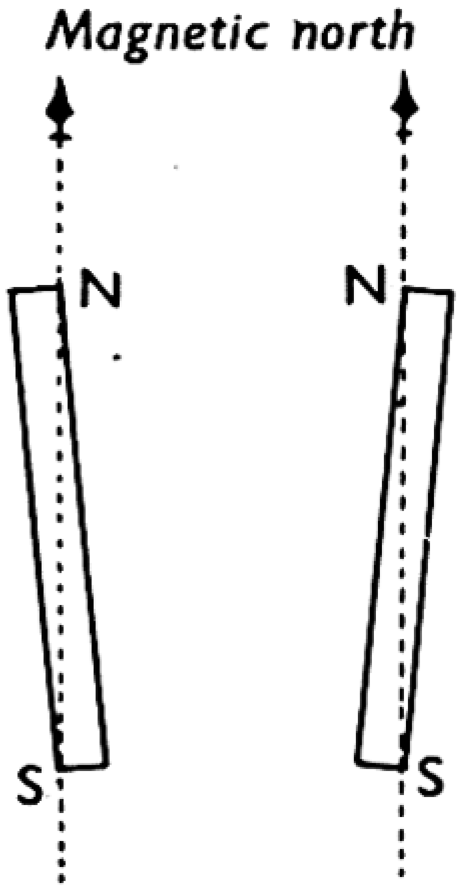


Fig. 68

Determination of declination

The determination of the angle of declination at a place involves finding two directions, geographic N. and magnetic N.

The former can be found accurately only by an astronomical method—observation of the sun or stars. It can be found with fair accuracy from the fact that the shadow of a vertical stick cast by the sun at mid-day is due N.

Magnetic N. is found by suspending a bar magnet freely on a vertical axis. Since the magnetic axis of the magnet may not coincide with its geometrical axis, the magnet must be turned over and the mean of its two directions found (Fig. 68).

Determination of dip

(1) *The dip circle.* The angle of dip may be measured by means of an instrument called a dip circle (Fig. 69). This consists of a magnetic needle freely pivoted on a horizontal axis at the centre of a vertical, circular scale marked in degrees. The whole instrument can be revolved about a vertical axis and the angle turned through measured on a horizontal scale.

The instrument is first levelled with the aid of a spirit level. This is important, since it is just as necessary to obtain a true horizontal as the true direction of the earth's lines of force.

The needle has now to be set so that it swings in the magnetic meridian. The dip circle is turned about its vertical axis until the needle is absolutely vertical. When this occurs the plane of the dip circle must be at right angles to the magnetic meridian. The vertical component of the earth's field pulls the needle into a vertical position, but the horizontal component exerts forces on the poles of the needle in directions N. and S., whereas they are free to move only E. and W. (Fig. 70); hence the horizontal component has no effect and the needle is vertical. The whole dip circle is now turned through exactly 90° (measured on the horizontal circular scale), and it is then in the magnetic meridian.

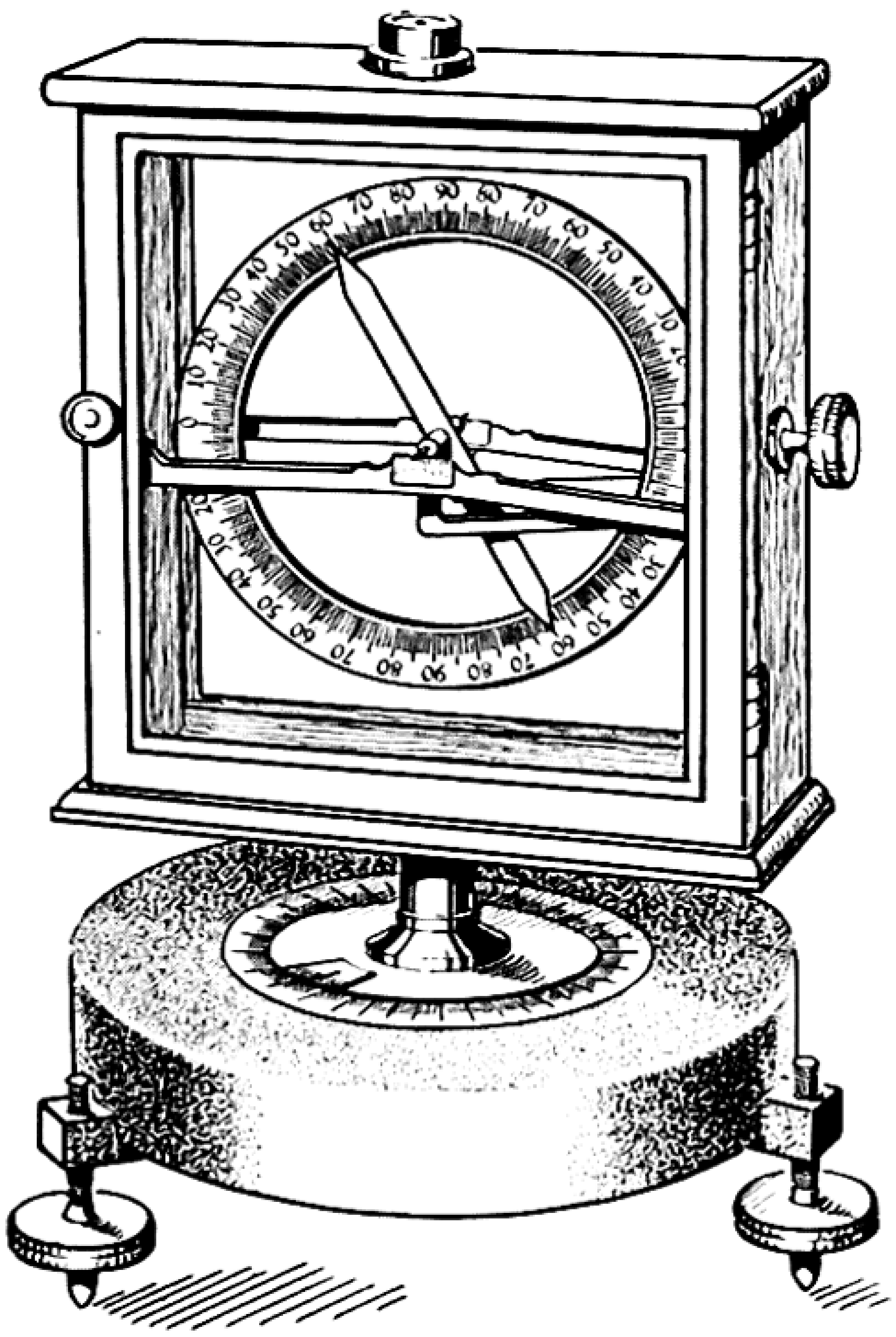


Fig. 69

The angle the needle makes with the horizontal gives the angle of dip.

The possible errors and the means by which they may be avoided are as follows (Fig. 71):

(*a*) The axis of rotation of the needle may not coincide with the centre of the scale; read both ends of the needle.

(*b*) The line joining the 0–0 readings on the scale may not be horizontal; turn the dip circle through 180° and again read both ends of the needle.

(*c*) The magnetic axis of the needle may not coincide with the geometrical axis; reverse the needle in its bearings and repeat (*a*) and (*b*).

(*d*) The centre of gravity of the needle may not coincide with its axis of rotation; remagnetise the needle to the same strength in the opposite direction, and repeat the eight readings of (*a*), (*b*) and (*c*).

The mean of the sixteen readings should be taken.

The reader should consider the possible errors in the deflection magnetometer.

(2) *The earth inductor.* Another method of finding the angle of dip is by means of the earth inductor (Fig. 72). The instrument consists of a coil of wire which can be rotated about an axis capable of being set in any direction by turning a movable frame.

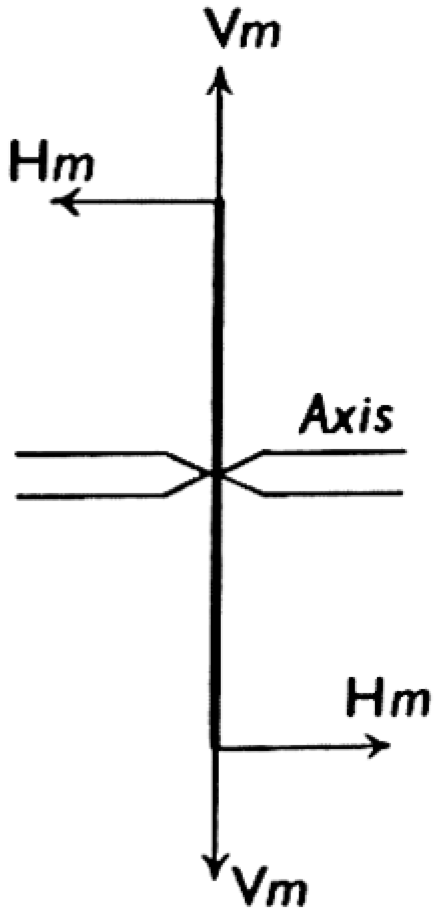


Fig. 70

The frame is made vertical and the whole instrument set magnetic E. and W. If the coil is turned through 90° (about its vertical axis) so that its plane changes from the direction of magnetic E. and W. to magnetic N. and S., a quantity of electricity is induced in it which is proportional to *H*, the horizontal component of the earth's magnetic field. In the N.–S. position none of the earth's horizontal lines of force thread the coil, and in the E.–W. position the maximum number thread it. The induced quantity of electricity can be measured by the throw of a ballistic galvanometer (see p. 148).

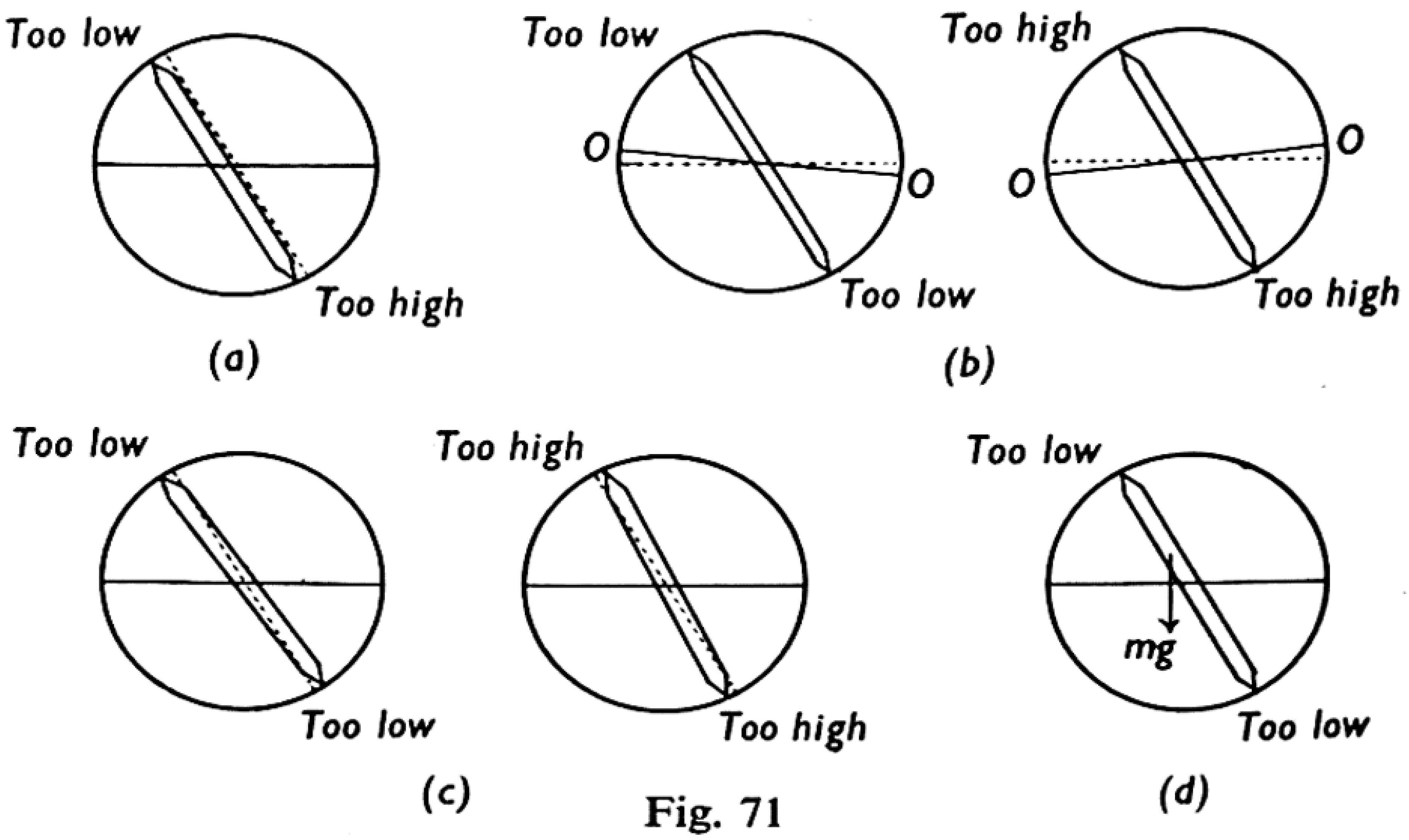


Fig. 71

The frame is now turned so that the coil can move about a horizontal axis in the magnetic meridian. When the coil is rotated through 90°, say from a vertical to a horizontal position, the quantity of electricity induced is proportional to V, the earth's vertical component.

If D = angle of dip,

θ_1 and θ_2 = throws of ballistic galvanometer,

$$\tan D = \frac{V}{H} = \frac{\theta_2}{\theta_1}.$$

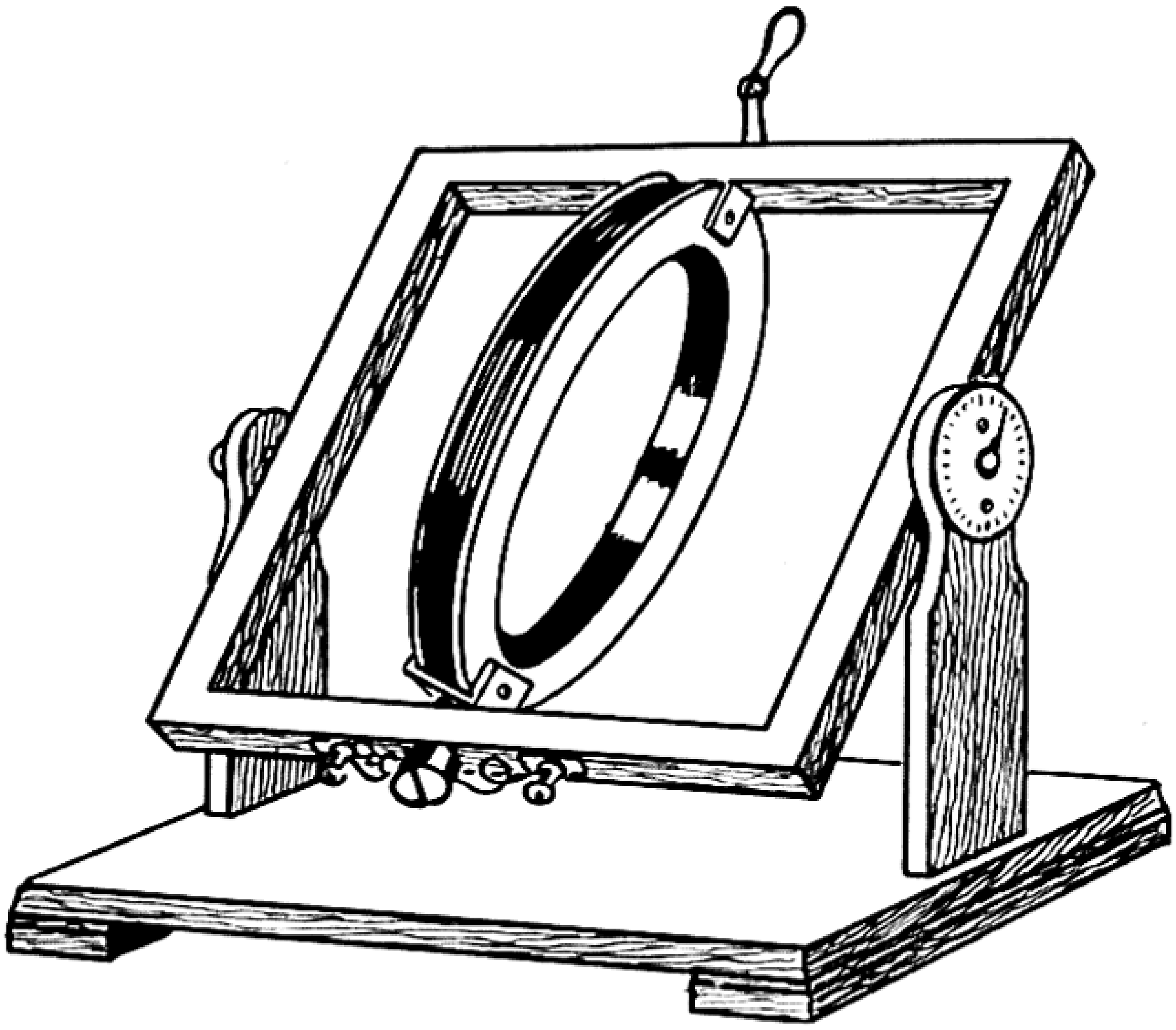


Fig. 72

Alternatively, the direction of the axis of the coil can be adjusted until no current is induced when the coil is rotated. The direction of the axis is then the direction of the resultant intensity of the earth's magnetic field.

The dip circle has now, in fact, been superseded by the earth inductor.

Variation of dip over the earth's surface

The angle of dip is 0° approximately at the equator. It increases steadily northward or southward, until it becomes 90° at the magnetic poles. In the northern hemisphere the N. pole dips, and in the southern hemisphere the S. pole dips. Fig. 73 is a map showing isoclinic lines, which are lines joining places at which the angle of dip is the same.

Gilbert constructed a model spherical magnet of lodestone, which he called a terrella or 'little earth', with poles as shown in Fig. 74. By means of this model he was able to show that the angle of dip should vary over the surface of the earth as described. The dotted lines in the figure represent the lines of magnetic force, and the arrows represent compass needles, the arrowheads being the N. poles.

The angle of dip at the magnetic poles is 90°; it is in this way that they are located. The N. magnetic pole was discovered in the far north of Canada in 1831 by Sir James Ross; recently, aircraft flights over the pole have shown that its position has shifted by some hundreds of mlies. The S. magnetic pole was discovered by Sir Ernest Shackleton in 1909.

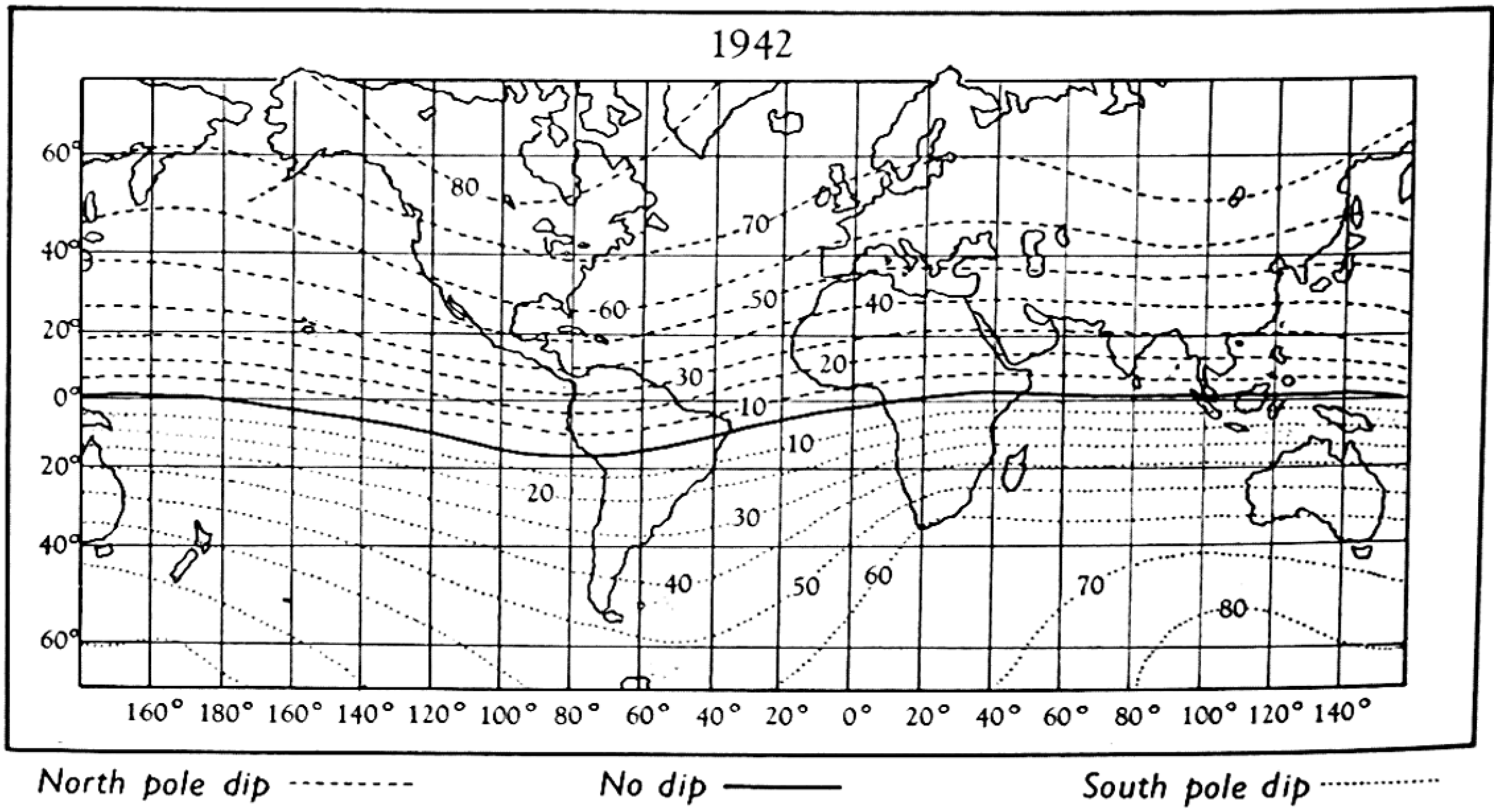


Fig. 73

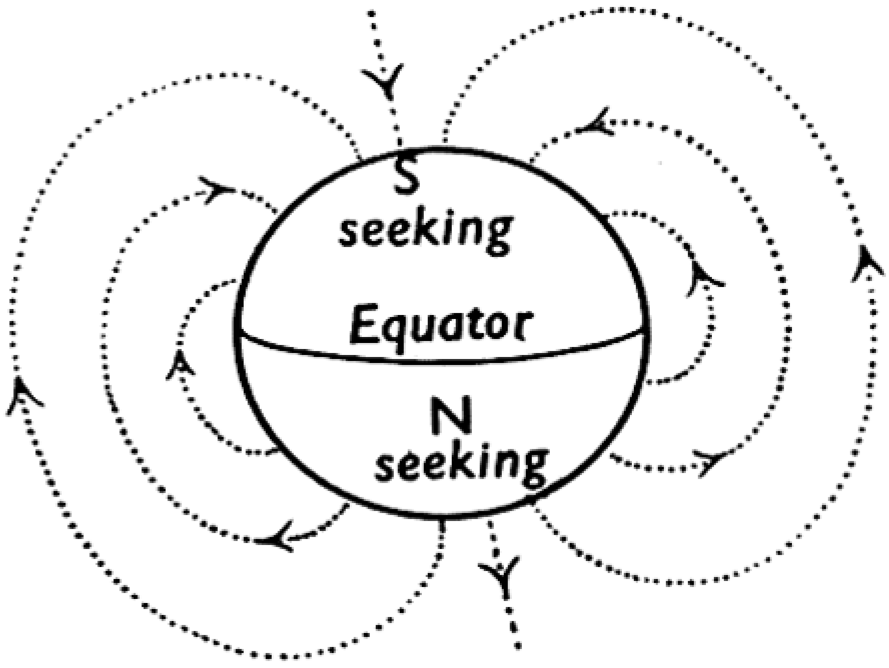


Fig. 74

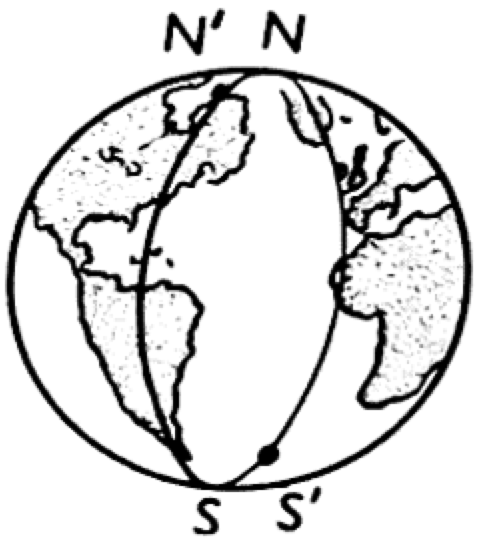


Fig. 75

Variation of declination over the earth's surface

Since the geographic and magnetic poles do not coincide, the declination varies over the earth's surface. Fig. 75 shows the great circle drawn through all four poles. The declination on this circle should be zero; on one side of it the declination should be E. of N. and on the other W. of N.

Fig. 76 shows isogonic lines, which are lines joining places at which the angle of declination is the same. The two halves of the great circle can be recognised, but one-half describes a large loop over Siberia, due to the irregular magnetisation of the earth's crust.

For many years in the seventeenth century it was hoped that changes in magnetic declination might be the clue to an accurate method of finding longitude, an urgent navigational problem for the solution of which the British Government offered a prize of £20,000. In 1671 Henry Bond expounded a method based on magnetic declination in a book *Longitude Found*, answered two years later by Peter Blackbarrow in *The Longitude Not Found*. The problem was solved not by a magnetic method but by the development of a suitable chronometer by Harrison.

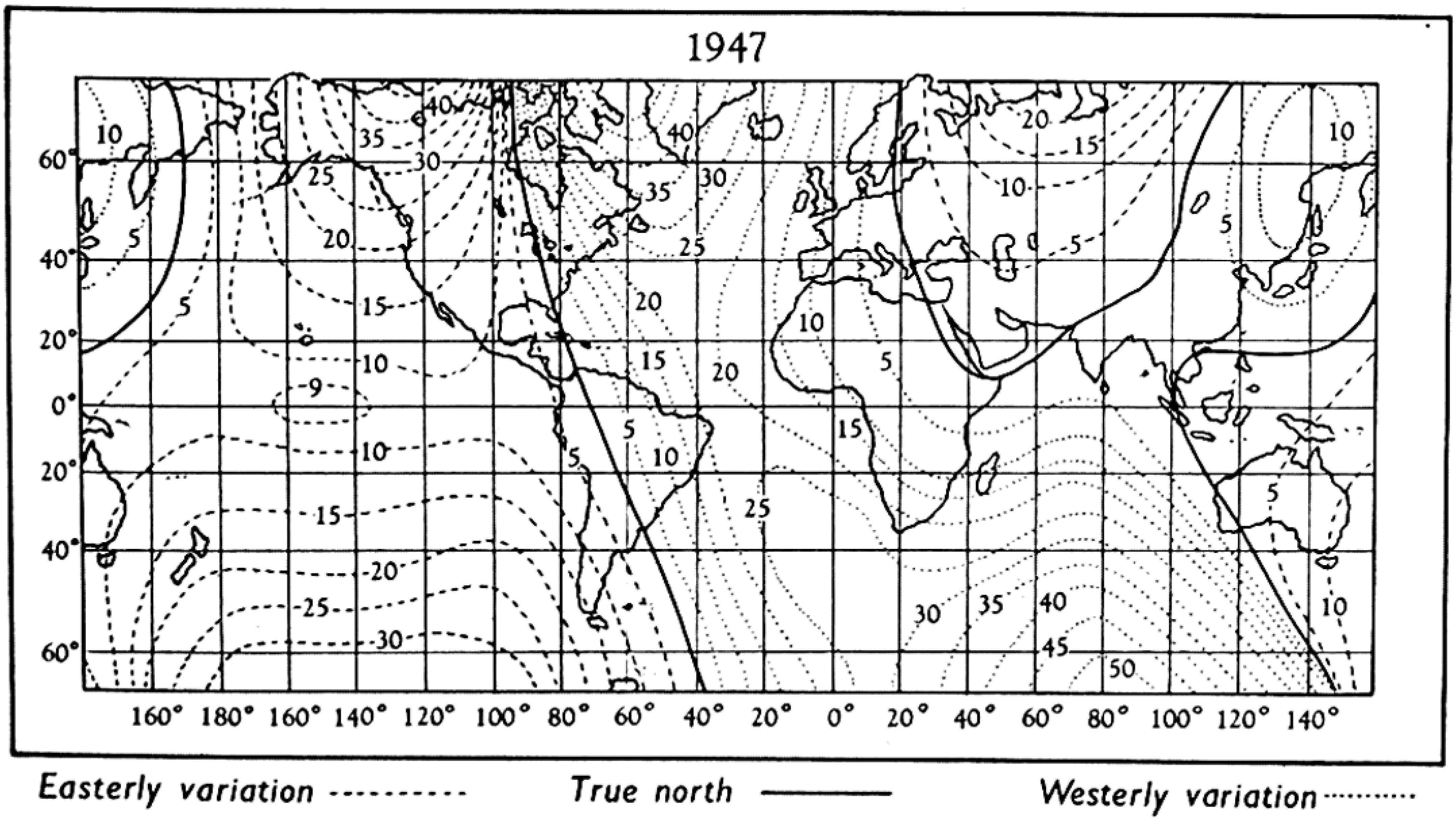


Fig. 76

Christopher Columbus is sometimes credited with the discovery of the change in declination when he sailed across the Atlantic to the Bahamas in 1492. It was probably known, however, to earlier navigators. Some of the portable sundials carried by travellers in the fifteenth century as time-pieces, had a compass in the base for setting, and a mark indicating the amount of declination. Edmund Halley, between October 1698 and September 1700, made two voyages in the North and

South Atlantic, the first sea voyages made for purely scientific purposes, which resulted in the first declination chart of the earth. There are now about seventy-five magnetic observatories distributed over the surface of the earth. In 1882–3 and 1932–3 many nations combined to set up magnetic observatories in the Arctic regions and results of great value were obtained.

Secular variations

Besides varying in space over the surface of the earth, the magnetic elements are slowly varying in time. Measurements of the angle of declination in Greenwich have been made since the year 1580. The following table shows the variation from that date until 1946:

1580	11° E.	1816	24° W.
1622	6° E.	1900	17° W.
1658	0°	1946	9° 51′ W.*

The declination is now decreasing, and it seems likely that it will, in due course, become easterly again. It has been estimated that a complete cycle will take about 480 years, that is, the value in the year 2060 will be the same as in 1580.

The angle of dip is also varying, but to a smaller extent than the declination. In 1580 its value in London was 72°, in 1658 74°, in 1816 71°, and in 1946 66° 44′.*

Besides these slow secular changes, as they are called, there occur at intervals magnetic storms, when the needles of all magnetic recording instruments are affected in a comparatively violent manner. Magnetic storms have been traced to electrons in the upper atmosphere, originating from sunspots. These are also responsible for the Aurora Borealis or Northern Lights, a beautiful and spectacular glow to be seen occasionally in the sky in northern latitudes. Hiorter, an observer in Upsala, who was one of the pioneers in this sphere, wrote in 1747: 'Who could have thought that the northern lights would have a connection and sympathy with the magnet, and that these northern lights, when they draw southwards across our zenith or descend unequally towards the eastern and western horizons, could within a few minutes cause considerable oscillations of the magnetic needle through whole degrees?'

Magnetic storms show an 11-year cycle of activity which corresponds with the 11-year sunspot cycle; they also tend to recur every 27·3 days, the period of rotation of the sun.

---

* The values for 1946 are those at Abinger, near Dorking, where the magnetic elements are now recorded.

The origin of the earth's magnetism

The main features of the earth's magnetic field can be accounted for by assuming that the earth's crust is a comparatively uniformly magnetised shell with some large irregularities. The shell cannot be thicker than 20 kilometres, since at depths greater than this the earth is too hot to be magnetic.

The sun has a magnetic field similar to that of the earth, and its magnetic moment bears the same ratio to its angular momentum as in the case of the earth. If M represents the magnetic moment and U the angular momentum of the earth or the sun,

$$M=\beta\frac{G^{\frac{1}{2}}}{c}U,$$

where G = gravitational constant, c = velocity of light, β = a constant of the order of unity.

The fact that the universal constants G and c can be fitted into an equation connecting M and U has prompted the suggestion that every large rotating body is a magnet. This offers an approach to the hitherto unsolved problem of the origin of the earth's (and the sun's) magnetism, of the approximate coincidence of its magnetic axis and axis of rotation and of the revolution of the magnetic poles round the geographic poles.

In 1947 the magnetic field of a star, 78 Virginis, was measured by means of its effect on the spectral lines, i.e. the Zeeman effect. Professor Blackett has shown that the magnetic moment and angular momentum of this star fit the above equation,* which has greatly increased the probability that the equation represents a universal law.

The mariner's compass

The earliest compasses were small hollow pieces of iron, often shaped like a fish, rubbed with lodestone and floating in a bowl of water.

The modern compass (Fig. 77) consists of a group of magnetic needles rigidly attached to a card, mounted on a jewelled bearing, in a bowl filled with liquid to damp any oscillatory movement of the card. The card is marked in degrees as well as with the old thirty-two points of the compass. Since the magnets tend to dip, the card is supported at a point above its centre of gravity so that its weight tends to keep it horizontal.

Some of the latest compasses, especially those in aircraft, have no cards. Instead the rim of the bowl is marked in degrees (Fig. 78). To

* P. M. S. Blackett, *Nature*, vol. 159, no. 4046, 17 May 1947.

set a course, two parallel lines on the glass cover, called grid lines, are turned to point to the course in degrees on the rim, and the compass needle is kept parallel to the two lines as the pilot steers.

In both ships and aircraft magnets are housed below the compass bowl to compensate for the deviations of the compass produced by the magnetism of the craft. Since some of the magnetism is temporary the ship or the aircraft must be 'swung' at frequent intervals. The craft is turned due N. and the appropriate compensating magnets are adjusted until the compass reads N. A similar procedure is followed with the craft pointing S., E. and W. The full electrical equipment of an aircraft must be switched on if it is being swung on the ground.

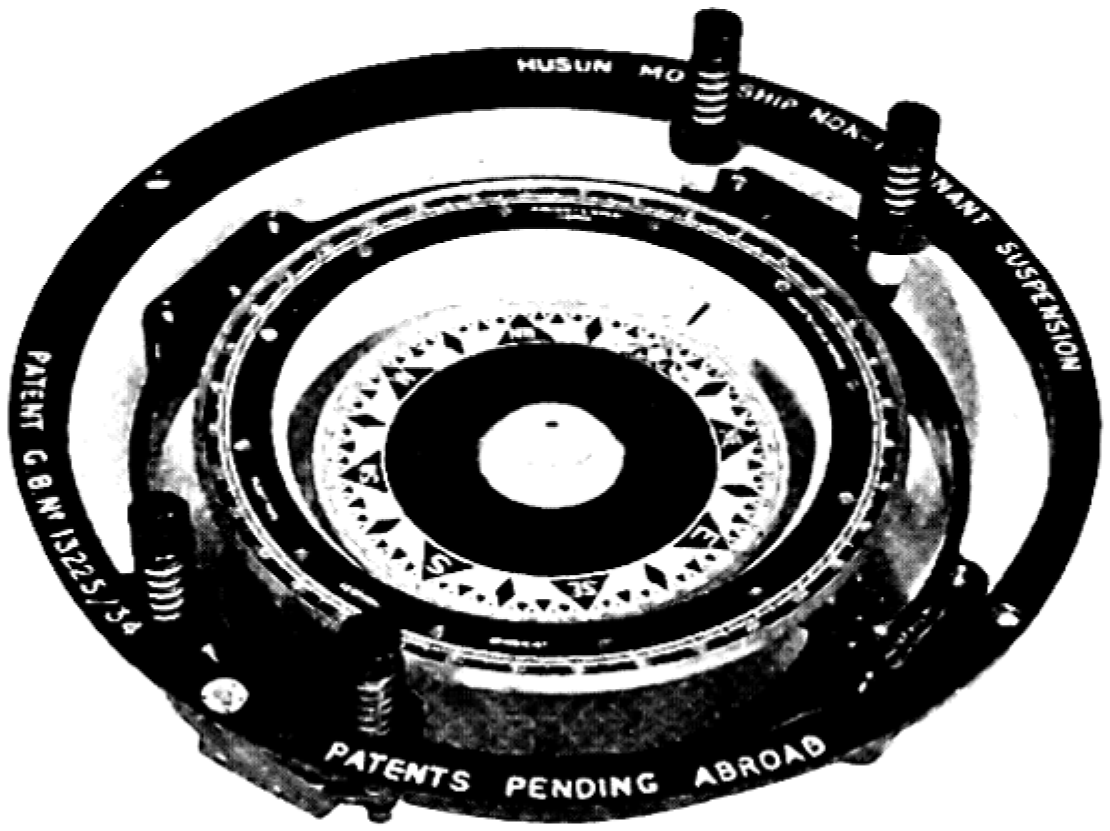


Fig. 77. Marine compass (fleet dead-beat type).

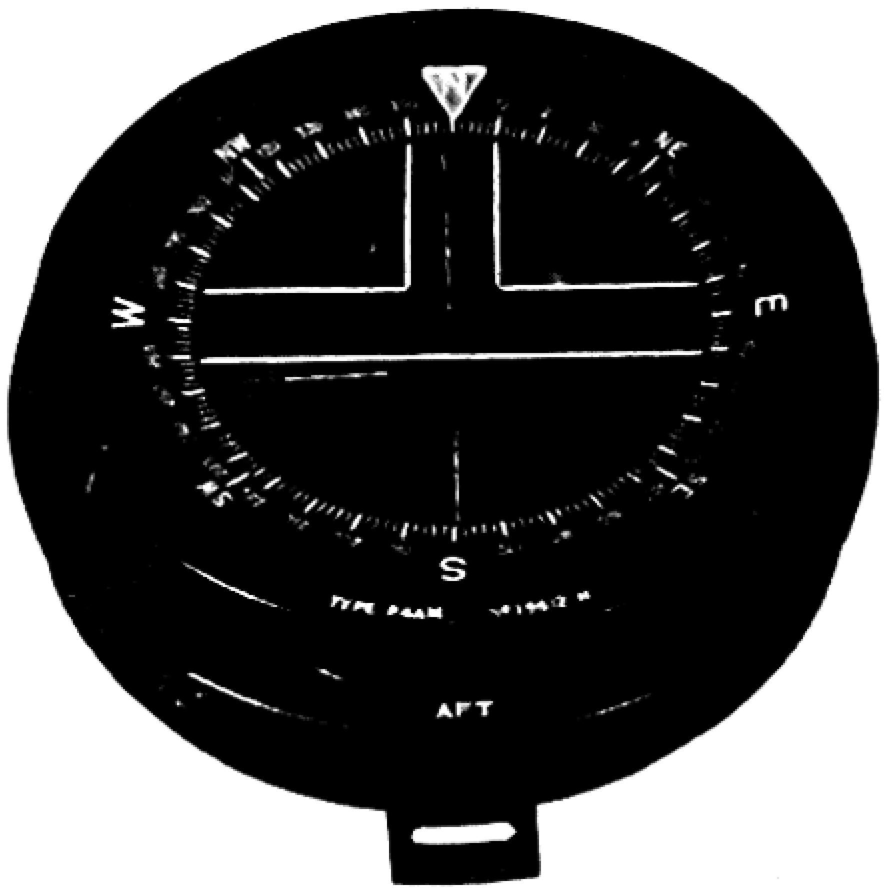


Fig. 78. Aircraft compass (type P 4)

Degaussing

On 23 November 1939 the first German magnetic mine to be recovered intact was rendered innocuous in the Thames Estuary, and taken to H.M.S. *Vernon* for examination. It was found to operate only on changes in the vertical magnetic field; it was therefore easy to counter.

The principle of the magnetic operation of the mine is shown in Fig. 79. A magnet pivoted on a horizontal axis—essentially a dip needle—is kept horizontal by a helical spring until a ship passes overhead and the intensified magnetic field causes it to dip, thus completing a circuit which sets off the detonator. It was found that an increase in the vertical magnetic field of 0·05 oersted was needed for firing the mine.

The mine can be set for use in any latitude in the northern hemisphere (in which the N. pole dips) by means of a clock. The clock winds up the helical spring by an amount depending on the angle of dip, in order to make the magnet horizontal, and the mine becomes live only after this winding operation has been completed. The apparatus is mounted in double gimbal rings to keep it horizontal.

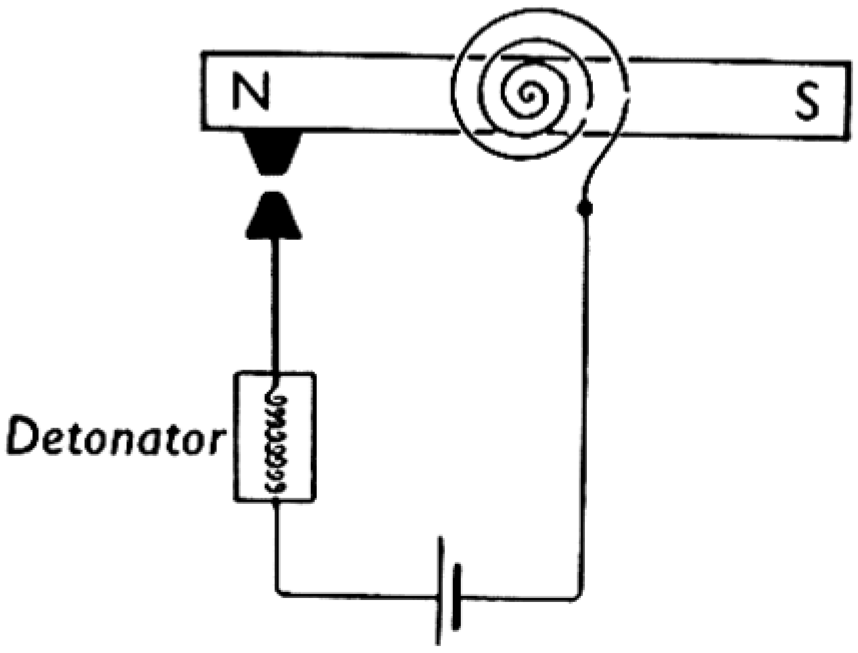


Fig. 79

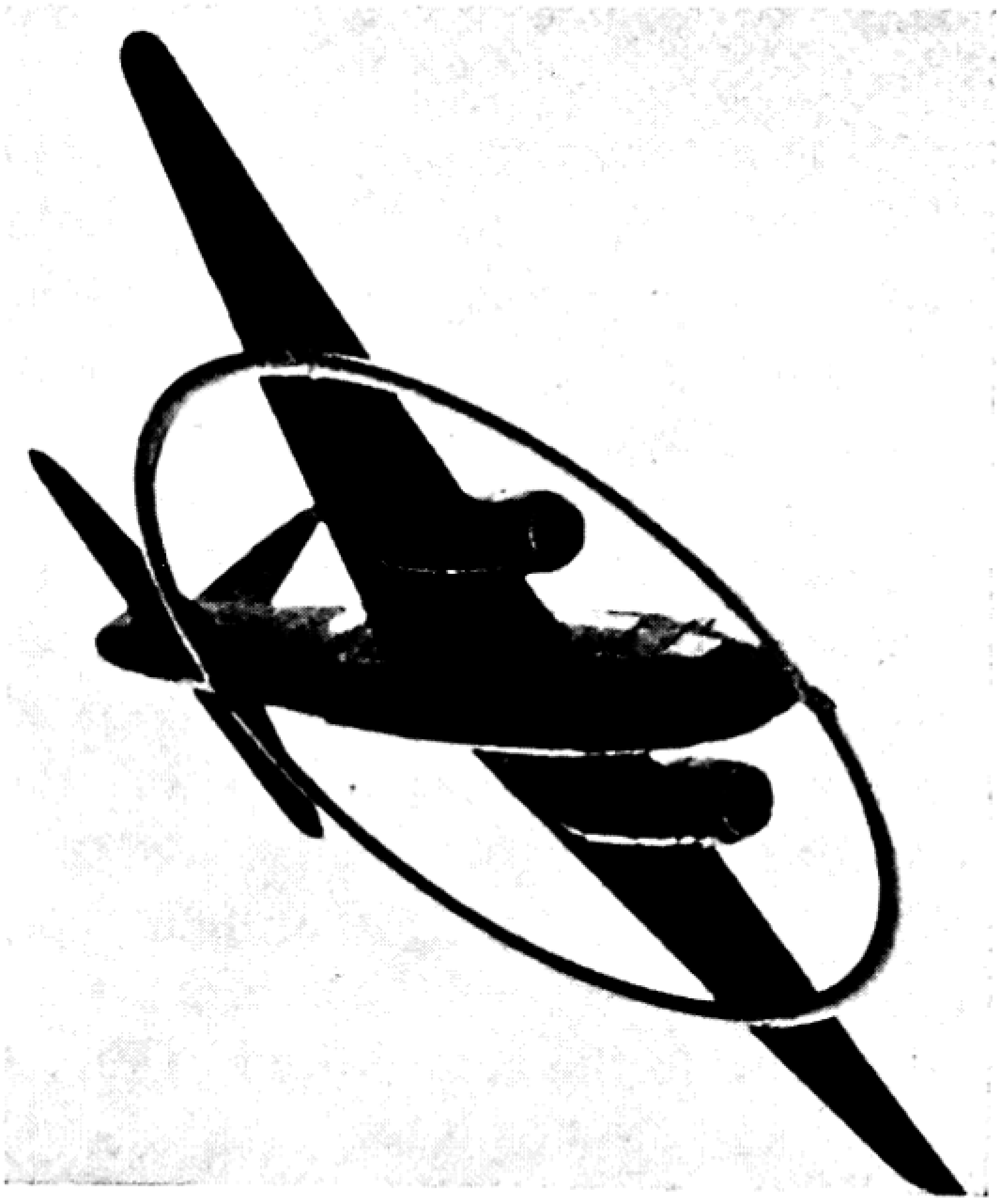


Fig. 80. A Wellington bomber fitted with a coil for detonating German magnetic mines.

The magnetic fields of ships had previously been measured by the British by sailing the ships over search coils (similar in principle to the earth inductor) on the sea bed. These magnetic fields were counteracted by 'degaussing' coils. The main coil girdled the ship and consisted, at first, of a single turn of copper strip, up to 6 inches wide and $\frac{3}{4}$ inch thick, bedded in a rubber channel round the outside of the ship; it carried a current of between 400 and 2400 amperes, depending on the class of the vessel. The life of these external coils was short owing to exposure, and it was soon found that a coil inside the ship would do equally well. Other coils on the forecastle and quarterdeck, and sometimes on the port and starboard sides, were necessary to compensate for poles at the ends of the ship and athwartship.

The magnetism of a ship consists of a comparatively constant 'permanent' part which was 'hammered in' during the ship's construction in the earth's field, and also a variable part caused by the inducing effect of the earth's field, and depending on the ship's course and position.

Thus the current through the degaussing coils had to be regulated to allow for change of course. At first this was done by hand, but later it became an automatic process.

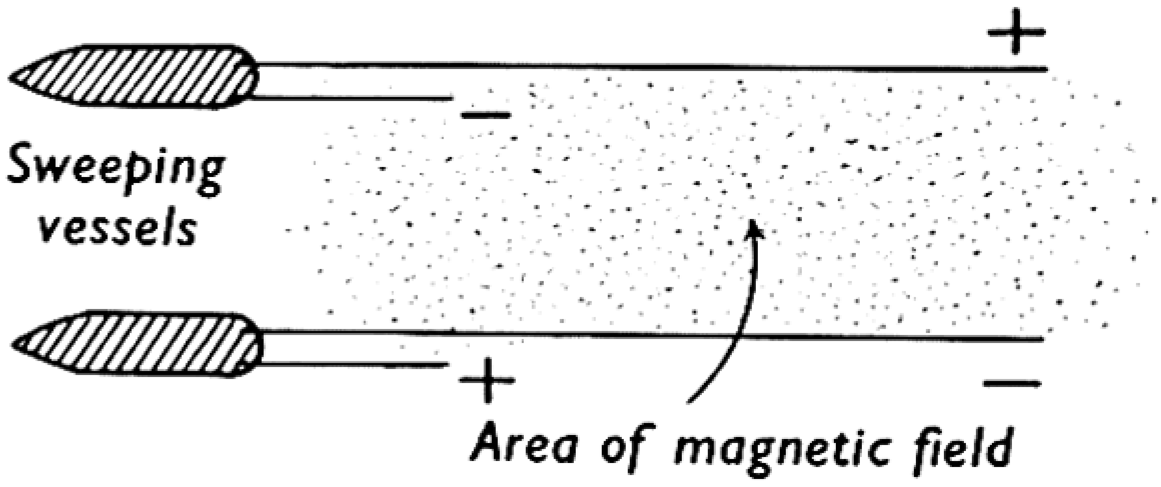


Fig. 81

The degaussing coils could cause errors in the compass of 30–40°, and therefore a system of coils near the compass was necessary to compensate for their effect.

In order to destroy the mines, sweeping was carried out by low-flying aircraft of Coastal Command, equipped with a large horizontal coil (see Fig. 80) through which passed a large electric current from a generator. Since the mines were set for the slow speed of ships the aircraft had passed out of danger before the explosion occurred.

The first sweeping ships were equipped with powerful magnets in the bows to explode the mines ahead. But it was a dangerous operation and the crews had to be protected with anti-shock boots and seats. The most successful device was the 'Double L' sweep shown in Fig. 81. The degaussed sweeping vessels worked in pairs. Each towed a long and a short self-buoyant electric cable, the longer 'tail' being about 500 yards in length. A current of about 2700 amperes, passed through the sea between electrodes at the ends of the cables for a few seconds, was effective in firing the mines in the area dotted in Fig. 81.

Geophysical prospecting

It is possible to locate the position of underground magnetic ores by finding the variation in the magnetic elements on the earth's surface. Fig. 82 shows the effect on the earth's vertical and horizontal components, V and H, of a deposit of ore, assuming AB to lie along the magnetic meridian. It can be seen, for example, how V increases over the S. pole of the deposit and decreases less over the N. pole, since the N. pole is deeper underground.

The differences in V and H from the normal values are found by means of an instrument called a variometer. One type of variometer for finding changes in V consists of a magnetic 'needle', comprising two elliptical magnets carried by an aluminium block provided with quartz knife edges (Fig. 83) which rest on horizontal pivots. The centre of gravity of the needle is below the knife edges and hence the equilibrium is stable, as in an ordinary beam balance. The instrument is set so that the vertical plane in which the needle swings is perpendicular to the magnetic meridian; in consequence, the motion of the needle is uninfluenced by H. An increase or decrease in V causes an alteration in

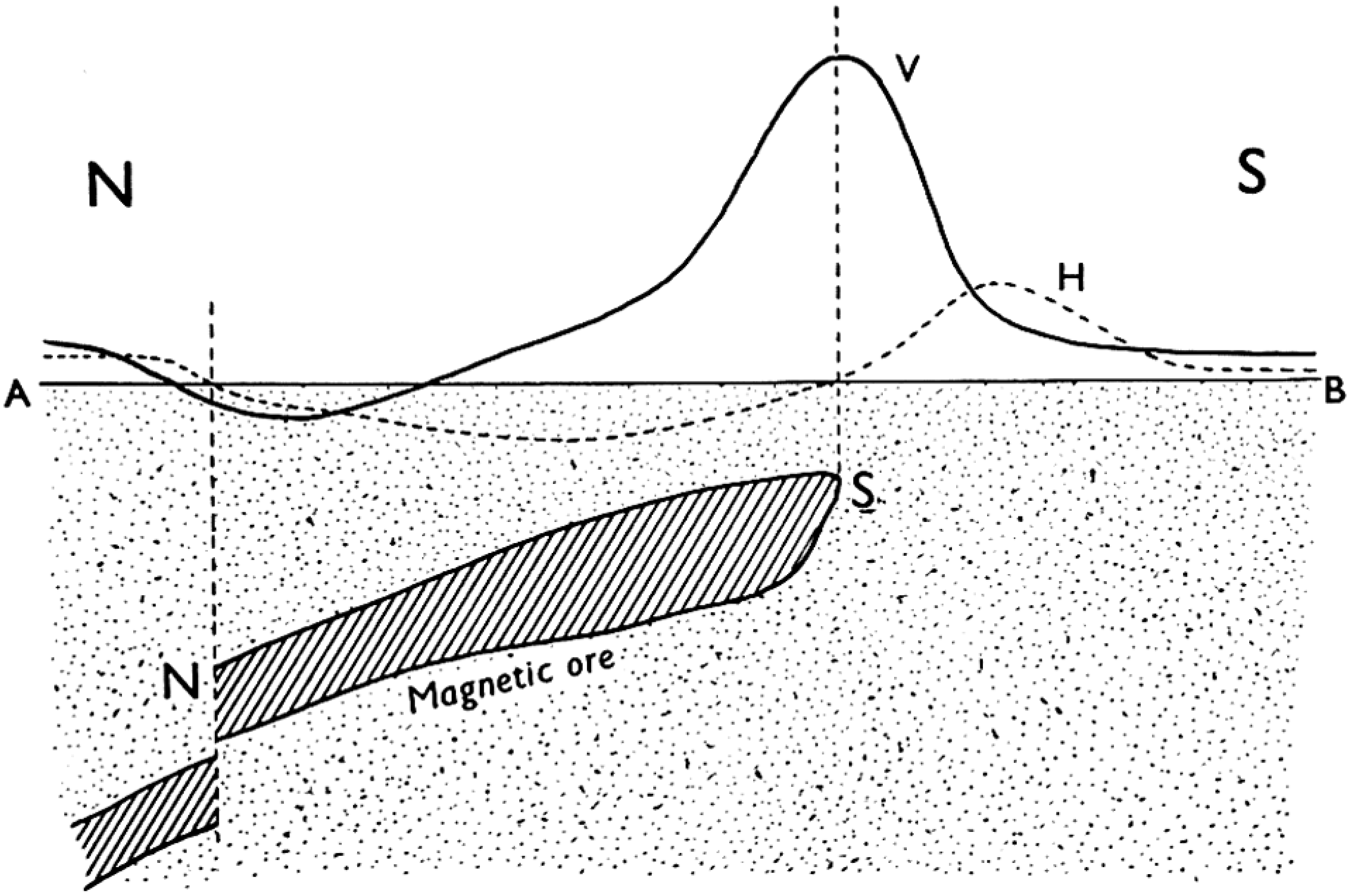


Fig. 82

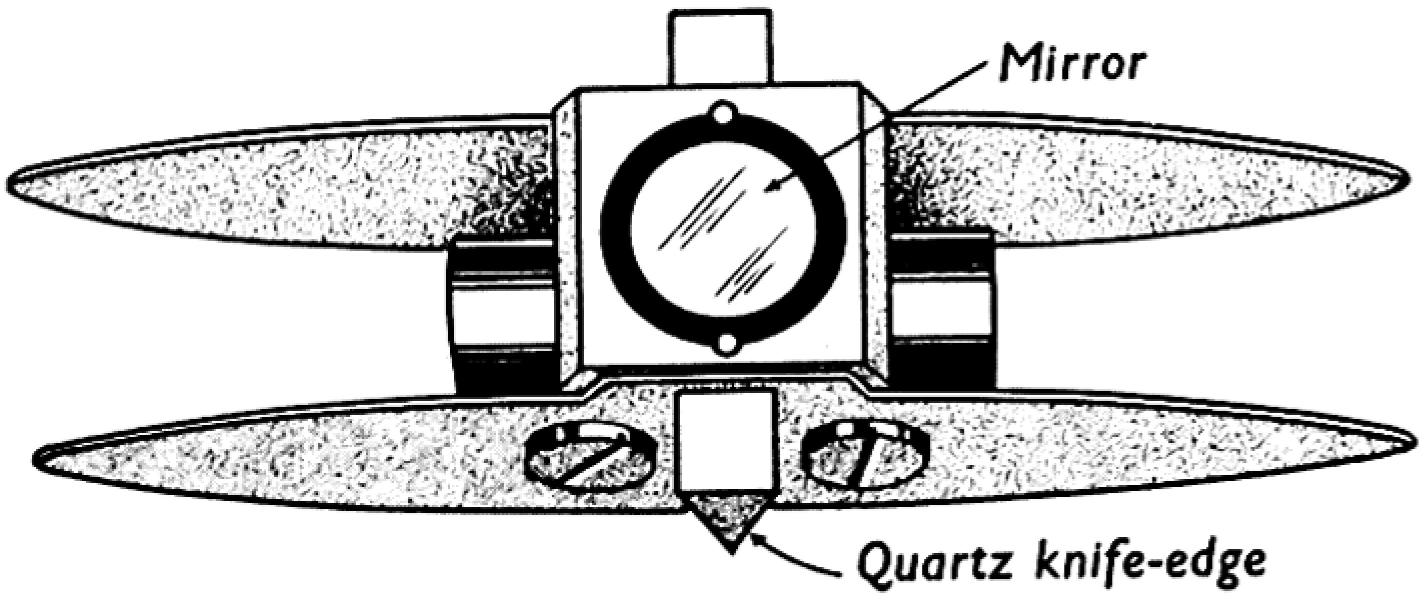


Fig. 83

the inclination of the needle to the horizontal; the angle is measured by reflecting a beam of light from a mirror attached to the needle. The sensitivity can be varied by altering the position of the centre of gravity by means of screws.

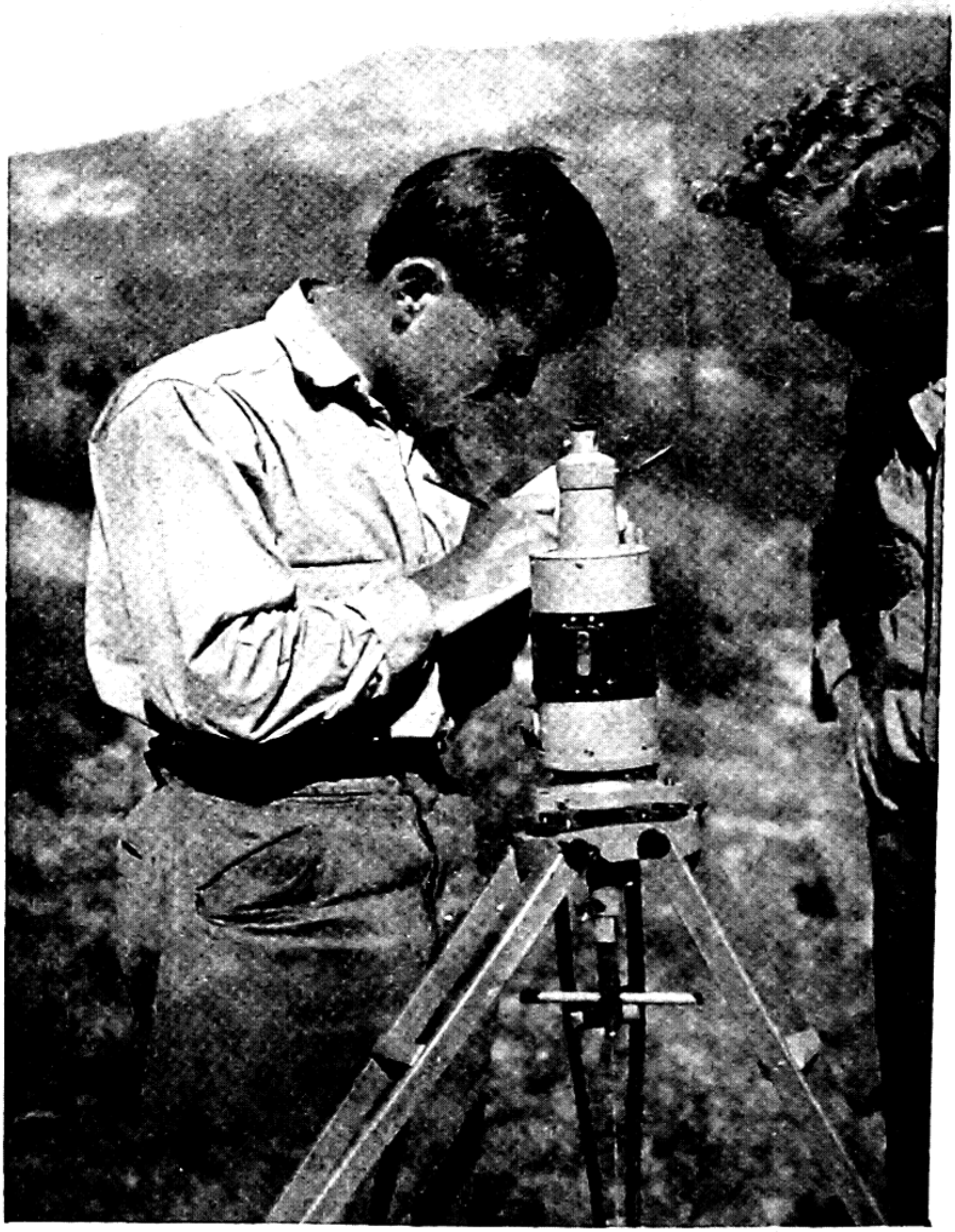


Fig. 84. Horizontal variometer being used in the field. Note the horizontal auxiliary magnet below the instrument.

The instrument is calibrated by fixing a short vertical magnet below the needle. The field at the needle due to the magnet is $2M/d^3$, where M

is the magnetic moment of the magnet and d is the distance of the centre of the magnet from the needle. If the magnet is reversed, the change in the field at the needle is $2\times 2M/d^3$. The difference in deflection with the magnet in the two positions is noted. Then

$$\text{Sensitivity} = \frac{4M}{d^3} \div \text{change in deflection.}$$

If during a survey there is a big variation in V, the sensitivity of the instrument may have to be altered constantly; three auxiliary magnets of different strengths are carried for the sensitivity measurements. The variations are referred to the value of H at a given station.

The horizontal variometer for measuring differences in H consists of a 'needle' similar to that in Fig. 83, but with the knife edges and centre of gravity arranged so that the needle is vertical when free to swing in the magnetic meridian. Changes in H cause the needle to make different small angles with the vertical.

The verticality of the needle is controlled, and the sensitivity is measured, by means of a horizontal magnet below the instrument, both magnet and needle being set in the magnetic meridian. Since the needle is in the broadside position with respect to this magnet, the field at the needle due to the magnet is M/d^3 (using the same nomenclature as before):

$$\text{Sensitivity} = \frac{2M}{d^3} \div \text{change in deflection when magnet is reversed.}$$

Changes in V will affect the deflection but to a much smaller extent than changes in H because the needle is nearly vertical; if V has been determined a correction for its effect can be made.

EXAMPLE

Assuming the magnetic field of the earth to be due to a small magnet at its centre, find an expression for the dip at a point, in terms of the angle, θ, which the radius to the point makes with the magnetic axis.

Suppose the magnet at the centre of the earth has a magnetic moment M (Fig. 85). It is equivalent to two magnets, of moments $M\cos\theta$ and $M\sin\theta$ along, and perpendicular to, OP respectively.

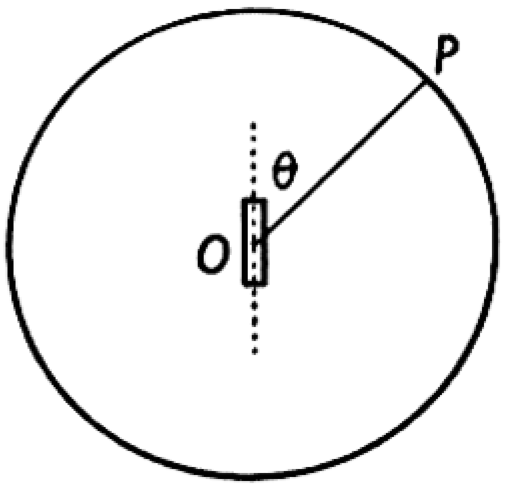


Fig. 85

$$\therefore \text{ Vertical component at } P = \frac{2M\cos\theta}{r^3}.$$

$$\text{Horizontal component at } P = \frac{M\sin\theta}{r^3}.$$

If ϕ is the angle of dip at P,

$$\tan\phi = \frac{2M\cos\theta}{r^3} \div \frac{M\sin\theta}{r^3}$$

$$= 2\cot\theta.$$

QUESTIONS

1. How would you attempt to verify experimentally the inverse square law of force between magnetic poles? (O.)

2. A *short* bar magnet lies in the magnetic meridian with its north pole pointing north. On plotting lines of force, neutral points are found 30 cm. from the centre of the magnet. How far from the centre will the neutral points be if the magnet is reversed? Draw rough diagrams to show where the neutral points are in each case. (C.)

3. A bar magnet of magnetic length 10 cm. is placed on a table so that a neutral point is found on its axis. This neutral point is found to be 15 cm. from the centre of the magnet at a place where the earth's horizontal field is 0·2 dyne per unit pole. Calculate the pole strength of the magnet. (N.)

4. A bar magnet is laid on a table. When the field is plotted neutral points are found on the line which bisects the axis of the magnet at right angles. Draw a map of the lines of force, marking clearly their directions and the direction of the magnetic meridian.

If the moment of the above magnet is 1000 c.g.s. units, the magnetic length 10 cm., and the horizontal intensity of the earth's field 0·18 dyne per unit pole, find the distance of the neutral points from the centre of the magnet. (N.)

5. A bar magnet lies on a table with its axis magnetic east and west. Draw the lines of force due to the combined field of the magnet and the earth, marking in the position of the neutral points.

If a neutral point is 18·0 and 14·5 cm. from the two poles of the magnet respectively, the magnetic length of the magnet is 6·0 cm. and the horizontal component of the earth's magnetic field is 0·18 oersted, find the pole strength of the magnet.

6. A magnet is placed with its N. pole at the midpoint E of the side CD of a rectangle $ABCD$, and its S. pole at D. Calculate the strength and direction of the field at B, if $AB=2BC$. (O. S.)

7. Two equal cobalt-steel magnets are of effective length 5 cm. and weight 10 g. One is fixed horizontally, and the other placed directly above the former, like pole over like; the latter can move freely in a vertical direction, but is constrained so that it cannot move in any other direction. In equilibrium the axis of the magnets are 2 cm. apart. Calculate their moments. ($g=981$ cm.sec.$^{-2}$.) (O. & C.)

8. Explain the method of representing the direction and intensity of a magnetic field by a system of lines of force. A bar magnet, 20 cm. in length, is placed vertically with its N. pole pointing downwards. Indicate by a diagram the nature of the magnetic field in a horizontal plane through the lower pole of the magnet. If a magnetometer, placed in this plane 15 cm. due E. of the lower pole, is deflected though 30°, calculate the pole strength of the magnet. ($H=0{\cdot}2$ oersted.) (C.)

9. Two *short* magnets of equal moment are placed on a table with the line joining their midpoints along the axis of one and perpendicular to the other. The earth's field is perpendicular to this line. When a magnetometer needle is placed midway

between the midpoints of the magnets it is deflected 45° from the meridian. If the distance between the midpoints of the magnets is 20 cm. and the horizontal component of the earth's field is 0·18 dyne per unit pole, what is the value of the moment of each magnet? Is there more than one answer to the question? (N.)

10. A deflection magnetometer is set up with its arms pointing magnetic N.E. and S.W. A short bar magnet is placed to the N.E. of the needle with its axis along the arm and with its S. pole pointing towards the needle. A deflection of 10° is observed. The arms are now set in the usual E. and W. position, and the same magnet is placed with its centre half as far from the needle as before, and with its axis along the arm. Find the deflection of the needle. (N.)

11. A sensitive balance has a steel pointer which is a magnet. How, if at all, does this affect the accuracy of the balance? Could you find the magnetic moment of the pointer without removing it from the beam of the balance? If so, how? (L.)

12. A magnet is suspended by a wire so as to move in a horizontal plane. In order to deflect the magnet through 30° from the magnetic meridian the upper end of the suspension has to be twisted through 90°. Find the angle through which the upper end of the wire must be twisted so that the magnet may be turned through 90° from the magnetic meridian. N.B. The couple required to produce a twist θ in the wire is proportional to θ. (D.)

13. A bar magnet, pivoted so that it can swing freely with its axis in a horizontal plane, is placed in a uniform, horizontal magnetic field. Calculate the couple required to deflect it through an angle θ from the direction of this field. If the magnet is released when in this position, what will be the maximum kinetic energy it will acquire, and in what units will it be measured?

14. Two short bar magnets have their centres at points A and B. The magnet whose centre is at A has its axis along AB, and that with centre at B has its axis perpendicular to AB. On the assumption that the lengths of both magnets are very much less than the distance AB, calculate (*a*) the couple which magnet A exerts on magnet B, (*b*) the couple which magnet B exerts on magnet A. Comment on your result. (C. S.)

15. A very small bar magnet is suspended so that it can oscillate freely in a horizontal plane. What factors determine its period of oscillation and how does the period depend on each factor?

Describe briefly what improvements would make the above apparatus into a useful vibration magnetometer.

How would you use a vibration magnetometer to compare the intensities of the magnetic fields at two places in the laboratory? Give experimental details and show how you would calculate the result. (N.)

16. The needle of a compass needle consists of a small magnet of moment of inertia 0·1 g.cm.^{2} When the compass needle is placed on the axis of, and 10 cm. from, one pole of another magnet, it oscillates twice per second if given a small angular displacement and released. If the poles of the second magnet are 10 cm. apart and of strength 200 c.g.s. units, what is the magnetic moment of the compass needle? (The earth's field may be ignored.) (C. S.)

17. A long magnet of pole strength 18 c.g.s. units, consists of a magnetised steel rod provided with spherical ends of soft iron. It is placed vertically with its N. pole

uppermost. A small compass needle which can oscillate freely in a horizontal plane is placed at points *A*, *B*, *C*, *D*, which are all 10 cm. from, and lie on a horizontal plane through, the N. pole of the magnet and are respectively N., E., S., W. of that pole. If $H=0{\cdot}18$ dyne per unit pole and the compass needle makes 10 oscillations per minute when in the earth's field alone, calculate the number of oscillations per minute when it is placed at *A*, *B*, *C*, *D*. (D.)

18. A short bar magnet, of magnetic moment 2000 units, was placed in the magnetic meridian with its centre 20 cm. east of a vibration magnetometer which then made 60 oscillations per minute. When the magnet was reversed, the magnetometer needle became reversed and then made 20 oscillations per minute. Calculate the value of *H*, the horizontal component of the earth's magnetic field. (C.)

19. How would you compare the moments of two magnets of the same dimensions by using (*a*) a deflection method, (*b*) a vibration method? Give the necessary experimental details. (O. & C.)

20. A short bar magnet in the end-on position produces a deflection of 14° when 20 cm. from a reflecting magnetometer. The same magnet makes 3 oscillations per minute when vibrating horizontally in the earth's magnetic field. Find the magnetic moment of the magnet if its moment of inertia is 300 c.g.s. units. (C.)

21. A bar magnet is freely suspended and swings in a uniform magnetic field. Show that the oscillations are simple harmonic and derive an expression for the periodic time.

What do you think would happen, in such a system, if the magnetic field were reduced from a value *H* to zero? Explain why your answer is consistent with conservation of energy.

If you had available a series of iron bar magnets of different lengths, all of intensity of magnetisation (I) equal to 20 e.m.u. and you wished to use them for making an accurate determination of the earth's magnetic field ($H=0{\cdot}2$ e.m.u.) in 1 hour, what length of magnet would you choose? Assume that the density of iron is $8{\cdot}7$ g.cm.$^{-3}$ and the square of the radius of gyration of a bar of length L is $\frac{1}{3}L^2$ when swinging about its centre.

Would you choose a different size if you had 24 hours available? (C. S.)

22. Explain what is meant by the *magnetic moment* of a magnet, and discuss the advantage gained by introducing this conception.

Two short bar magnets, of magnetic moment 60 and 80 units, can be fastened together with their axes horizontal and their centres in a vertical line about which the system is free to oscillate. Compare the times of oscillation in the earth's field when the axes of the magnets are (*a*) in the same direction, (*b*) in opposite directions, (*c*) at right angles. (O. & C.)

23. Give an account of the way in which the earth's magnetism varies (*a*) in place, and (*b*) in time. (O. & C.)

24. Describe the main features of the earth's magnetic field.

Part of the field may arise from vertical electric currents in the earth's atmosphere. How could the field from such causes be differentiated from that arising from subterranean and celestial origins? (O. S.)

25. What is meant by saying that the horizontal component of the earth's magnetic field, at a given place, is 0·2 oersted?

How would you determine the horizontal component of the earth's magnetic field? (O. & C.)

26. The true value of the angle of dip at a certain place is 68° 24′. If the plane of the dip circle is turned through 45° from the meridian, what will be the apparent angle of dip? (D.)

27. A dip circle, when set in turn in two perpendicular vertical planes, registers dips of α_1 and α_2. Find an expression for the true angle of dip. (C.)

28. A short bar magnet, of magnetic moment 2000 c.g.s. units, is placed with its N. pole pointing north at a distance of 100 cm. due S. of a dip circle so that its axis is horizontal and passes through the centre of the circle. If the undisturbed reading of the dip circle was 70°, what will the new reading be? The horizontal component of the earth's field may be taken as 0·20 oersted. (C.)

29. An unmagnetised compass needle of mass 10 g. is freely pivoted about the point of a vertical support 1 mm. above its centre of mass, so that it takes up a horizontal position. It is then magnetised so that its magnetic moment is 200 c.g.s. units. Find its inclination to the horizontal at a place where the horizontal magnetic field is 0·18 oersted, and the vertical 0·43 oersted. (O. & C.)

30. Describe the precautions necessary in measuring magnetic dip.

If a dip needle, whose plane of rotation is in the magnetic meridian, is set in vibration and makes 30 small oscillations in 50 seconds at A where the dip is 60°, and 25 oscillations in 40 seconds at B where the dip is 70°, find the ratio of the vertical component of the earth's magnetic field at A to that at B. (L.)

31. A very short bar magnet of moment M is placed at a point O with its axis horizontal and perpendicular to the magnetic meridian. If P is a neutral point, show that the angle between OP and the axis of the magnet is $\tan^{-1}\sqrt{2}$, and the distance OP is $2^{\frac{1}{6}}(M/H)^{\frac{1}{3}}$, where H is the horizontal component of the earth's magnetic field. (O. & C.)

32. Show that, if the force between magnetic poles were inversely proportional to the nth power of the distance between them, the strength of the field on the axis of a short bar magnet would be n times that at an equal distance from the centre of the magnet on the perpendicular bisector of the axis.

33. Magnetic potential is exactly analogous to electric potential. Hence define magnetic potential and state the value of the magnetic potential at a distance of r cm. from a point magnetic pole of m c.g.s. units.

Show that the magnetic potential at a point distant d from the centre of a bar magnet along the axis is $M/(d^2-l^2)$, where M is the magnetic moment of the magnet and $2l$ is its magnetic length.

CHAPTER 5

ELECTROMAGNETISM

An electric current creates a magnetic field. The lines of magnetic force due to an electric current in a straight wire consist of concentric circles round the wire (Fig. 86), whose positive direction is given by *Maxwell's corkscrew rule. Imagine a corkscrew screwed along the wire in the direction of the current. The direction in which the thumb rotates is the positive direction of the lines of magnetic force.*

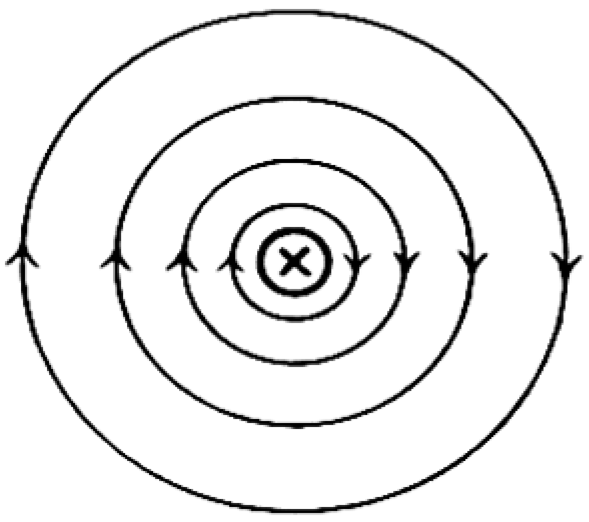


Fig. 86. Magnetic field due to a current in a straight wire passing down into the paper.

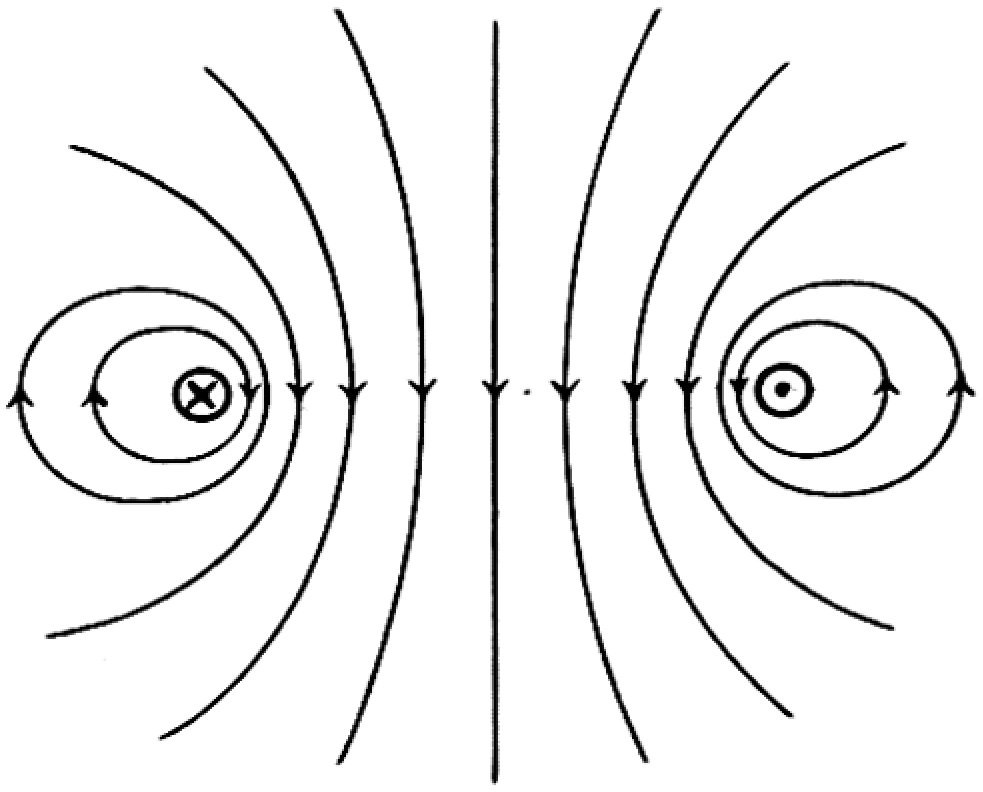


Fig. 87. Magnetic field due to a current in a circular coil. The current is passing down into the paper on the left and up out of the paper on the right.

A current flowing in a circular coil creates a magnetic field, a plane section of which is shown in Fig. 87. The field is similar to that of a flat magnet, and each face of the coil behaves like a magnetic pole (Fig. 88). The field at the centre is at right angles to the plane of the coil.

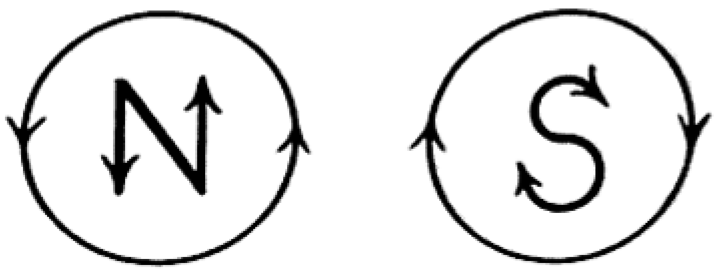


Fig. 88

Strength of the magnetic field due to a current

To investigate most simply the strength of the magnetic field due to a current a circular coil is used, since every point on it is equidistant from the centre. The strength of the magnetic field at the centre may be measured with either a deflection or vibration magnetometer.

It is found that, if the same current, i, is passed through coils of 1 turn and n turns of the same radius, it produces at the centres magnetic fields in the ratio $1:n$. With n turns the current may be regarded as ni units in one turn, and hence the magnetic field $H \propto i$.

By taking different lengths l of wire in a coil of fixed radius (not necessarily an integral number of turns) and the same current, it can be shown that $H \propto l$. Again, using coils of different radius r with the same values of i and l, it can be shown that $H \propto 1/r^2$. Thus

$$H \propto \frac{il}{r^2},$$

i.e. $$H = A\frac{il}{r^2}, \text{ where } A \text{ is a constant.}$$

If the unit of current is such that $H=1$ when $i=1$, $l=1$, and $r=1$, then $A=1$ and

$$\mathbf{H} = \frac{\mathbf{il}}{\mathbf{r^2}}.$$

Definition of electromagnetic unit of current

The electromagnetic unit of current is that current which, flowing in an arc 1 *cm. long, of a circle of radius* 1 *cm., creates a magnetic field of* 1 *oersted at the centre of the circle* (Fig. 89).

The electromagnetic unit of current, sometimes called the abampere (an abbreviation of 'absolute ampere'), is a large unit. The practical unit, the ampere, is defined as $\frac{1}{10}$ electromagnetic unit.

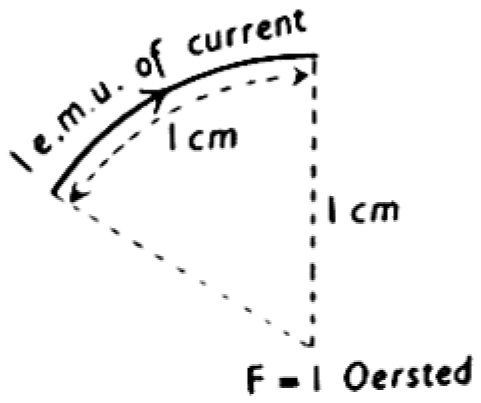


Fig. 89

The tangent galvanometer

The tangent galvanometer was historically the first instrument for the absolute measure of electric current. It is an absolute instrument in the sense that it does not require calibration by another instrument.

It consists of a circular coil in which flows the current to be measured (Fig. 90); the strength of the magnetic field due to the current at the centre of the coil is compared with the horizontal component of the earth's magnetic field by means of a deflection magnetometer.

The galvanometer must be set before use with the plane of its coil in the magnetic meridian so that the magnetic field due to the current at the centre is at right angles to the earth's field. Then, as proved on p. 61, if H is the strength of the magnetic field due to the current at the centre of the coil, H_e is the horizontal component of the earth's magnetic field and θ is the deflection of the magnetometer,

$$H = H_e \tan\theta.$$

But
$$H = \frac{il}{r^2}$$
$$= \frac{2\pi rni}{r^2},$$

where n is the number of turns and r the radius of the coil.

$$\therefore \frac{2\pi ni}{r} = H_e \tan \theta,$$
$$i = \frac{H_e r}{2\pi n} \tan \theta \text{ e.m.u.}$$
$$= \frac{10H_e r}{2\pi n} \tan \theta \text{ amperes.}$$

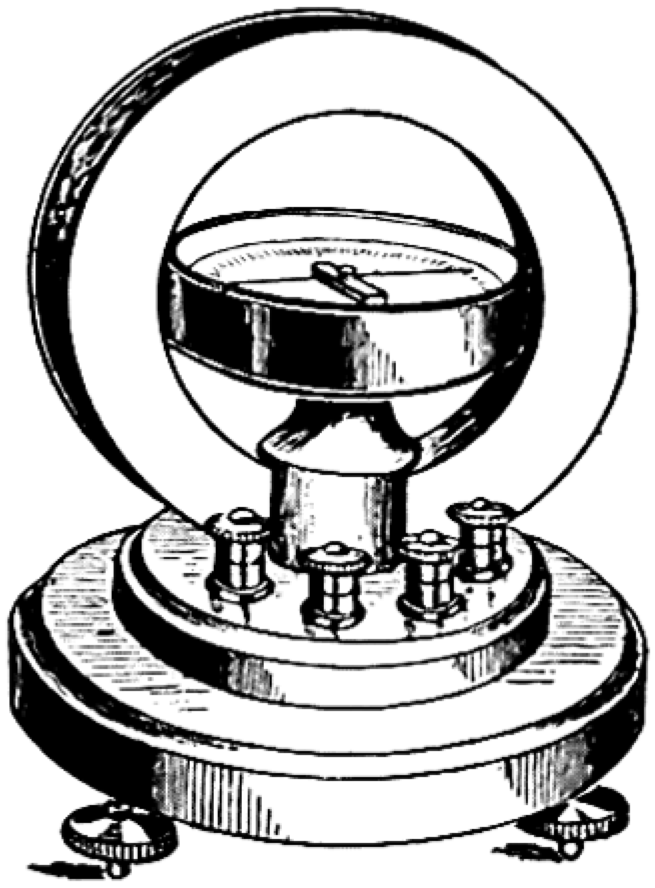


Fig. 90

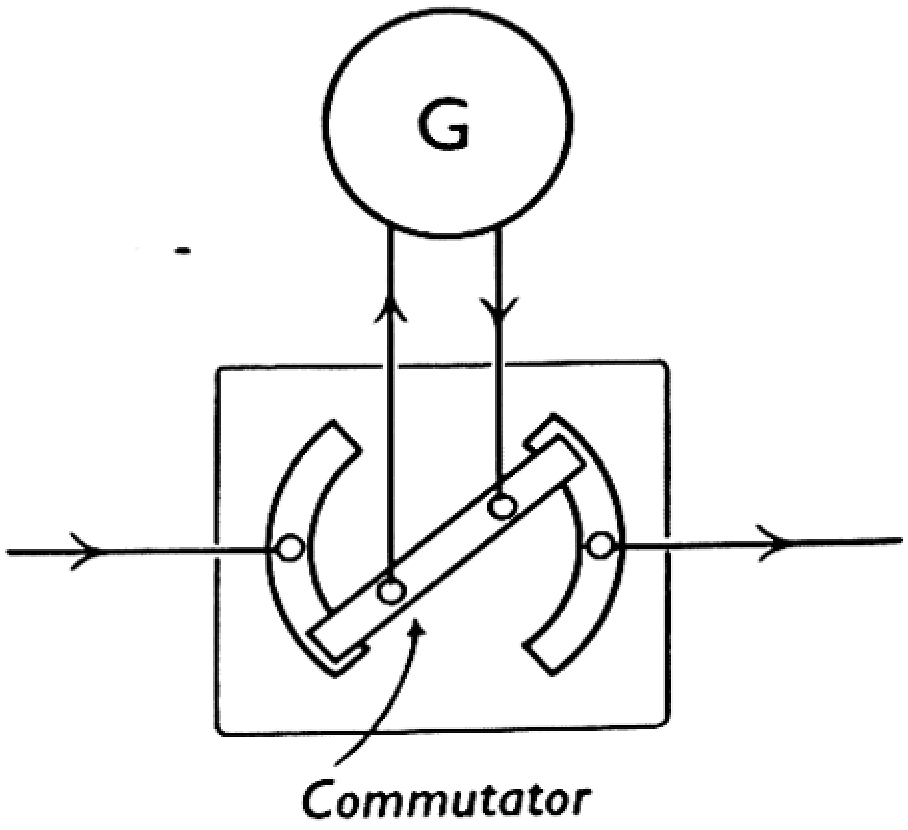


Fig. 91

This is often written
$$i = k \tan \theta, \quad \text{where } k = \frac{10H_e r}{2\pi n};$$

k is called the reduction factor of the galvanometer.

When in use the galvanometer should be connected to a commutator (Fig. 91), so that the current can be reversed in the coil without changing the direction of the current in the rest of the circuit. By taking readings of the deflection when the current is flowing in opposite directions through the coil, various errors, for example, that due to imperfect setting of the coil in the magnetic meridian, may be eliminated.

The force on a wire carrying a current in a magnetic field

A wire carrying a current at right angles to a magnetic field experiences a force at right angles to itself and to the field. This can be demonstrated as in Fig. 92.

The direction of the force can be predicted by means of *Fleming's left-hand rule*. Hold the thumb and first two fingers of the left hand mutually at right angles (Fig. 93). Then if the *f*irst finger points in the direction of the *f*ield, the se*c*ond finger in the direction of the *c*urrent, the thu*m*b will point in the direction of *m*otion (force).

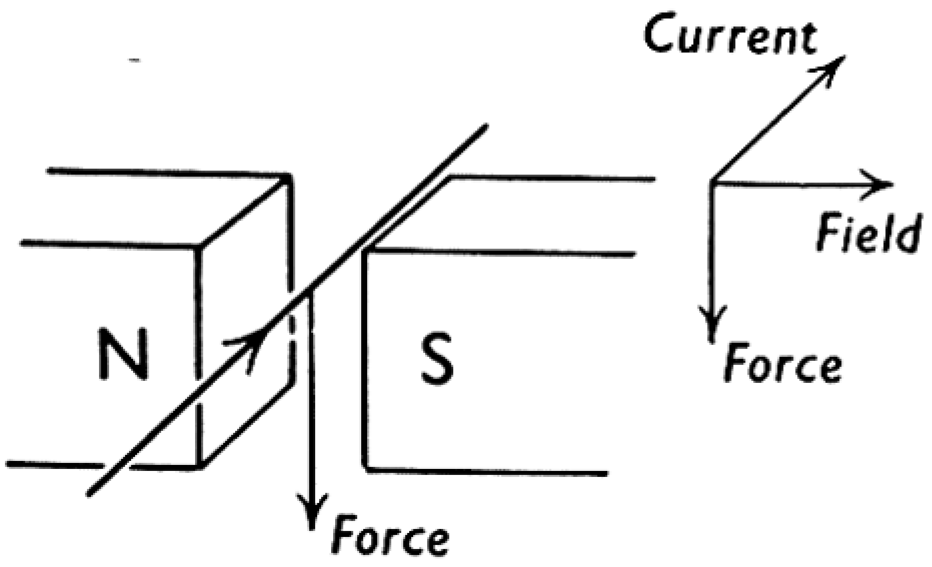


Fig. 92. Force on a wire in a magnetic field due to a current in the wire.

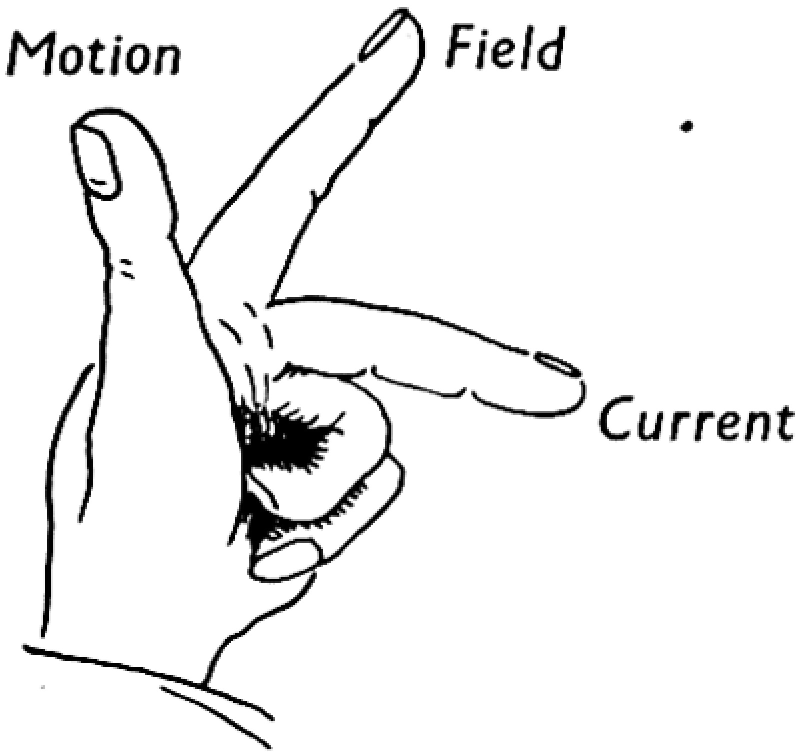


Fig. 93

If the magnetic field is not at right angles to the current, the component of the field at right angles to the current will be effective in producing the force. A current parallel to a magnetic field experiences no force.

The force can be explained as due to the interaction of the magnetic field due to the current and of the external magnetic field (see Fig. 94), assuming that lines of magnetic force are in tension and tend to shorten.

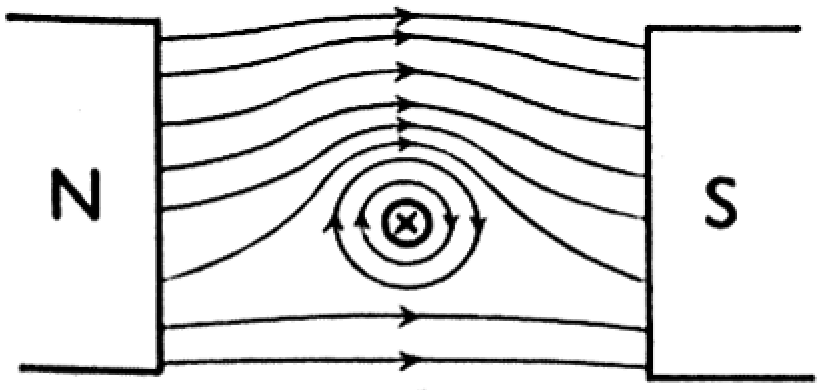


Fig. 94. Combined magnetic field due to a current flowing through a straight wire into the paper and also to the N. and S. poles.

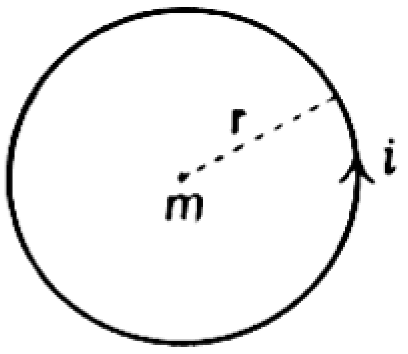


Fig. 95

Magnitude of the force on a wire carrying a current

A current i e.m.u. flowing in a single circular coil of radius r cm. creates at the centre a magnetic field of strength $2\pi i/r$ oersted (see p. 86). A circular coil has the advantage that every point on it is equidistant from the centre.

If a pole of strength m c.g.s. units is placed at the centre (Fig. 95) it will be acted upon by a force equal to the product of the pole and the field strength, $2\pi im/r$ dynes.

Since action and reaction are equal and opposite (Newton's third law of motion), the pole will exert an equal and opposite force on the wire.

Force on each cm. of wire

$$= \frac{2\pi im}{r} \div 2\pi r = i\frac{m}{r^2}.$$

But m/r^2 = field at the coil due to the pole $= H$.

∴ Force on each cm. of coil $= Hi$ dynes.

Note that since the magnetic field due to the pole is radial, the coil is everywhere at right angles to the field.

We can generalise this expression and state that a straight wire of length l cm., at right angles to a uniform magnetic field of strength H oersted, and carrying a current i e.m.u., experiences a force of

Hil dynes.*

If i is expressed in amperes the expression becomes

$$\frac{Hil}{10} \text{ dynes.}$$

Force on an electron moving in a magnetic field

An electric current in a wire is conventionally regarded as a flow of positive charge, although it consists in fact of a flow of negative electrons in the opposite direction.

Suppose an electron of charge e e.m.u. is moving with velocity v cm. per sec. at right angles to a magnetic field of strength H oersted.

The electron moves a distance l cm. in a time t sec. where $t = l/v$ sec., and constitutes a current i e.m.u.

Current = flow of charge per second.

$$\therefore\ i = \frac{e}{t} = \frac{e}{l/v} = \frac{ev}{l}.$$

$$\therefore\ il = ev.$$

But force on a current $= Hil$ dynes.

∴ Force on a moving electron $= Hev$ dynes.

Fig. 96 represents the direction of the force on the electron which should be verified by Fleming's left-hand rule, remembering that the direction of the positive current is opposite to the direction of motion of the electron.

---

* If the wire is in a magnetic medium, the expression should be Bil, where B is the magnetic induction or flux density (see p. 118).

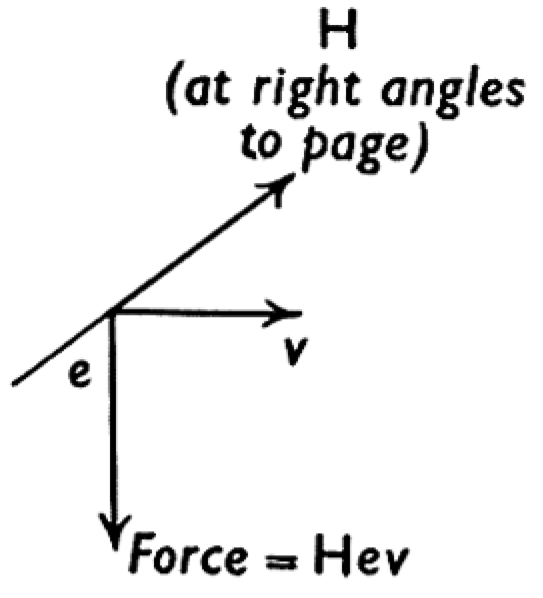


Fig. 96

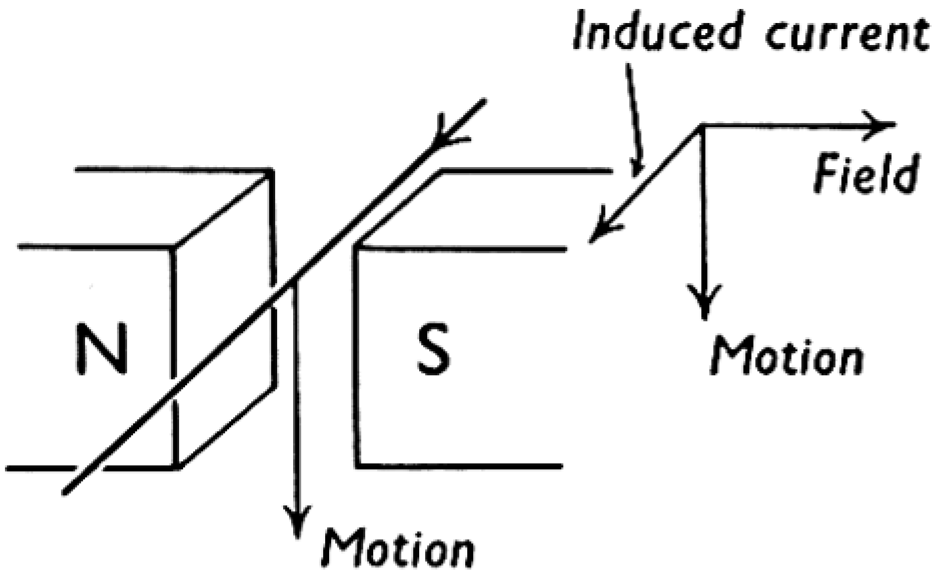


Fig. 97. Induced current in a wire due to its motion in a magnetic field.

Electromagnetic induction

When a wire is moved in a magnetic field as in Fig. 97 so that it cuts lines of magnetic force, an induced electromotive force is set up in it, and if the circuit is complete an induced current flows. The phenomenon is known as electromagnetic induction, and was discovered by Faraday after ten years of repeated efforts to 'convert magnetism into electricity'. The induced current flows only while the wire is moving.

Had it been known that an electric current consists of a flow of electrons the phenomenon could have been predicted. When the wire and its contained electrons are moved down, the vertical movement of the electrons is equivalent to a vertical current. The electrons will therefore experience a force at right angles to the vertical and to the magnetic field, i.e. along the wire.

Magnitude of the induced E.M.F.

Suppose the wire in Fig. 97 is being moved down with a velocity v cm. per sec. and the strength of the magnetic field is H oersted. The force on each electron is Hev dynes.

Suppose the wire consists of a short length l cm. in the middle of the field. Electrons will be urged to one end of the wire until the opposing electric field prevents further motion of the electrons.

Let the induced electromotive force or potential difference between the ends of the wire be E e.m.u.

Then the electric field in the wire is E/l and the force on an electron due to this field is $(E/l)\,e$ dynes. Thus when equilibrium is attained,

$$\frac{E}{l}e = Hev.$$

$$\therefore\ E = Hvl.$$

But vl represents the area swept out by the wire in 1 sec. and H equals the number of lines of magnetic force per unit area. Thus *the induced E.M.F. is equal to the number of lines of magnetic force cut per second.*[*]

The E.M.F. calculated in this way is in e.m.u. This can be converted to volts from the fact that 1 volt $= 10^8$ e.m.u. (see p. 168).

Thus

$$E = Hvl \times 10^{-8} \text{ volts.}$$

The conception of cutting of lines of force is due to Faraday. No induced E.M.F. is set up if a conductor moves along lines of magnetic force—only if it cuts across them.

Note that, as the wire in Fig. 97 moves down, the positively charged atoms of the wire which have lost electrons also experience a force but in a direction opposite to the electrons. However, they cannot move in the wire, since they constitute the body of the wire, so that no variation in potential is set up by them.

The direction of the induced current

The direction of the induced current is such that it tends to oppose the motion of the wire. This fundamental law, known as Lenz's law, is a consequence of the Principle of the Conservation of Energy. If the induced current in Fig. 97 tended to urge the wire down, instead of up, a perpetual motion machine could be constructed.

Thus the currents in Figs. 97 and 92 are in opposite directions. A right-hand rule, also due to Fleming, instead of a left-hand rule, is used to predict the direction of the E.M.F. The thumb and first two fingers of the right hand are held mutually at right angles. Then if the *f*irst finger points in the direction of the *f*ield and the thu*m*b in the direction of *m*otion, the se*c*ond finger will point in the direction of the induced *c*urrent or E.M.F. The reader is reminded that the direction of the conventional positive current determined by this rule is opposite to the flow of the (negative) electrons.

Summary of the forces on the electrons in the wires in Figs. 92 *and* 97

Fig. 92. The electrons are caused to move along the horizontal wire by a source of E.M.F.—a battery. They therefore experience a vertical force. The wire moves down as a result of this force, and the electrons now also constitute a vertical current in space; they are therefore urged along the wire in a direction opposite to the E.M.F. of the battery. The battery must do work to drive the electrons against this back E.M.F. and the work it does, by the Principle of the Conservation of Energy, must be equivalent to the work done in moving the wire.

---

[*] If the wire is in a magnetic medium, the term 'lines of magnetic induction or flux' should replace 'lines of magnetic force' (see p. 118).

Fig. 97. The wire is moved down by an external force, and the electrons in it constitute a vertical current. They are therefore urged along the wire, and their motion along the wire constitutes an induced current. Since they are now moving along the wire they experience a vertical force which is in a direction opposite to that of the motion of the wire. The work done in pushing the wire down against this resisting force is equivalent to the electrical energy of the induced current.

The circuital law

A magnetic pole tends to revolve in a circle round a straight wire carrying a current, following a line of magnetic force.

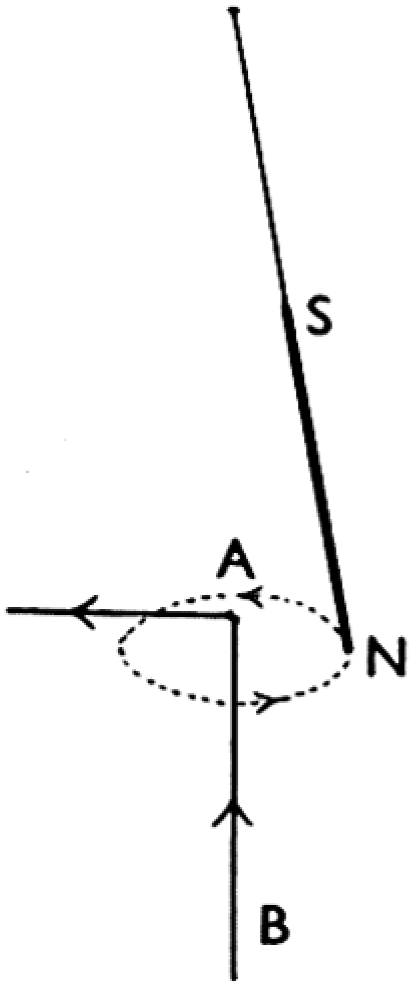


Fig. 98

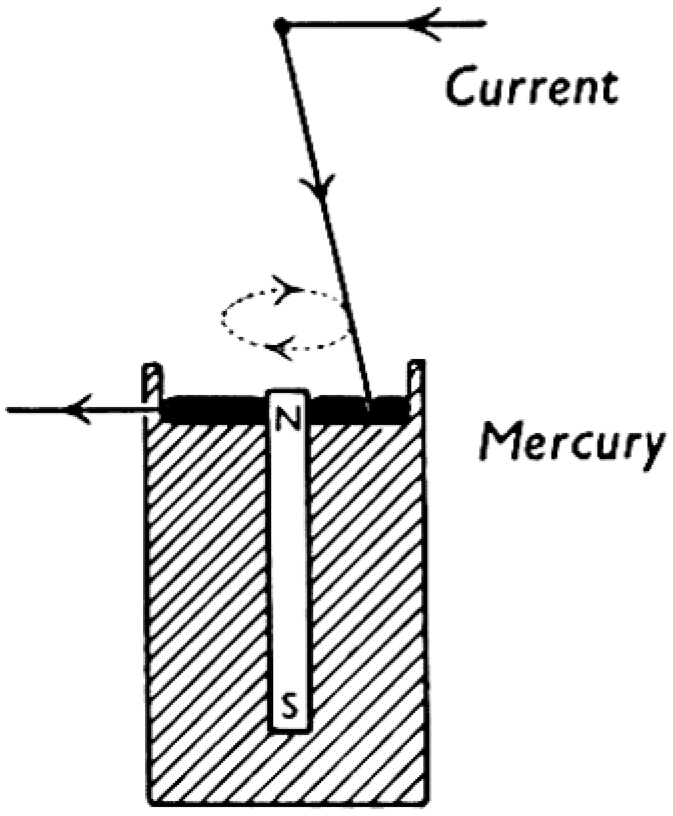


Fig. 99

This can be demonstrated as in Fig. 98 by suspending a magnetised knitting needle from a thread so that its N. pole is near to a vertical wire *BA* carrying a current. The contact at *A* is made by touching the vertical wire with the horizontal wire, and this contact must be broken temporarily, when the needle comes round, to allow the latter to rotate. The vertical wire cannot be continued upwards much farther than in the figure because the S. pole of the knitting needle would tend to rotate in a direction opposite to that of the N. pole.

As the pole rotates its lines of magnetic force are cut by the wire, thereby setting up an induced back E.M.F. The battery supplying the current must do work in driving the current against this back E.M.F., and this is equal to the work done in driving the magnetic pole round the wire.

If the pole is forced round the wire in a direction opposite to that in which it tends to rotate, work must be done by an external force. The induced E.M.F. now aids the battery.

The work done when a unit pole moves in a closed path completely round a wire carrying a current i e.m.u. is $4\pi i$ *ergs.* The operation is sometimes called 'threading the circuit' and the work done is called the magnetomotive force, by analogy with electromotive force, which is the work done when a unit charge moves along a path.

A unit magnetic pole has 4π lines of force and all these lines are cut by a wire when the pole rotates completely round the wire. Suppose the time taken is t sec.

Induced E.M.F. in the wire, E,

$$= \text{Number of lines of force cut per sec.}$$

$$= \frac{4\pi}{t} \text{ e.m.u.}$$

$$\text{Work done by battery} \quad = Eit \quad \text{(see p. 128)}$$

$$= \frac{4\pi}{t} it$$

$$= 4\pi i \text{ ergs.}$$

Thus the work done in moving the unit pole round the wire is $4\pi i$ ergs.

So long as the pole moves in a closed path the actual length and nature of the path makes no difference to the net work done.

If the magnetic pole is fixed and the wire is free, then the wire will tend to rotate round the pole. This can be demonstrated as in Fig. 99.

The circuital law is useful for deducing the strengths of magnetic fields.

Magnetic field due to a current in a long straight wire

Suppose a wire is straight and very long so that its magnetic field is symmetrical when a current, i e.m.u., flows through it.

Take a unit pole round the wire in a circle of radius a having the wire for its centre (Fig. 100).

The magnetic field H oersted will be constant along this path.

$$\text{Force on the pole} = H \text{ dynes.}$$

$$\therefore \text{ Work done} = H \times 2\pi a$$

$$= 4\pi i \quad \text{circuital rule.}$$

$$\mathbf{H} = \frac{\mathbf{2i}}{\mathbf{a}} \text{ oersted.}$$

Note that if i is in amperes the expression becomes $2i/10a$ oersted.

That the field is inversely proportional to the distance from the wire can be verified experimentally as in Fig. 101. A magnet rests on a horizontal cardboard disc which is suspended by threads and is free to rotate about a vertical wire carrying a current. The disc shows no tendency to rotate because the moments of the forces on the N. and S. poles of the magnet are equal and opposite. If F is the force on a pole at distance a from the wire, the moment of the force is Fa. Since the experiment shows that the moment is independent of a, F must be proportional to $1/a$.

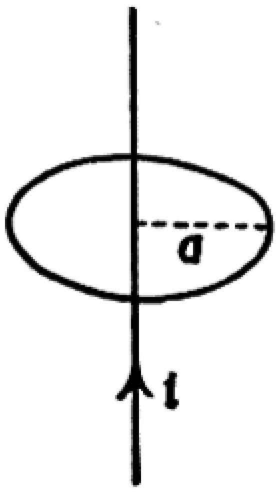


Fig. 100

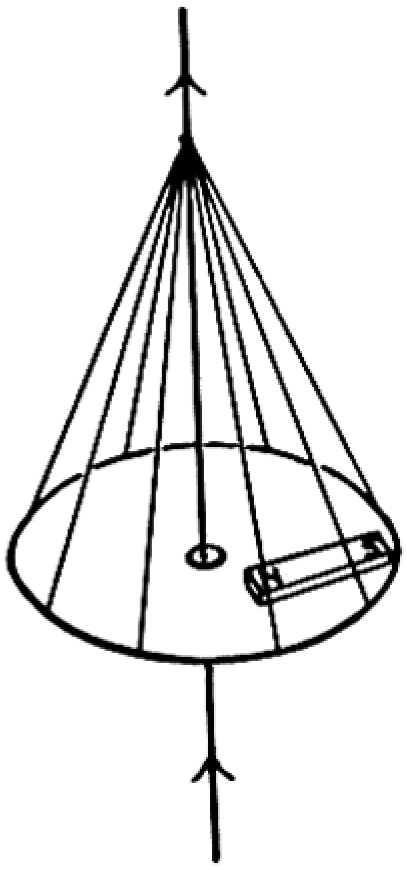


Fig. 101

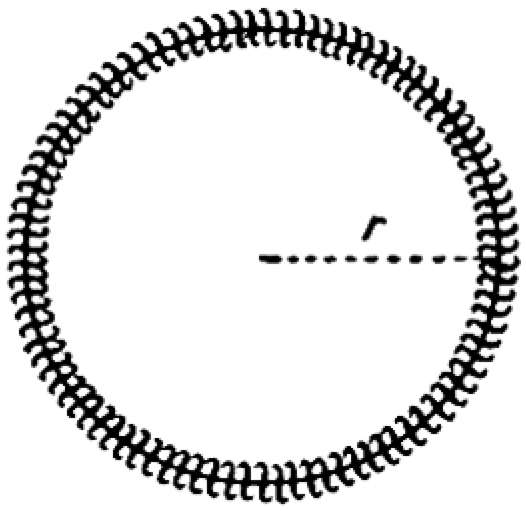


Fig. 102

Magnetic field inside an endless solenoid

Fig. 102 represents an endless solenoid in which a current i e.m.u. flows. Suppose it has n turns *per cm. length*, and its mean radius is r.

Let the field on the circular path inside the solenoid shown in Fig. 102 be of strength H oersted. It will exert a force H dynes on a unit pole.

Work done in taking unit pole round the circular path

$$= 2\pi r H \text{ ergs.}$$

The total number of turns threaded by the pole is $2\pi rn$.

Hence by the circuital rule,

$$\text{Work done} = 4\pi i \times 2\pi rn \text{ ergs,}$$

$$\therefore \; 2\pi rH = 4\pi i \times 2\pi rn,$$

$$H = 4\pi ni \text{ oersted.}$$

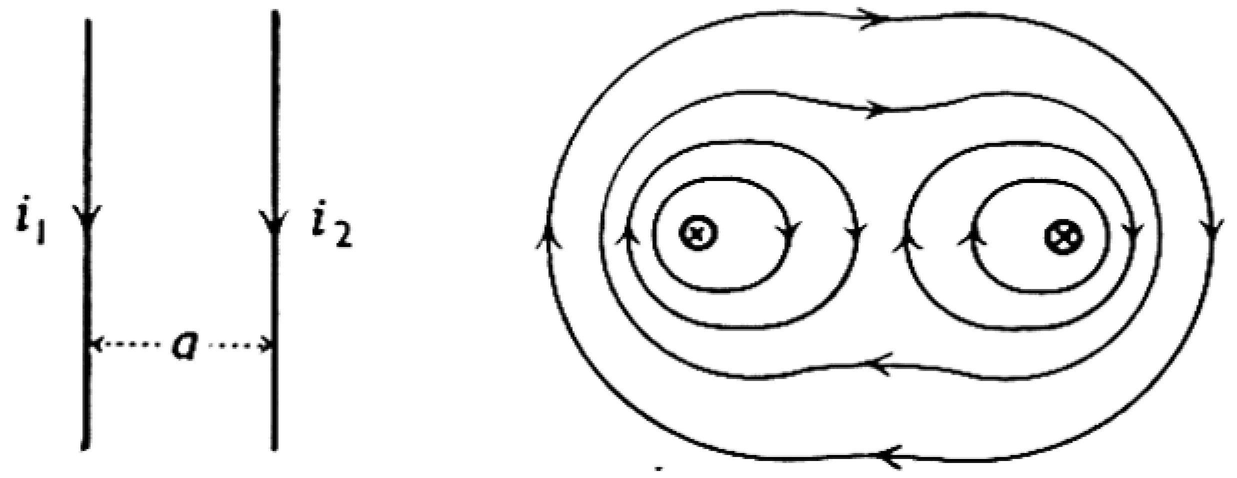
H is independent of r, and if r becomes infinite the solenoid becomes straight.

Hence the field inside an infinite straight solenoid

$$= \mathbf{4\pi ni} \text{ oersted.}$$

If i is in amperes, $H = \dfrac{4\pi ni}{10}$ oersted.

The field at a point inside a long solenoid may be considered as due to the two halves of the solenoid on either side of it. If one-half of the solenoid is removed the field at the exposed end will become $2\pi ni$ oersted. Thus the field at the end of a long solenoid is $2\pi ni$ oersted.



(a) (b)

Fig. 103. Attraction of two like currents.

Force between two long straight wires

Two parallel currents flowing in the same direction attract one another, and flowing in opposite directions repel. This can be understood from the combined magnetic field (see Fig. 103 (*b*)), assuming the lines of force to be in tension and to repel each other laterally. It can also be deduced from Fleming's left-hand rule.

Suppose two long parallel wires carrying currents i_1 and i_2 e.m.u. are at a distance a apart.

$$\text{Field due to } i_1 \text{ at distance } a = \frac{2i_1}{a}.$$

This field is at right angles to the current i_2.

$$\therefore \text{ Force per cm. on wire carrying } i_2 = \frac{2i_1 i_2}{a} \text{ dynes.}$$

The force on the other wire is equal and opposite.

If i_1 and i_2 are in amperes the formula becomes $\dfrac{2i_1 i_2}{100a}$ dynes.

Force between two circular coils

The force between two identical coils which are close together (see Fig. 104) is $\dfrac{2i_1 i_2}{a} \times l$, where l is the length of wire in each of the coils and a is their distance apart.

The force between two coils is a fundamental calculation in experiments with the current balance; the formula needed there, however, is more elaborate than the simple approximation given above.*

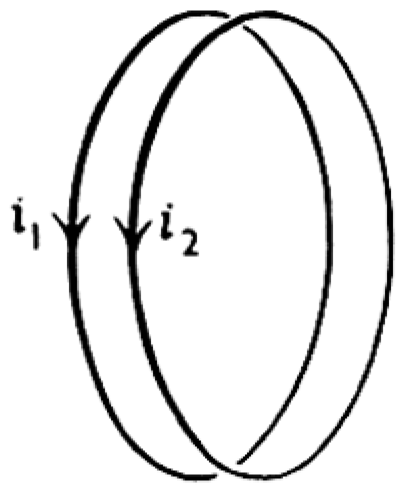


Fig. 104

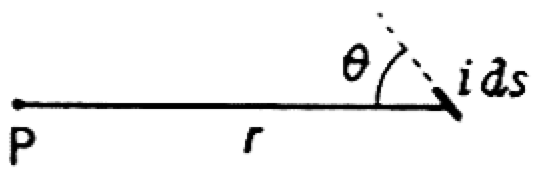


Fig. 105

Alternative development from the general formula

We have developed the elementary mathematical theory of electromagnetism without the aid of the calculus. We shall now cover some of the same ground using the calculus which enables problems to be tackled which are insoluble by the foregoing simple methods.

We start by assuming a formula, known as Laplace's or Ampere's formula, for the strength of the magnetic field, H, due to a current, i, in a very short length of wire, ds, at the point P in Fig. 105.

$$H = \frac{i\,ds \sin\theta}{r^2}.$$

This formula cannot be verified directly by experiment but is justified by the many verifiable deductions which can be made from it. It is, of course, a reasonable assumption since $i\,ds \sin\theta$ may be regarded as the component of 'the current element', $i\,ds$, at right angles to the line joining it to P.

Field due to a long straight wire

To find the field due to a current i e.m.u. in a long straight wire at a point P distant a cm. from the wire (Fig. 106), Ampère's formula may be used.

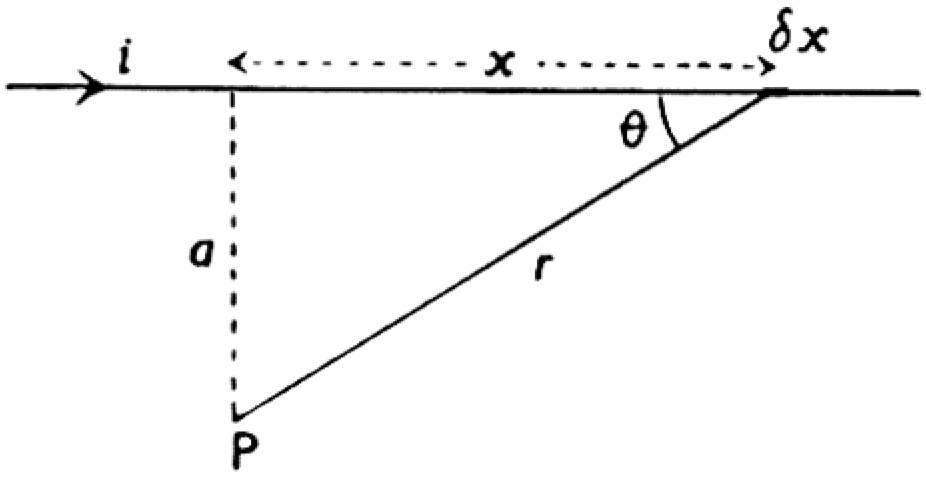


Fig. 106

Let the field at P due to the very short current element, $i\delta x$, be δH. By Ampere's formula

$$\delta H = \frac{i\delta x \sin\theta}{r^2}.$$

---

* The rest of this chapter can be omitted on the first reading of the book.

But $x=a\cot\theta$. $\therefore\ dx=-a\operatorname{cosec}^2\theta\,d\theta$. Also $r=a\operatorname{cosec}\theta$.

$$\therefore\ \delta H=\frac{i(-a\operatorname{cosec}^2\theta\,d\theta)\sin\theta}{a^2\operatorname{cosec}^2\theta}=-\frac{i\sin\theta\,d\theta}{a}.$$

$$\therefore\ \text{Total field, } H=\int_\pi^0-\frac{i\sin\theta\,d\theta}{a}=\frac{i}{a}\Big[\cos\theta\Big]_\pi^0$$

$$=\frac{2i}{a}.$$

Field at any point on the axis of a circular coil

Consider a circular coil of radius a cm. carrying a current i e.m.u. (Fig. 107). The magnetic field, δH, at P due to a short current element $i\delta s$ will be $\dfrac{i\delta s\sin\theta}{x^2+a^2}$. Since $\theta=90°$, $\delta H=\dfrac{i\delta s}{x^2+a^2}$.

The direction of the field is at right angles to the line joining P to the current element. The field can be resolved into a component along the axis $\dfrac{i\delta s}{x^2+a^2}\sin\phi$ and one at right angles. By considering pairs of current elements at opposite ends of a diameter of the coil it is clear that the field at right angles to the axis vanishes.

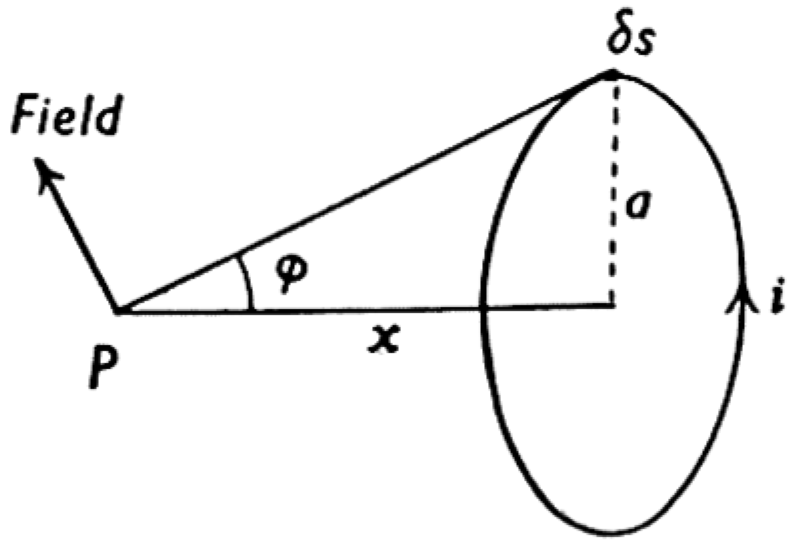


Fig. 107

If the coil has n turns, $i\delta s$ for the whole coil can be replaced by $i2\pi na$.

$\therefore$ Field along the axis due to the whole coil

$$=\frac{i2\pi na}{x^2+a^2}\sin\phi$$

$$=\frac{2\pi a^2ni}{(x^2+a^2)^{\frac{3}{2}}}.$$

Field along the axis of a solenoid

Consider a point P on the axis of a solenoid carrying a current i e.m.u., having n turns per cm. length, and of radius a cm. (see Fig. 108).

The field, δH, at P due to a short length of the solenoid δx is

$$\frac{2\pi a^2n\,\delta x\,i}{(x^2+a^2)^{\frac{3}{2}}}.$$

But $x=a\cot\theta$. $\therefore\ dx=-a\operatorname{cosec}^2\theta\,d\theta$.

Also $x^2+a^2=a^2 \operatorname{cosec}^2 \theta$.

$$\therefore \quad \delta H=\frac{2\pi a^2 n(-a \operatorname{cosec}^2\theta\, d\theta)\, i}{a^3 \operatorname{cosec}^3 \theta}$$

$$=-2\pi ni \sin \theta\, d\theta.$$

$\therefore$ Field, H, at P due to whole solenoid

$$=\int_{\theta_1}^{\theta_2} -2\pi ni \sin \theta\, d\theta$$

$$=2\pi ni(\cos \theta_2-\cos \theta_1).$$

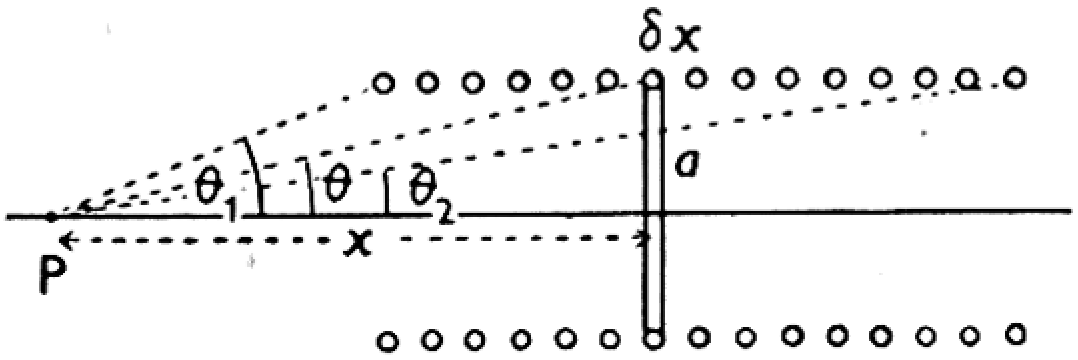


Fig. 108

If the solenoid is infinitely long, $\theta_1=\pi$ and $\theta_2=0$.

$$\therefore \quad H=4\pi ni.$$

If P is at one end of a long solenoid, $\theta_1=90°$ and $\theta_2=0$.

$$\therefore \quad H=2\pi ni.$$

QUESTIONS

1. Explain the principle of the tangent galvanometer. A battery of negligible resistance when connected to the 500-turn coil of a tangent galvanometer produces a deflection of 45°. What deflection will it produce when connected to the 50-turn coil of the same instrument, assuming that both coils have the same radius and are wound with the same wire? (C.)

2. In reading the deflection of a tangent galvanometer it is usual to read both ends of the pointer for both directions of the current. Explain carefully the advantages of this procedure and indicate clearly what errors it minimises and which it leaves unchanged. (O. S.)

3. A careless schoolboy measured a current by means of a tangent galvanometer, the plane of whose coil was +20° from the meridian. The observed deflection was +45°. Calculate graphically, or otherwise, the percentage error in the value of the current indicated by the galvanometer. (O. S.)

4. Define the electromagnetic unit of current. A circular coil of 9 cm. radius and having 50 turns is placed with its plane at right angles to the meridian. A small magnet is suspended at the centre of the coil, and with the current in one direction it is found to make 12 vibrations per minute. With the current reversed it makes 15 vibrations per minute. Given that the earth's field (0·18 c.g.s. units) is smaller than that due to the coil, calculate the current in the coil. (O. & C.)

5. A circular coil of wire of 100 turns, of mean radius 10 cm., is set with its plane in the magnetic meridian. A freely suspended magnet at its centre makes $4\sqrt{15}$ vibrations per minute when a current flows through the coil, and 12 vibrations per minute when no current flows. If the strength of the horizontal component of the earth's magnetic field is 0·18 gauss, find the current in the coil. (O. & C.)

6. The coil of a tangent galvanometer is placed at right angles to the magnetic meridian. The needle is made to oscillate gently about a vertical axis and is found to have the same periodic time when the current passing through the coil is 0·270 ampere and when the current is 0·195 ampere, the direction of the current being the same in each case. Explain these results and indicate by a clear diagram the direction of the current.

If the coil has 9 turns of radius 7·5 cm., calculate the horizontal intensity of the earth's field. (N.)

7. A tangent galvanometer having a coil of 100 turns of mean radius 10 cm. is first placed so that the plane of the coil is perpendicular to the magnetic meridian. When a current of 0·1 ampere is passed it is found that the plane of the coil must be turned through 73° in order to give a deflection of 90°. Calculate the strength of the earth's horizontal magnetic field. (N.)

8. In an experiment to find the intensity (H) of the horizontal magnetic field of the earth a circular coil is placed with its plane vertical and its axis making a small angle θ with the magnetic meridian. A current is passed round the coil so that the field produced tends to oppose that of the earth. Describe and explain the effect on a small horizontal magnet, suspended at the centre of the coil, of increasing the strength of the current. If the magnet sets E. and W. when the current is 0·046 ampere, $\theta = 10°$, $n = 50$ and $r = 8$ cm., calculate the value of H. (N.)

9. A given current through a tangent galvanometer deflects the magnet through an angle θ. The plane of the coil is then slowly rotated round the vertical axis through the centre of the magnet. Prove that if $\theta > \frac{1}{4}\pi$ the magnet will describe complete revolutions. If $\theta < \frac{1}{4}\pi$, find the angle through which the magnet oscillates. (O. S.)

10. Calculate the vertical component of the force due to the earth's magnetic field on an overhead cable 40 metres long which carries a current of 400 amperes and runs perpendicular to the magnetic meridian. ($H = 0·18$ oersted.) (B.)

11. A rectangular coil of wire is suspended from a spring balance so that its lower side is horizontal between the poles of an electromagnet and at right angles to the lines of force of the magnet. A known current is led in and out of the coil by flexible leads. Explain how you would use this apparatus to determine the field between the poles of the magnet for different currents in the field coils. Indicate the general nature of the results you would expect to obtain. (B.)

12. Find the difference of potential between the two ends of a horizontal wire 1 metre long pointing E. and W., 2 sec. after it has been dropped from rest. ($H=0{\cdot}2$ c.g.s. units.) (C. S.)

13. A straight copper wire forming part of an electrical circuit lies across the lines of force between pole pieces of a magnet. Discuss what happens (*a*) if the wire is free to move and a current is passed through it, (*b*) if the wire is connected to a circuit without a battery and is moved across the lines of force.

An all-metal aeroplane with a wing span of 30 metres is flying horizontally in a straight line at a speed of 300 km. per hour. What will be the difference of potential in volts between the wing-tips? (The horizontal component of the earth's magnetic field is 0·18 gauss and the angle of dip is 67°. 1 volt $=10^8$ e.m.u.) (C.)

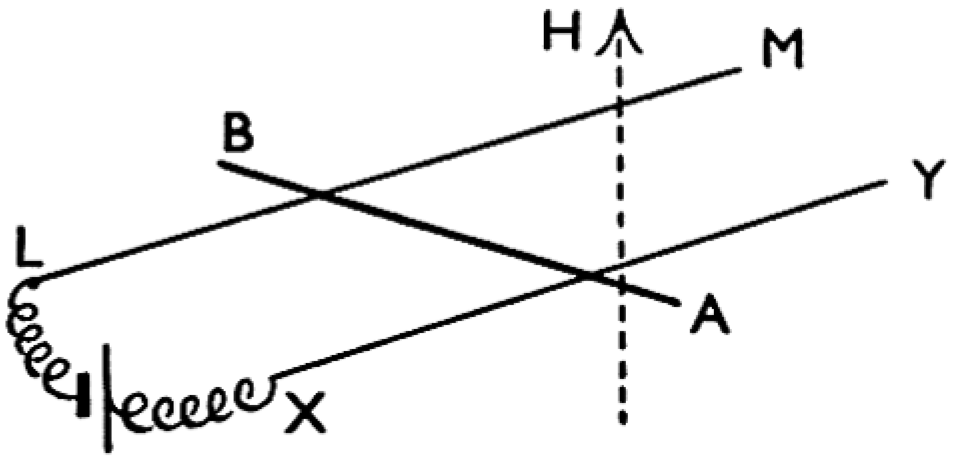


Fig. 109

14. *AB* is a wire free to roll or slide without friction along *LM*, *XY*, two horizontal parallel wires *d* cm. apart (Fig. 109). All have no resistance. A magnetic field of *H* gauss is perpendicular to the plane *LBAX*. The E.M.F. of the battery is *e*, its resistance *r*. Corresponding instantaneous values of the current and the velocity of *AB* (due only to interaction of *H* and *i*) are *i* and *v*. All are in c.g.s. units.

Prove from energy considerations, stating your assumptions carefully, that *i* is less than *e*/*r*, and find by how much.

Show how this illustrates the laws of electromagnetic induction. (C.)

15. A solid wheel 1 metre in diameter spins on an axis which lies in the magnetic meridian, and makes 300 revolutions per minute. If the horizontal component of the earth's field is 0·18 oersted, calculate the potential difference between the centre of the wheel and its circumference.

16. Calculate the work done in taking unit magnetic pole round a closed path surrounding a current-bearing wire. Explain what becomes of the energy expended, and describe illustrative experiments. (O. & C.)

17. Give an account of the magnetic field in the neighbourhood of a long straight wire carrying an electric current. How would you verify your statements experimentally?

An isolated magnetic pole in the neighbourhood of an electric current would continue to move as long as the current flowed. What would be the source of energy of the moving pole? (O. & C.)

18. Explain how you would investigate experimentally the relation between the magnetic intensity at a point due to a current flowing in a long straight wire and the distance of the point from the wire. Indicate graphically the result to be expected.

Draw a diagram showing the combined horizontal magnetic field due to a long straight wire held vertically, carrying a downward current of 4 amperes, and the earth's horizontal field ($H=0{\cdot}2$ dyne per unit pole). Indicate on the diagram the exact position of (*a*) a neutral point, (*b*) a point where the resultant field is equal to $2H$. (N.)

19. The same current is passed in turn through a long straight vertical wire and a single circular loop of wire radius 10 cm. lying with its plane in the magnetic meridian. Compare the deflections of a magnetometer needle when placed (*a*) 10 cm. N. or S. of the straight wire, (*b*) at the centre of the circle. (C.)

20. A uniformly wound cylindrical solenoid 60 cm. long and 6 cm. in diameter with 600 turns, has a current of 2 amperes passed through it. Obtain an approximate value for the intensity of the magnetic field on the axis (*a*) at the centre, (*b*) at the ends. Establish any formulae you use and explain why the value obtained is approximate. Find approximately the moment of the bar magnet to which the coil is equivalent.

Indicate briefly how you would investigate experimentally the variation of the intensity of the field along the axis of the coil. (N.)

21. A long solenoid is placed horizontally with its axis magnetic N. and S., and a small magnet at its centre is found to make 20 oscillations per minute when a current is passed round the solenoid, and 6 oscillations per minute when the earth's field alone acts. If the solenoid has 15 turns per cm., calculate the possible values of the current passing round it. (Assume $H=0{\cdot}18$ oersted.) (N.)

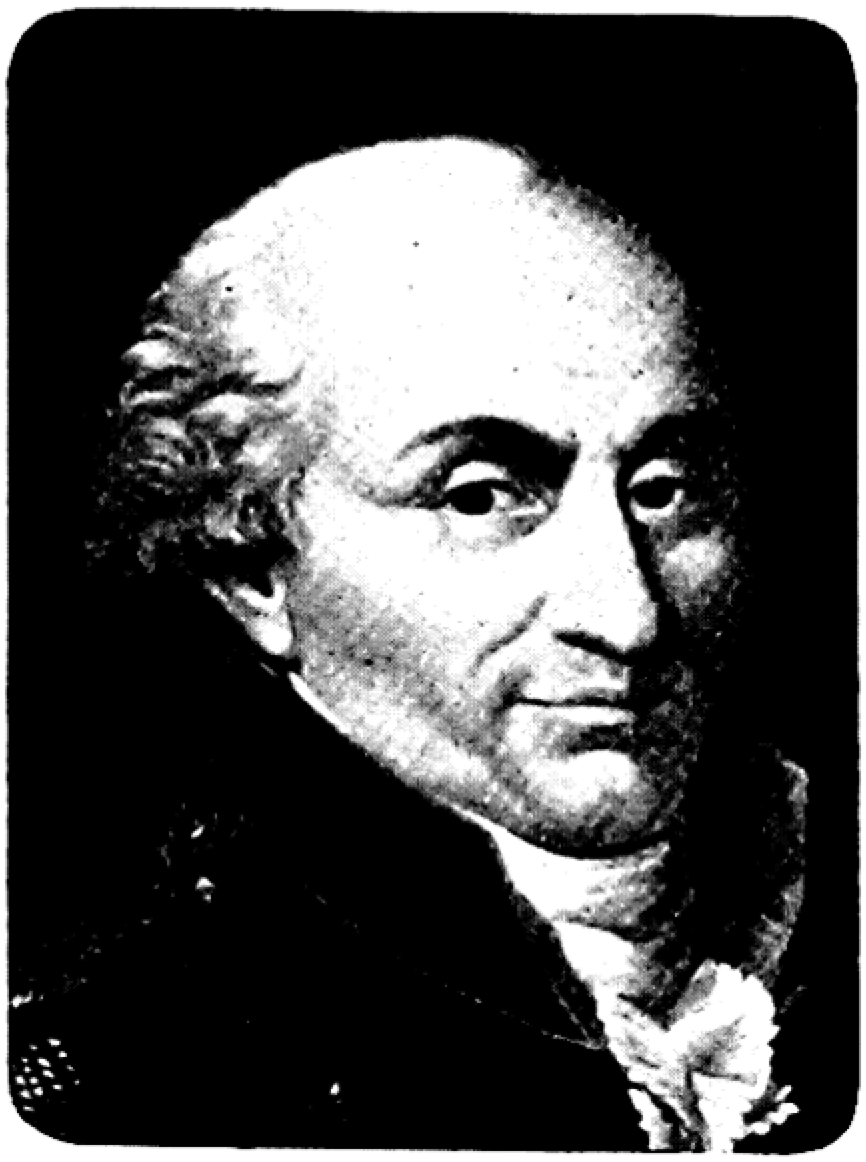
22. State Laplace's formula for the magnetic field intensity at any point due to a current element, and apply it to find the magnetic field (*a*) at the centre of a circular coil of n turns of radius a carrying a current of I electromagnetic units, and (*b*) on the axis of the same coil at a distance from the centre equal to its radius.

A current of 0·1 ampere passes through a circular coil 15 cm. in diameter and consisting of 50 closely wound turns of fine insulated wire. Calculate the intensity of the field (i) at the centre of the coil, and (ii) at a point on its axis distant 7·5 cm. from the centre. (L.)

23. State the main laws connecting the electric current in a circuit and the magnetic field it produces. How would you verify these results experimentally?

A mine-sweeping aeroplane carries a horizontal circular circuit 10 metres in diameter. If the maximum current from the dynamos is 1000 amperes, what is the minimum number of turns in the loop if the field from the coil has to neutralise the vertical component of the earth's field (0·1 oersted) 30 metres below the surface of the sea? (O. S.)

24. Find an expression for the magnetic force at points on the axis of a circular coil of wire carrying a current. Show that if two equal coils carrying the same current are placed in parallel positions on the same axis, the field at a point on the axis midway between them is most uniform when the separation of the coils is equal to the radius of each. (O. S.)



CHARLES AUGUSTIN
DE COULOMB

ANDRÉ MARIE
AMPÈRE

CHARLES AUGUSTIN DE COULOMB (1736–1806) *was the first to apply mathematics to the phenomena of electricity and magnetism. His early experiments on the twisting of fine wires and invention of the torsion balance were put to use in his investigations resulting in the discovery of the inverse square laws in both electrostatics and magnetism. His experimental skill was of the highest order; the forces he had to measure were small and in the case of electrostatic charge, apt to be evanescent because of faulty insulation. Coulomb was an officer in the French army and spent some years supervising the building of the fortifications in the island of Martinique. During the Revolution he retired to his estate, but was later recalled to Paris by Napoleon.*

ANDRÉ MARIE AMPÈRE (1775–1836) *was called by Maxwell 'the Newton of electricity'. Ampère heard of Oersted's discovery, that an electric current in a wire causes a magnetic compass to set at right angles to the wire, by letter in Paris on 11 September 1820 and in a short time he created a new subject, electromagnetism. He formulated a clear idea of electric current—Oersted had spoken of the 'electric conflict' in the wire—and also of electromotive force, coming quite near to a discovery of Ohm's law. He discovered that a force exists between parallel wires carrying currents, that a solenoid behaves like a magnet and can be used to magnetize iron rods, and put forward the suggestion that all magnetism might be due to electric currents. Ampère was a man of peculiar temperament, religious, often tortured by doubts. As a young man he suffered two severe blows, his father being guillotined, and his young wife dying shortly after their marriage.*

CHAPTER 6

MAGNETIC MATERIALS

IRON and its alloys are capable of strong magnetisation; if this were not so the dynamo and electric motor could never have become commercial possibilities, and electricity could not have rivalled steam. The magnetic properties of materials are thus of great practical importance.

Intensity of magnetisation

The extent to which a material is magnetised may be measured by its *magnetic moment per unit volume*, known as the *intensity of magnetisation*, J. If the material is in the form of a rod, uniformly magnetised,

$$J=\frac{M}{V}=\frac{2lm}{2lA}=\frac{m}{A},$$

where M = magnetic moment, V = volume, $2l$ = length, m = pole strength, and A = area of cross-section of the rod. Thus the intensity of magnetisation is also equal to the pole strength per unit area. Note, however, that this is only an approximation, since the magnetic length (in the numerator of the above expression) is always somewhat less than the geometrical length (in the denominator).

Magnetisation curve

The way in which the intensity of magnetisation of a specimen of iron in the form of a thin rod depends on the strength of the magnetising field may be investigated by the apparatus shown in Fig. 110. The rod is magnetised inside a vertical solenoid S, the magnetic field of which is proportional to the current, i, measured by the ammeter A and varied by a rheostat. The commutator enables the current to be reversed and the rod to be magnetised in the opposite direction.

The pole strength of the lower end of the specimen is JA, where J is the intensity of magnetisation and A the area of cross-section; it is measured by means of a deflection magnetometer, D. Provided that the specimen is long compared with the distance SD the effect of the upper pole of the specimen may be ignored.

The magnetometer must first be set magnetic east or west of S. so that the field due to the specimen and the earth's magnetic field are mutually at right angles. A compensating coil C in series with S, is arranged on the other side of D from S at such a distance that, when the specimen is removed from the solenoid, the magnetic fields of S and C exactly neutralise at the centre of D. Then the deflection of D is due to the magnetisation of the specimen only.

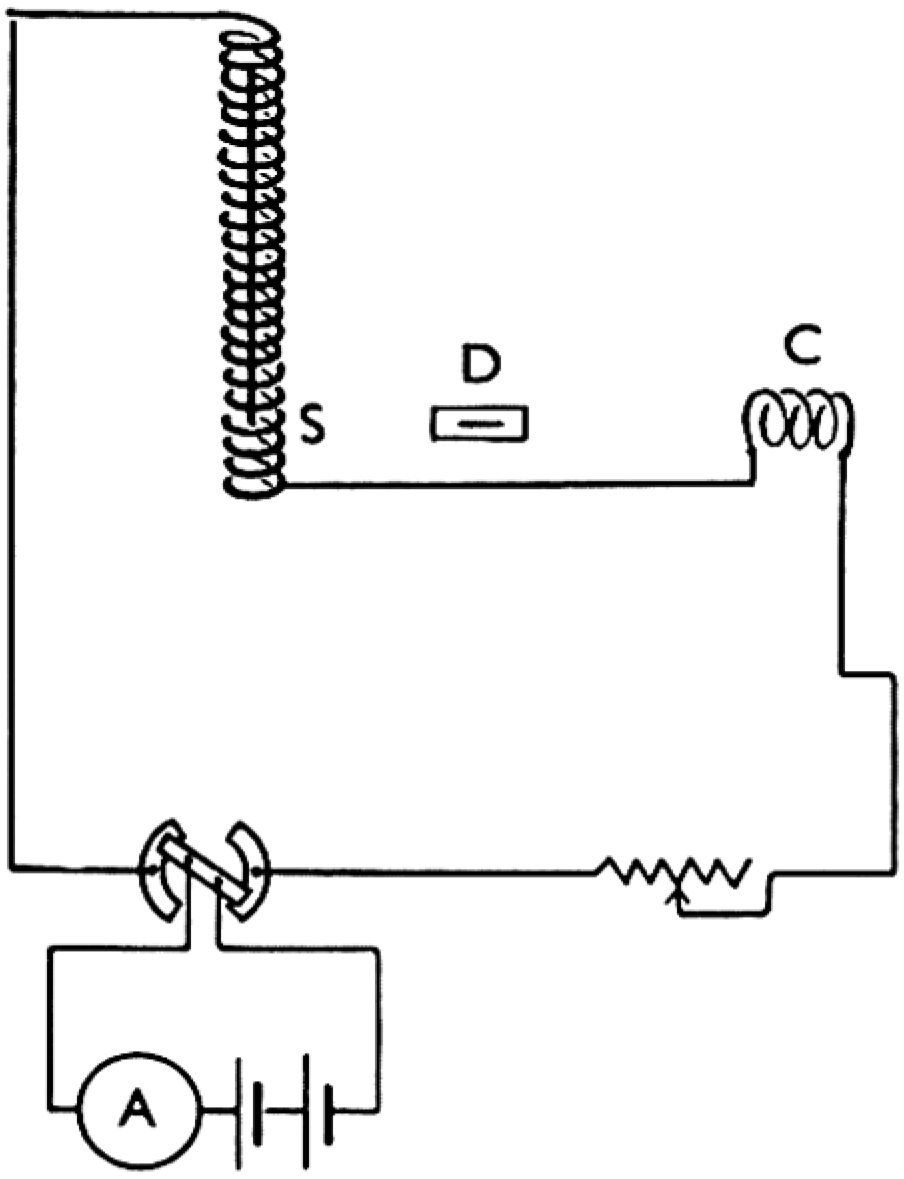


Fig. 110

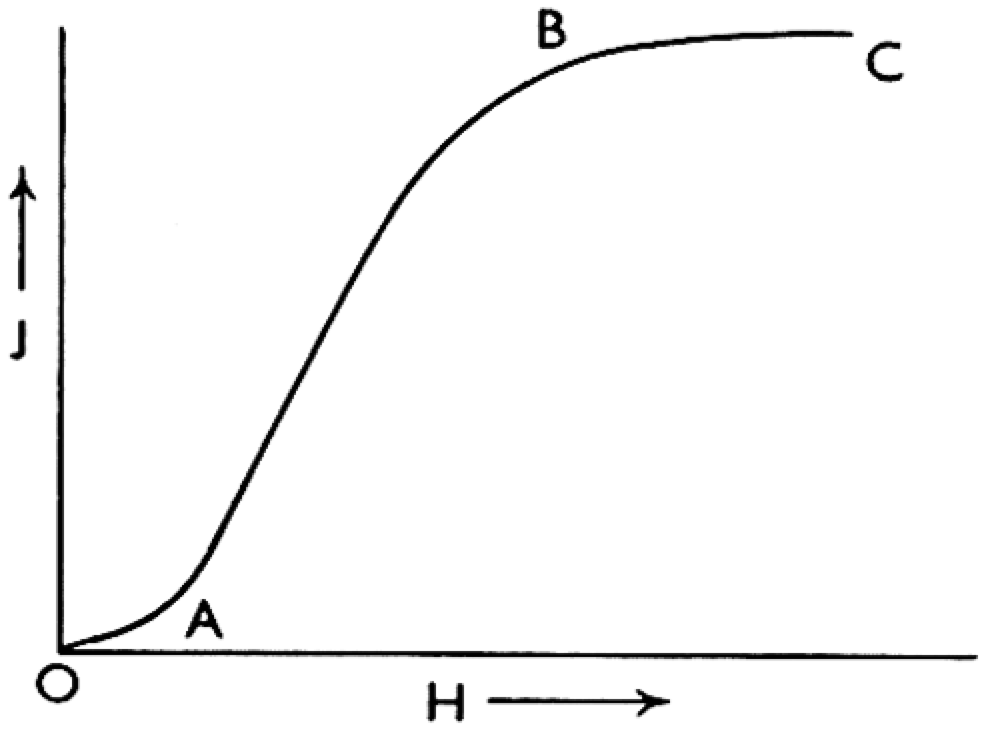


Fig. 111

The field at D due to the lower pole of the specimen is JA/d^2, where d=distance between lower pole of specimen and centre of D. Then, if H_e=earth's horizontal component and θ=the deflection of the magnetometer,

$$\frac{JA}{d^2}=H_e \tan\theta.$$

Hence J can be found.

The object of the experiment is to find how J varies with the strength of the magnetising field H inside the solenoid.

Now $H \propto i$, the current through the solenoid.

Also $J \propto \tan\theta$.

Hence by plotting $\tan\theta$ against i a curve is obtained similar to the J-H curve (Fig. 111).

A second solenoid with a small number of turns is sometimes wound on the outside of S. A sufficient current must be passed through this to neutralise the vertical component of the earth's magnetic field, particularly when dealing with a specimen capable of high magnetisation in a weak magnetic field. Otherwise the magnetising field is not proportional to *i*.

The poles at the ends of the specimen set up a demagnetising field in opposition to that of the solenoid; this may be minimised by making the specimen thin.

The curve in Fig. 111 may be divided into three parts, *OA*, *AB*, *BC*, representing three fairly distinct stages of magnetisation. In the first stage, *OA*, the field produces comparatively small magnetisation, and it may be regarded as breaking up stable configurations of the elementary magnets or domains inside the specimen. In the next stage, *AB*, the configuration of the domains is unstable and an increase in *H* produces a large increase in *J*. In the last stage, *BC*, the magnetic axes of the domains are almost completely aligned, and when alignment is complete the curve becomes horizontal. The term *saturation* is applied to this state.

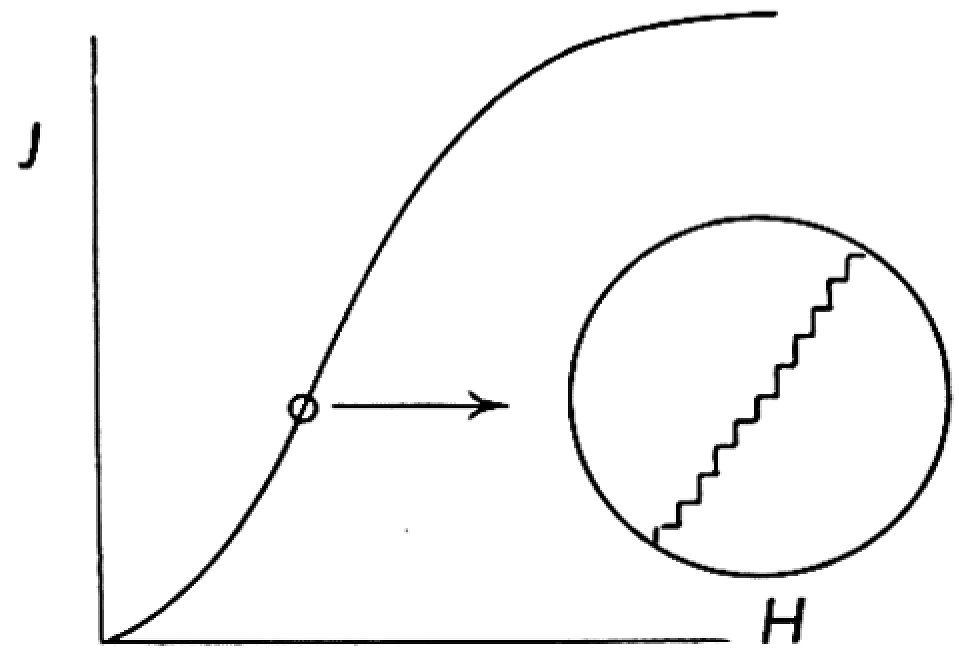


Fig. 112

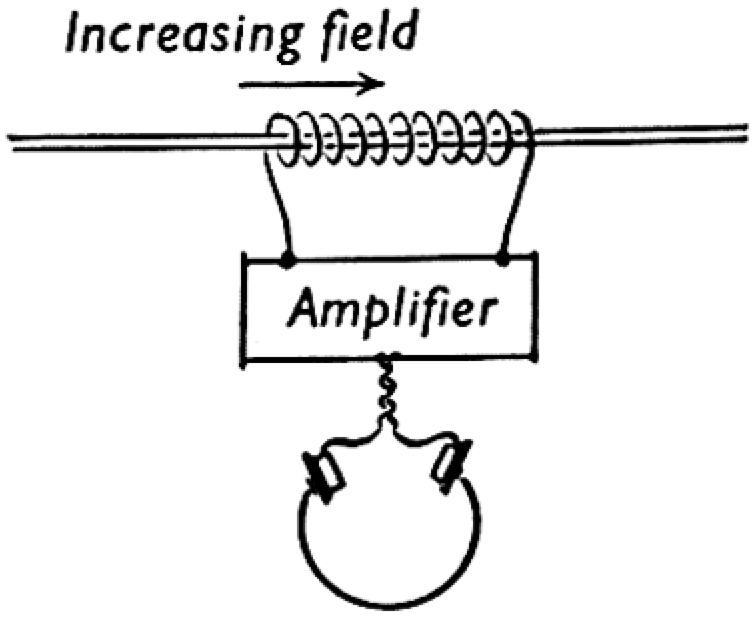


Fig. 113

If the magnetisation curve could be enormously magnified it would be seen to be made up of steps (Fig. 112) each representing a sudden change of magnetisation due to the alignment of the magnetic axis of a domain. This can be demonstrated by wrapping round an iron rod a coil connected to an amplifier and telephones (see Fig. 113). As the magnetising field is slowly increased a series of clicks is heard in the telephones, each click being due to a tiny current induced in the coil when the magnetic lines through it are increased by the alignment of a domain. The phenomenon is known as the Barkhausen effect.

Susceptibility

The intensity of magnetisation, *J*, divided by the strength of the magnetising field, *H*, is called the *susceptibility* of the material, *k*.

Thus $$k = J/H.$$

If the magnetisation curve in Fig. 111 were a straight line k would be constant. But the magnetisation curve is not straight, and k varies with H as shown in Fig. 114. The maximum value of k in Fig. 114 corresponds to the steepest part of the J-H curve in Fig. 111.

The values of k for different values of H can be calculated from the readings of the magnetometer experiment described above. Assuming the solenoid to be infinitely long, $H = \frac{4\pi ni}{10}$ (see p. 94). Also

$$J = \frac{H_e \tan \theta d^2}{A} \text{ (see p. 103).}$$

$$\therefore \; k = \frac{J}{H} = \frac{10 H_e \tan \theta d^2}{4\pi n i A}.$$

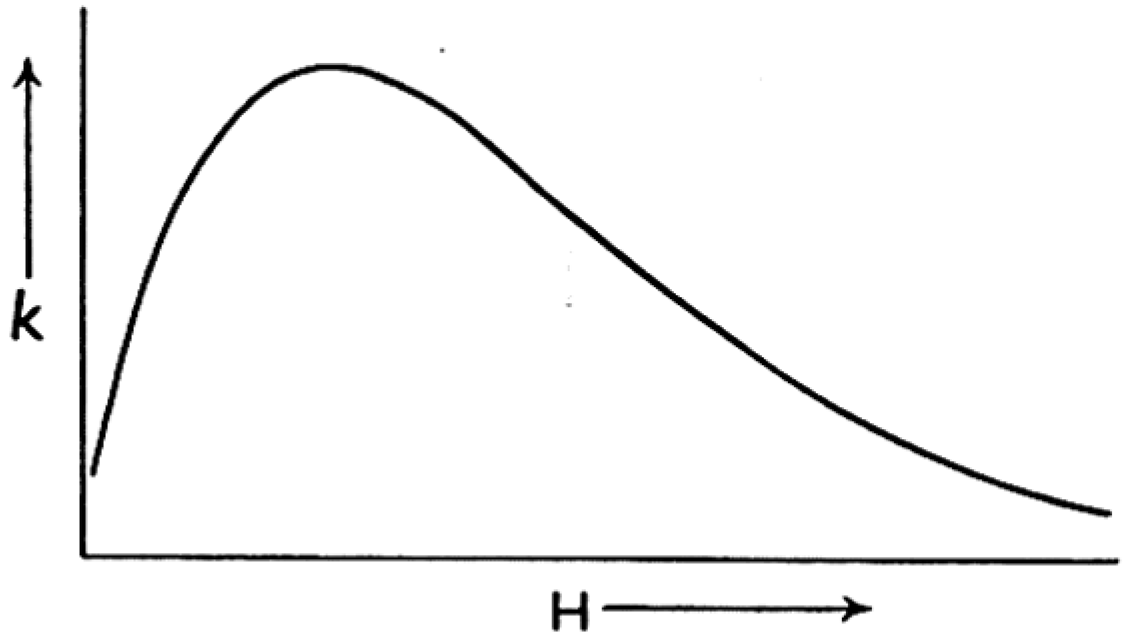


Fig. 114

Besides the approximation introduced by assuming the solenoid to be infinitely long, the field inside the solenoid is reduced by the demagnetising effect of the poles at the end of the specimen.

Hysteresis

If, after the specimen has been magnetised until the point C in Fig. 111 has been reached, the magnetising current is gradually varied through a complete cycle from the positive maximum through zero to an equal negative maximum and back again to its positive value, the J-H curve obtained is shown in Fig. 115.

OB represents the *residual magnetism* or *remanence* of the specimen when the field is removed; the ability to retain magnetism is called the *retentivity*. OC represents the field required to demagnetise the specimen and is called the *coercive force.*

The curve $ABCDEFA$ is known as the *hysteresis* loop or cycle. The word 'hysteresis' means 'to lag behind', and it will be seen that J becomes zero after H has done so.

The shape and size of the loop depends on the nature of the material of the specimen; the loop is narrow for soft iron and wider for steel (Fig. 116). The area of the loop can be shown to represent the energy expended in the material in going through the cycle. The resistance to re-alignment of the magnetic axes of the domains is akin to friction and the work done against this resistance appears in the form of heat in the specimen.

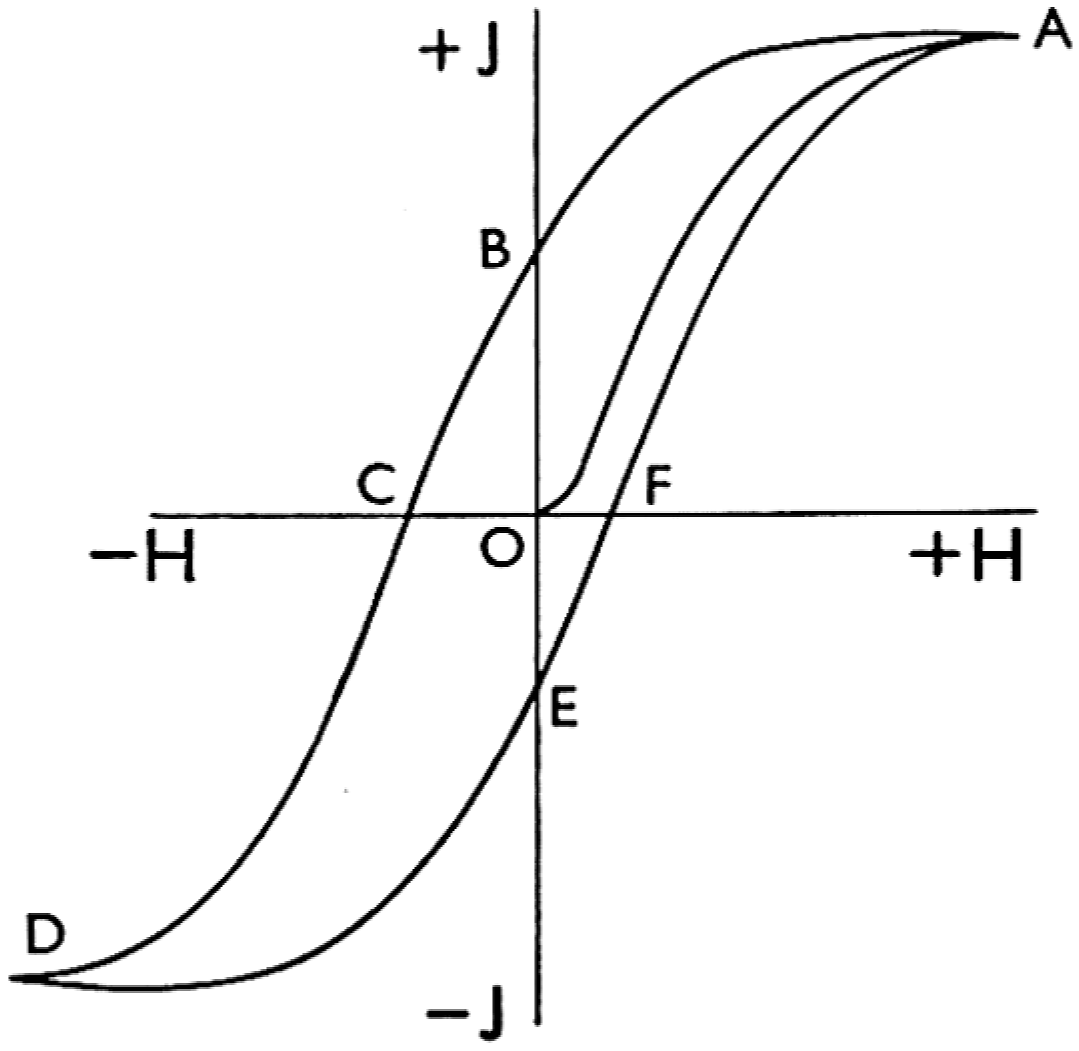


Fig. 115

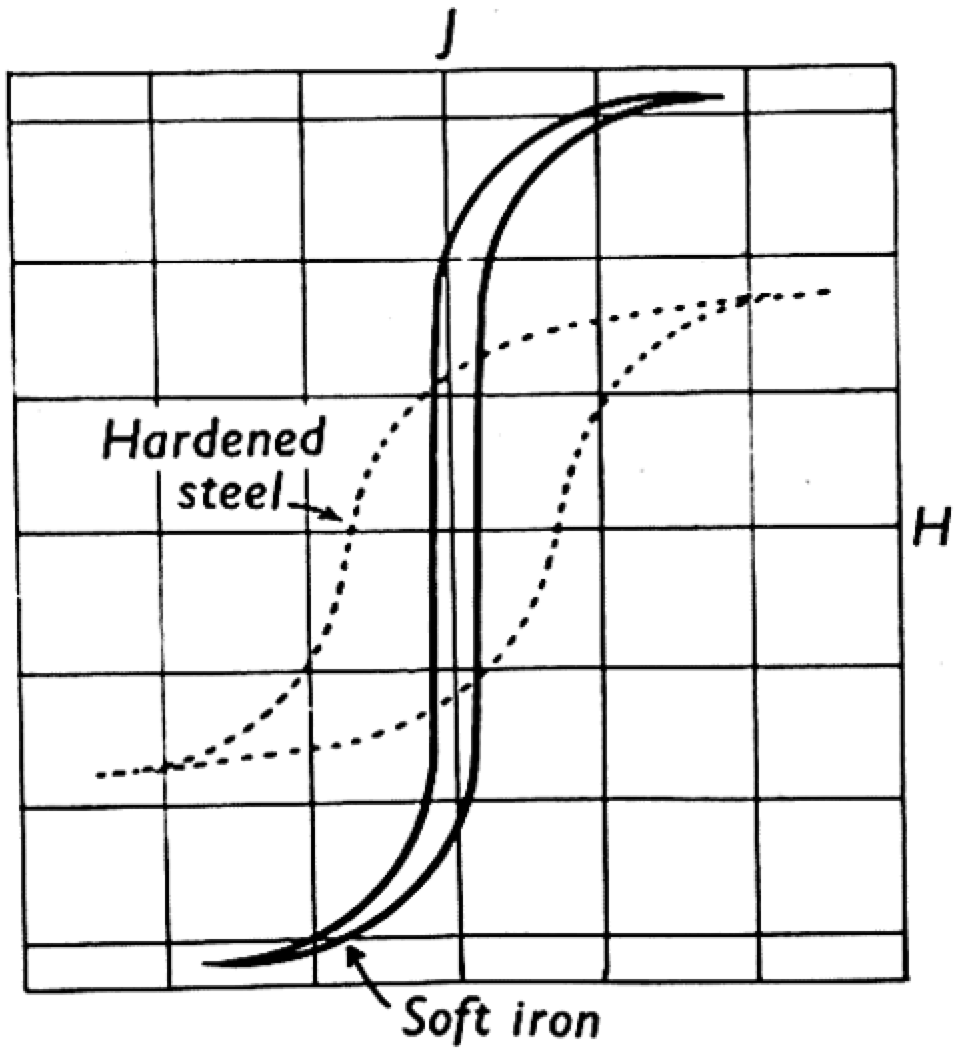


Fig. 116

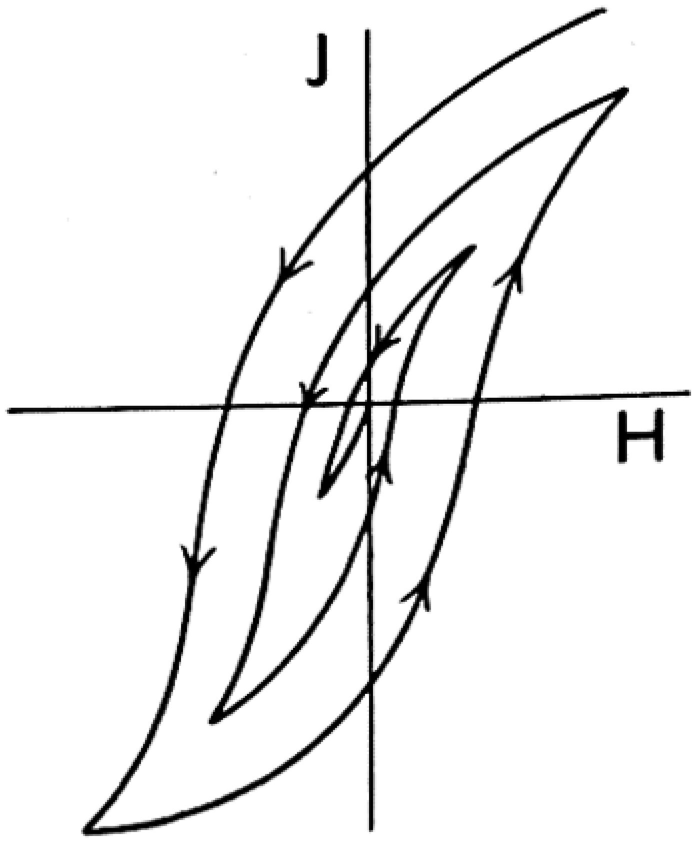


Fig. 117

It is clear from Fig. 115 that the magnetic condition of a material depends on its previous history. Thus at *C* and *F* the specimen is unmagnetised but it is not in a natural condition; if the field is annulled it ceases to be unmagnetised. It can be brought back to its natural condition most conveniently by magnetising and demagnetising it in a field which is gradually reduced so that the material passes through diminishing cycles as in Fig. 117. This can be done by drawing it slowly out of a solenoid connected to the A.C. mains. When the hysteresis loop is being obtained by the method of Fig. 110 the current must not be switched off in the middle of the cycle. Should this happen by mistake the specimen must be demagnetised and the cycle restarted.

The ballistic method and magnetic induction

The demagnetising effect of the free poles at the ends of the specimen, which has been noted as a defect of the magnetometer method, may be overcome by using a closed ring of the material.

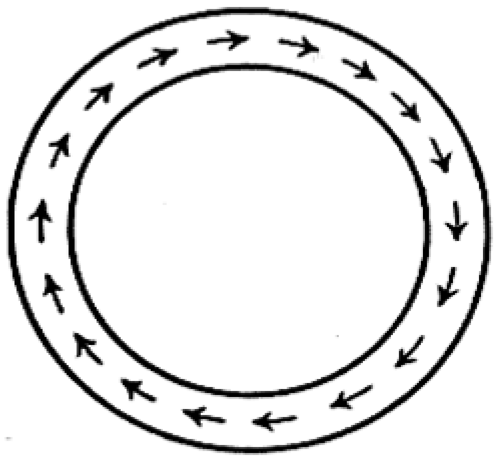


Fig. 118

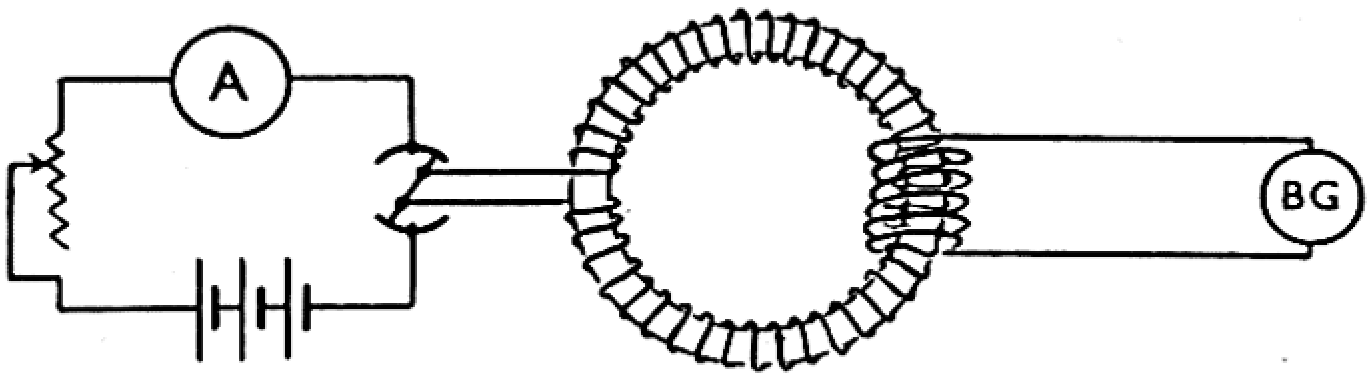


Fig. 119

Suppose a ring of iron is wound with an endless solenoid (sometimes called a toroidal winding) through which a current is passed. The elementary magnets or domains in the ring will be aligned in circles as in Fig. 118, and there will be no free poles.

The magnetisation of the iron may be investigated by means of electromagnetic induction. Suppose a secondary coil is wound round part of the ring (Fig. 119) and connected to a ballistic galvanometer, which is a galvanometer modified to measure the electric charge which passes when an induced current flows for a very short time. When the current in the magnetising ring solenoid is changed, a current will be induced in the secondary coil.

The induced current in the secondary coil is found to be enormously greater with an iron core than with an air core. This is due to the extra magnetic lines produced by the magnetised iron. Suppose the field due to the solenoid is H, and suppose that the intensity of magnetisation of the iron, that is, the pole strength per unit area at any cross-section of the ring, is J. We have seen that a unit pole may be regarded as having 4π lines of magnetic force (p. 56) and a field H may be represented by H lines per sq.cm. The total number of lines of magnetic force per sq.cm.

in the ring will therefore be $(H+4\pi J)$.* This is called the *magnetic induction* and denoted by B. Thus

$$\mathbf{B=H+4\pi J.}$$

The view taken in this book is that B and H are similar quantities with the same dimensions. But another possible view is that they are different in nature with different dimensions and hence they have been given different units (which we shall use). The unit of B is the *gauss*, which is one line of induction per sq.cm., and the unit of H is the oersted, which is one line of magnetic force per sq.cm.

A number of lines of magnetic induction is often called a *magnetic flux*; B, the number of lines of induction per sq.cm., is called the *flux density*. The unit of magnetic flux is the *maxwell*; 1 maxwell is equal to 1 line of induction.

The quantity of electricity induced in the secondary in Fig. 119 is proportional to the change of B. Thus, if the ballistic galvanometer has been calibrated, the change of B can be found. By suitable variation of the current in the primary solenoid and corresponding measurements of the change of B, the whole hysteresis loop can be plotted. It is customary to plot B against H instead of J against H; since H is very small compared with B, B is nearly equal to $4\pi J$ and B-H curves have almost exactly the same shape as J-H curves.

Permeability

B/H is called the *permeability* of the material and is denoted by μ. Thus

$$B=\mu H.$$

In view of the concept susceptibility, μ may seem somewhat unnecessary, but its use will be considered shortly.

There is a simple relation between μ and k:

$$B=H+4\pi J,$$

$$\therefore\ B/H=1+4\pi J/H,$$

i.e.

$$\mu=1+4\pi k.$$

Permeability, like susceptibility, is not constant for iron but depends on H; a μ-H graph is very similar to the k-H graph in Fig. 114. Since μ for iron is of the order of 1000 it is almost equal to $4\pi k$.

---

* To elucidate the term $4\pi J$ imagine the iron ring to be sawn across without altering its magnetisation. A N. and a S. pole, each of strength J per sq.cm., will be facing each other and there will be $4\pi J$ lines of magnetic force per sq.cm. passing between them. This is similar to the case of a polarised dielectric (p. 31).

Magnetic alloys

The suitability of a specimen of iron for a particular magnetic purpose can be seen at a glance from its hysteresis curve. Thus the steel in Fig. 116 is more suitable than the soft iron for a permanent magnet since it has a higher coercive force; the soft iron, on the other hand, is the more suitable for the core of a transformer or the motor of an electric motor or dynamo, which is continually being magnetised and demagnetised, since its hysteresis loss is low and it has a high permeability.

The magnetic properties of iron can be varied very considerably by alloying it with other metals. The initial permeability of pure iron, for example, can be increased from 250 to 400 by adding 4% silicon. The resistivity is thereby raised five times, with the advantage of reducing eddy currents. The silicon-iron alloy, Stalloy, is widely used for transformer cores, and has a hysteresis loss of about 3000 ergs per c.c. per cycle.

Alloys of nickel and steel are called permalloys. A 78% Ni permalloy has the large maximum permeability of about 60,000, and the low hysteresis loss of about 200 ergs per c.c. per cycle.

For permanent magnets steels containing tungsten or cobalt were employed until lately. Tungsten steels have a coercive force of up to 80 oersteds and cobalt steels of up to 250 oersteds. In recent years extremely powerful Alni and Alnico magnets have been available. Alni (Ni 25–30, Al 10–15, Fe 65–55) has a coercivity of about 500 oersteds; Alnico (Ni 18, Al 10, Cu 6, Co 12, Fe 54) is more expensive but rather better than Alni. The new K.S. steel invented by Honda has a coercive force of 900 oersteds.

The addition of manganese makes steel intensely hard and almost non-magnetic. No-mag is a nickel manganese cast iron used for cable boxes, meter cases, etc., where a strong non-magnetic material is required.

It has recently been found that by treating iron in hydrogen at 1500° C. to remove non-metallic impurities, the maximum permeability increases from 10,000 to 340,000, and there is a parallel reduction in mechanical hardness. The coercive force is reduced to 0·03 oersted.

The concept of permeability

The increase in magnetic flux when air inside a solenoid is replaced by iron was explained on p. 107 as due to the contribution of the elementary magnets composing the iron. The atoms of the iron contain revolving electrons which constitute tiny circular currents and when aligned in the field may be regarded as increasing the ampere-turns, thereby increasing the flux.

The phenomenon is often regarded in an entirely different way, however, in order to justify a simple method of calculating flux. The granular structure of the iron with its myriads of tiny magnets is ignored and the iron looked upon as a homogeneous medium which is far more permeable to magnetic flux than air. Taking this view we regard the magnetising field H as a stress and the flux density or induction B as a strain resulting from the stress. The permeability of a medium, strain divided by stress, is then analogous to the reciprocal of a modulus of elasticity. A given stress or magnetising field produced by a solenoid results in a greater strain or flux in iron than in air because the permeability of iron is greater. To be consistent the stress and strain in a vacuum and the permeability of a vacuum must also be considered. The Electrotechnical Commission in 1930 decided that 'for electrotechnical purposes the convention should be established that in free space the quantities flux density, B, and magnetising force, H, should be taken as physically different'. Thus different units, the gauss and the oersted, were chosen for B and H respectively. On this view μ is regarded as having dimensions and, since $B=\mu H$, B and H have different dimensions (see p. 180). We have taken the view that B and H are physically identical and that μ $(=B/H)$ is a pure ratio.

Lines of force and induction in a ring solenoid

It is desirable, before considering the calculation of flux in a magnetic circuit, to be quite clear about the distinction between lines of magnetic force due to a magnetising field and lines of induction. The facts concerning an iron ring will therefore be reconsidered.

Suppose a ring solenoid produces a magnetic field of 50 oersteds inside itself. There are 50 lines of magnetic force per sq.cm. inside the ring. If the ring has an air core the induction is 50 gauss and there are 50 lines of magnetic induction per sq.cm. inside the solenoid. If, however, the ring has an iron core with a permeability at this field strength of 1000, there will be 50,000 lines of magnetic induction per sq.cm., although the number of lines of magnetic force due to the solenoid is still 50 per sq.cm.

The magnetic circuit

The electrical engineer, when designing machines, requires to calculate the magnetic flux, i.e. the lines of magnetic induction, in combinations of iron core and air core circuits—as, for example, the circuit including the field magnet and armature core of a dynamo or electric motor. He does this by a simple but effective method.

Consider first the case of an iron ring, of cross-section A, wound with a uniform ring solenoid having a total of N turns, the mean circumference of the ring being l.

$$\text{Total flux through the ring} = BA$$
$$= \mu HA.$$

But
$$H = \frac{4\pi Ni}{l} \quad \text{(see p. 93).}$$
$$\therefore \ \text{Flux} = \mu \frac{4\pi Ni}{l} A$$
$$= \frac{4\pi Ni}{l/\mu A}.$$

The expression $4\pi Ni$ is called the magnetomotive force and is the work done in taking a unit pole round the inside of the ring (see p. 92). Note the analogy with electromotive force, which is the work done in taking a unit charge round an electric circuit. The expression $l/\mu A$ is called the magnetic reluctance. Note the analogy with electrical resistance, which is equal to $\rho l/A$ (see p. 155). Thus the formula may be written in words, as a complete analogy with Ohm's law,

$$\text{Magnetic flux} = \frac{\text{Magnetomotive force}}{\text{Magnetic reluctance}}.$$

Moreover, we can add reluctances in a magnetic circuit just as we can add resistances in series. For example,

$$\text{Reluctance} = \frac{l_{\text{air}}}{A_{\text{air}}} + \frac{l_{\text{iron}}}{\mu A_{\text{iron}}}.$$

It should, perhaps, be mentioned that the flux, despite its name, does not flow but consists of closed, stationary lines of magnetic induction.

Suppose now that the iron ring is wound with a concentrated solenoid in part of the ring only (Fig. 120). It is found by experiment that the magnetic flux round the ring is almost the same as if the ring were uniformly wound with the same

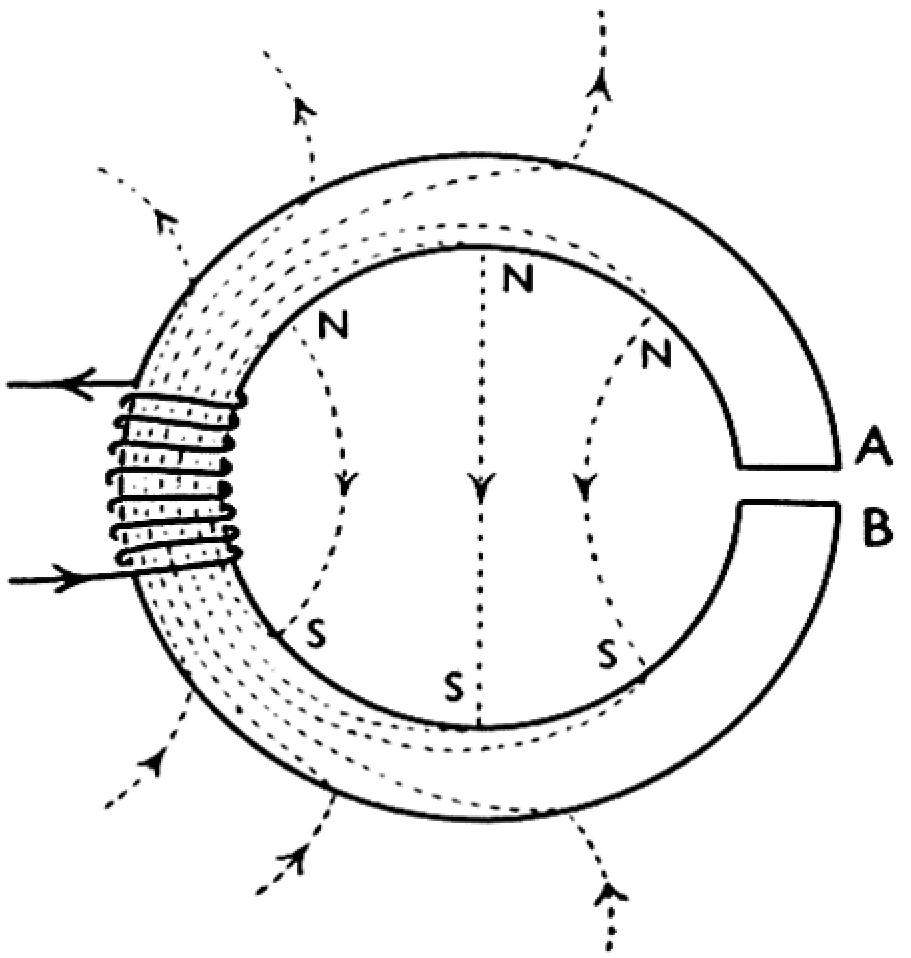


Fig. 120

number of turns. A small fraction of the flux, it is true, 'leaks' out of the ring as shown in Fig. 120 and free poles appear, but this has little effect on the flux in the ring.

If an air gap is cut in the ring at AB the flux in the ring is found to be considerably reduced. This is, in fact, caused by the demagnetising effect of the free poles which appear at A and B and an increase in the poles on the curved surface of the ring due to an increased leakage flux. Alternatively, it can be explained by saying that the magnetic reluctance of the air gap is large compared with that of the iron.

Owing to the fortunate circumstance that the leakage can be neglected if the gap is not too large the formula

$$\text{Magnetic flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}}$$

can still be successfully applied.

Calculation of the field between the poles of an electromagnet

We will apply the formula to calculate the field between the poles of an electromagnet. The following figures are based on the Stenzl electromagnet (Fig. 121):

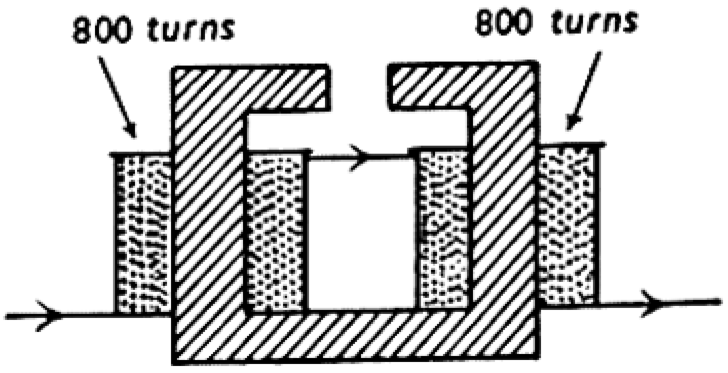


Fig. 121

Total number of turns, $N = 1600$

Current, $i = 2$ amperes

$= 0{\cdot}2$ e.m.u.

A_{iron} and A_{air} both $= 3{\cdot}5^2$ sq.cm.

Air gap between poles $= l_{\text{air}}$

$= 2$ cm.

Total length of iron core $= l_{\text{iron}}$

$= 70$ cm.

$\mu_{\text{iron}} = 2000$

$$\text{Flux} = \frac{4\pi Ni}{\dfrac{l_{\text{air}}}{A_{\text{air}}} + \dfrac{l_{\text{iron}}}{\mu A_{\text{iron}}}}$$

$$= \frac{4\pi \,.\, 1600 \,.\, 0{\cdot}2}{\dfrac{2}{3{\cdot}5^2} + \dfrac{70}{2000 \,.\, 3{\cdot}5^2}} = \frac{1280\pi}{0{\cdot}1632 + 0{\cdot}00286}$$

$$= 24{,}700 \text{ lines.}$$

$$\therefore \text{ Flux density} = \frac{24{,}700}{3{\cdot}5^2} = 2000 \text{ gauss.}$$

Thus the field in the air gap is 2000 oersteds.

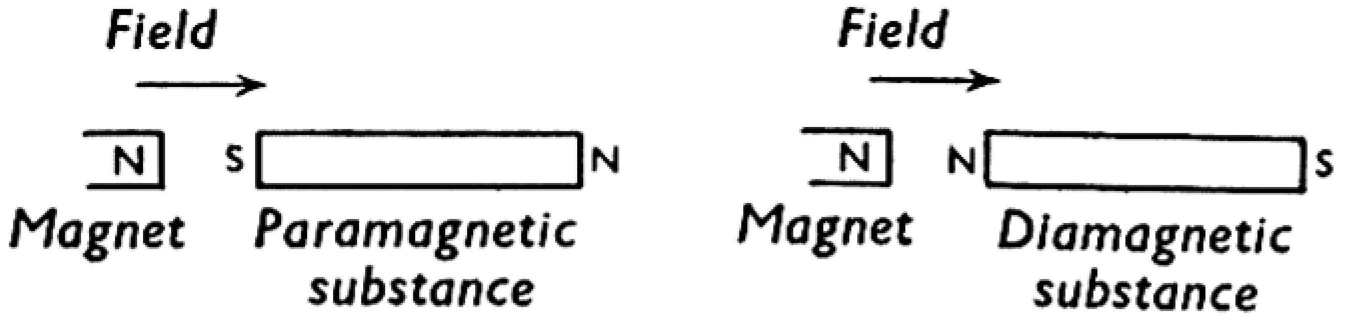
It will be seen that the reluctance of the iron is small compared with that of the air. This shows the importance of keeping the air gap as small as possible in a dynamo, electric motor, or moving-coil galvanometer.

Diamagnetism and paramagnetism

All substances are found to have magnetic properties although these are usually very slight. In 1845 Faraday found that a rod of glass suspended by a fine silk fibre between the poles of a powerful electromagnet set itself at right angles to the lines of force. The same happens to bismuth which exhibits the effect comparatively strongly, and to many other substances including the tissues of the human body. As Faraday remarked, 'If a man could be in the magnetic field, like Mohammed's coffin, he would turn until across the magnetic line.'

Rods of platinum and aluminium set themselves parallel to the lines of force. Faraday coined two new words to classify the two kinds of substances: *diamagnetics* set themselves across, and *paramagnetics* parallel to, the lines of force.

The fundamental difference between the two is that paramagnetics become induced magnets in the direction of an external field, as iron does, whereas diamagnetics become induced magnets in the opposite direction (Fig. 122). Thus for paramagnetics the susceptibility k is positive, and the permeability $\mu > 1$; for diamagnetics k is negative and $\mu < 1$.



(a) (b)

Fig. 122

Some typical values are as follows:

Paramagnetics	k	μ
Platinum	0·000020	1·00025
Aluminium	0·0000018	1·000023
Saturated solution of $FeCl_3$ (at room temperature)	0·000051	1·00064
Air (1 atmosphere)	0·000000032	1·00000038
Diamagnetics		
Bismuth	−0·000013	0·999824
Silver	−0·0000015	0·999981
Glass	−0·0000009	0·999987
Water	−0·00000072	0·999991

The values of μ and k for paramagnetics and diamagnetics differ from those for iron in that they are very much smaller and also constant, being independent of the field.

Paramagnetic substances are attracted by a magnet and diamagnetic substances are repelled, as we should expect from Fig. 122. A rod of a diamagnetic substance sets itself at right angles to the lines of force between the poles of an electromagnet because its ends are repelled by the poles.

Theories of magnetism

Ampère suggested that magnetism might be due to circulatory electric currents in the molecules. These currents can now be identified as revolving electrons. Thus magnetism is a product of electricity; it cannot exist apart from electricity.

Diamagnetism was accounted for by Langevin in 1905 as follows. When a substance is placed between the poles of an electromagnet and the current is switched on, the change in magnetic flux through the molecular circuits of the substance will give rise to induced E.M.F.'s. These induced E.M.F.'s, by Lenz's law, will be in such a direction as to cause changes in the magnetic fields of the molecular currents which oppose the field of the electromagnet. The speeds of the revolving electrons can be regarded as being increased or decreased, according to the sense of their rotation, and since there is no resistance to their rotation, the effect will persist after the field of the electromagnet has become constant. When the current in the electromagnet is switched off or the substance is taken from between the poles, the molecular currents will return to their original values.

Thus all substances should exhibit diamagnetism; when placed in a magnetic field they should become magnetised in a direction opposite to that of the field.

In some substances, however, the effect is masked by another which gives rise to paramagnetism. Suppose the revolving electrons in the molecule do not counterbalance each other, and the molecule has a resultant magnetic moment. Each molecule will align itself in a magnetic field, like a magnetic compass, so that it increases the field.

As would be expected on the theory, diamagnetism is independent of temperature but paramagnetism decreases with temperature, since more violent thermal agitation tends to impede alignment of the molecules.

The comparatively enormous susceptibilities of ferromagnetic substances may be explained by the mutual aligning influence of neighbouring molecules when the latter have a certain size and constitution. Groups of the order of a million molecules, in domains with the dimensions of a cube of side 0·001 inch, act as unit elementary magnets.

The modern theory of magnetism, which has made great strides since the time of Langevin, is too advanced to be given here.

The law of force between poles in a magnetic medium

The force F between two poles of strengths m_1 and m_2 distant r apart, in a medium of permeability μ, is given by

$$F=\frac{m_1m_2}{\mu r^2}.$$

Thus in the case of paramagnetic media, for which $\mu>1$, the force is less than in a vacuum; in the case of diamagnetic media, for which $\mu<1$, the force is greater than in a vacuum.

When a magnetic pole is placed in a paramagnetic medium a pole of opposite polarity is induced on the cavity in which the pole is contained; this results in a partial neutralisation of the pole. In a diamagnetic medium the induced pole on the walls of the cavity is of the same polarity as the contained pole and hence reinforces it.

In order to prove the formula it is necessary to determine the induction B produced by a pole in a magnetic medium since μ is defined in terms of B. Consider a small spherical pole of strength m_1, placed in a magnetic medium, such as a solution of ferric chloride which is paramagnetic, of permeability μ. Let the strength of the induced pole on the walls of the spherical cavity of the medium in which the pole is contained be $-m_1'$. Then the induction B at distance r from the pole will be the number of lines of magnetic force per sq.cm. of the magnetising field due to the combined pole m_1-m_1', which is $\frac{m_1-m_1'}{r^2}$, plus the lines per sq.cm. due to the magnetisation of the medium, $4\pi J$, where J is the intensity of magnetisation of the medium at a distance r from the pole.

$$\therefore\ B=\frac{m_1-m_1'}{r^2}+4\pi J.$$

The medium is magnetised radially from the induced pole $-m_1'$ and we can imagine that its molecules, acting as elementary magnets, are aligned in spoke-like formations from the pole. Hence the total pole strength on the surface of any sphere of the medium, with centre the pole, must be equal to $+m_1'$. In the case of a sphere of radius r it is equal to $4\pi r^2 J$, since J is the pole strength per unit area at distance r from the pole.

$$\therefore\ 4\pi r^2 J = m_1',$$

$$4\pi J = \frac{m_1'}{r^2}.$$

$$\therefore\ B = \frac{m_1 - m_1'}{r^2} + \frac{m_1'}{r^2}$$

$$= \frac{m_1}{r^2}.$$

It should be noted that the induction in the magnetic medium is equal to the induction which would be produced in a vacuum, a fact to which reference will be made later.

From the definition of μ, the magnetising field required to produce B is $H = \frac{B}{\mu} = \frac{m_1}{\mu r^2}$; the magnetising field, as we have seen, is due to the magnetic pole m_1 and to the induced pole on the cavity $-m_1'$. If another pole m_2 is placed at a distance r from m_1 there will be an induced pole on the cavity in which it is placed, but this induced pole acts equally in all directions on m_2 and exerts no resultant force on it. (Similarly, the electric field inside a charged, hollow, spherical conductor is zero.) Thus the force F exerted on m_2 is $m_2 H$ and hence

$$F = \frac{m_1 m_2}{\mu r^2}.$$

In iron μ varies with H and hence with the distance from the pole; the formula can be used for iron, therefore, only as a crude approximation.

Magnetic induction due to a permanent magnet and to a solenoid

Inside a permanent magnet a state of induction has been set up by a magnetising field no longer present. The lines of induction are closed loops as in Fig. 123, similar to the lines of magnetic force of a solenoid carrying a current. There is, however, an important difference between a permanent magnet and a solenoid. A permanent magnet may be regarded as providing always a fixed quantity of induction. We have seen in the previous section that each pole of the magnet produces the same amount of induction in a magnetic medium surrounding the magnet as it does in a vacuum, whereas its magnetic field in a magnetic medium of permeability μ is $1/\mu$ times that in a vacuum. A solenoid, on the other hand, may be regarded as a constant source of magnetising field H. The induction it produces depends upon the permeability μ of the medium in which it is placed and is equal to μH.

The difference in the effects of a permanent magnet and of a solenoid is due to the induced poles of the medium which surround the poles of the magnet; in the case of a solenoid there are no free induced poles of the magnetic medium in which it is placed.

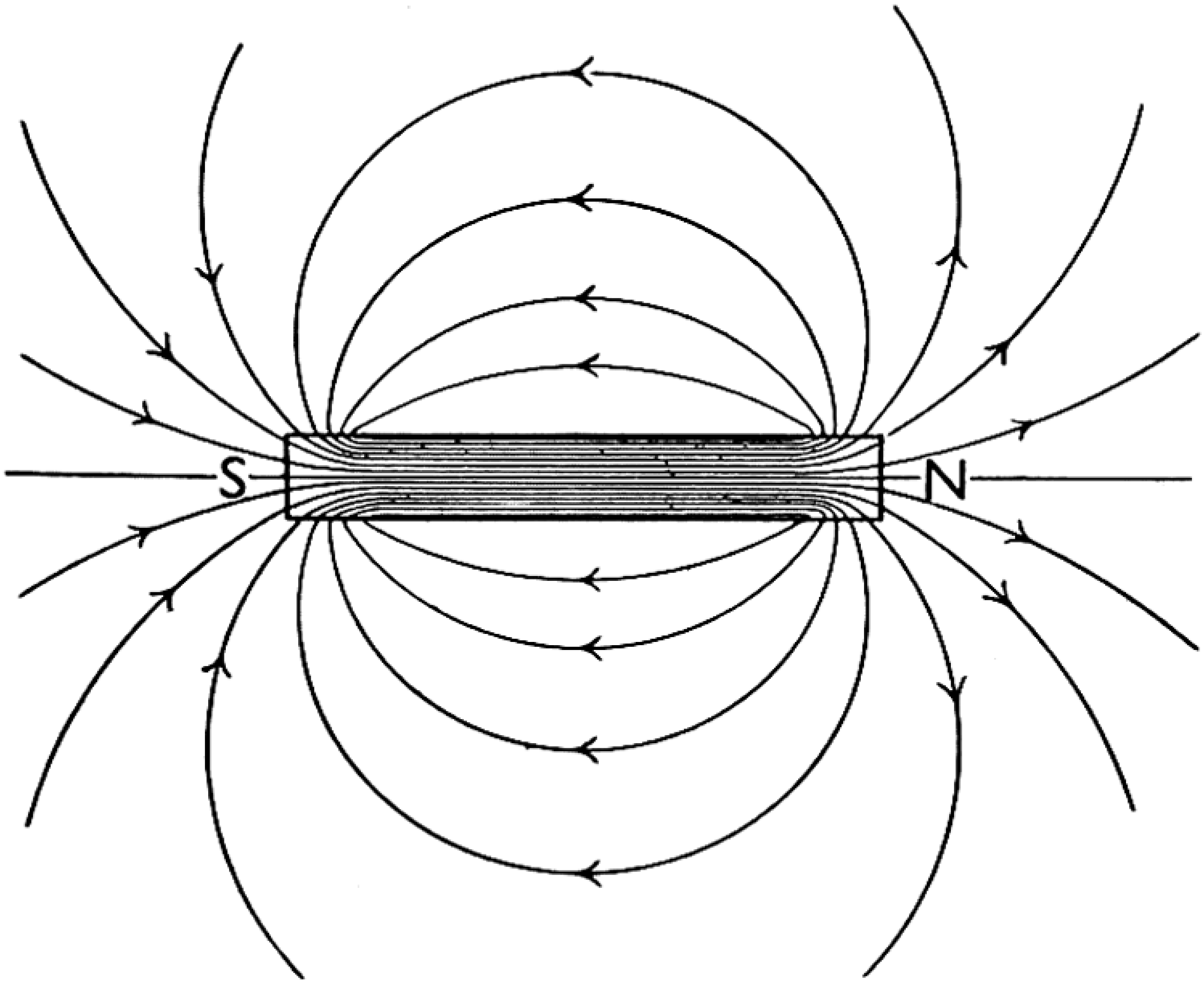


Fig. 123

The demagnetising field due to the free poles of a bar magnet

The free poles at the ends of a bar magnet set up a magnetic field, known as the demagnetising field, inside the bar from the N. to the S. poles in a direction opposite to the induction. This explains the shape and distribution of the lines of induction in Fig. 123. Assuming the bar to have been magnetised by a uniform external field along its length, the lines of induction would have been parallel to its length had they not been modified by the action of the demagnetising field. The induction is therefore less than it would have been had there been no free poles and hence it is preferable to use an iron ring, with no free poles, rather than a straight rod, when performing experiments to determine susceptibility and permeability (see p. 104).

Lines of force and lines of induction

This is perhaps a convenient point to elucidate the difference between lines of magnetic force and lines of magnetic induction. Lines of magnetic force are a graphical representation of the inverse square law as applied to free magnetic poles and the electric current. A unit magnetic pole has 4π lines of force in all media and the apparent change in the force it exerts in a magnetic medium on another pole is accounted for by the lines of force of a free, induced, magnetic pole of the medium.

Lines of magnetic induction give a fuller picture than lines of magnetic force because they represent not only the effects of free poles but also of the magnetisation of the interior of a magnet or of a magnetic medium. The picture of 4π lines of force radiating from a unit N. pole is a useful device but artificial; a more complete representation is that of 4π lines of induction both leaving the pole and entering it as well. It should be remembered that a magnetic pole is due to circular electric currents in the atoms of a substance.

Force on a wire carrying a current in a magnetic medium

The force on a wire carrying a current in a magnetic medium is Bil. In the derivation of the expression Hil on p. 88, suppose the coil and the pole to be immersed in a magnetic medium of permeability μ instead of in air or *in vacuo*. The magnetic field due to the pole at the circumference of the coil then becomes $\frac{m}{\mu r^2}$ and hence the force on the coil is μHil or Bil; (the magnetic field due to the current in the coil is independent of the medium).

Induced E.M.F. in a wire in a magnetic medium

The induced E.M.F. in a wire in a magnetic medium is equal to the rate of cutting of lines of magnetic induction or flux. In the derivation of the expression for the induced E.M.F. on p. 89, imagine the wire to be immersed in a magnetic medium of permeability μ instead of *in vacuo*. The force on an electron is μHev or Bev, and hence the induced E.M.F. is equal to the rate of cutting of lines of magnetic induction rather than of lines of magnetic force.

Summary

In all media H must be used to calculate the force exerted on a magnetic pole and B must be used to calculate the force on a wire carrying a current or the magnitude of an induced E.M.F. *In vacuo* $B=H$.

QUESTIONS

1. Explain the meaning of the terms *magnetic susceptibility, magnetic hysteresis.* Describe in detail how you would investigate the susceptibility and hysteresis of a specimen of iron in the form of a rod, and indicate the results you would expect to obtain (*a*) with soft iron, (*b*) with steel. (O. & C.)

2. Define *intensity of magnetisation* and *magnetic susceptibility*, and describe how you would measure the latter for soft iron.

A cylindrical iron bar of length 4·3 cm. and 5 mm. diameter is placed horizontally and magnetic N. and S. It is then found that there is a neutral point situated 12·8 cm. from each end of the bar. Find the magnetic susceptibility of the material of the bar, assuming that it is uniformly magnetised by the earth's field. (L.)

3. Define the terms *magnetic intensity, magnetic moment* and *susceptibility.*

Four metres below a bench in a laboratory is a steel girder 4 metres long and of total cross-sectional area 60 sq.cm. lying horizontally in the magnetic meridian. If the susceptibility of the steel is 50 and $H=0{\cdot}180$ gauss, what will be the apparent value of H at a point on the bench vertically above the centre of the girder. (O. & C.)

4. Write a short essay on magnetic hysteresis.

What type of curve representing the relation between the magnetic force and the magnetic induction when the force is taken through a complete cycle would correspond to a kind of iron suitable (*a*) for using with dynamos, (*b*) for making permanent magnets? (C. S.)

5. Define the terms *intensity of magnetisation, permeability* and *susceptibility.*

A long straight iron wire 1 metre long and 1·1 mm. diameter is placed vertically in a long solenoid with 10 turns per cm. A mirror magnetometer is placed 20 cm. due W. of the upper pole of the wire. After demagnetisation of the iron, a current of 0·24 ampere is reversed several times in the solenoid and then kept constant. The magnetometer then shows a deflection of 12·5 cm. at 1 metre scale distance. It is assumed that the vertical component of the earth's magnetic field and the direct effect of the current in the solenoid on the magnetometer have been compensated.

Describe briefly how the experiment can be carried out and calculate the intensity of magnetisation, the susceptibility and the permeability of the iron.

(Horizontal component of the earth's magnetic field = 0·18 oersted.) (B.)

6. An iron ring has a circumference of 1000 cm. and a cross-sectional area of 10 sq.cm. How many turns of wire carrying a current of 1 ampere are required to produce a total flux of 10,000 maxwells in the iron, assuming the permeability in these circumstances to be 1000? (C.)

7. An iron rod, 20 cm. long, 1 cm. in diameter and of permeability 1000, is placed inside a long solenoid wound with 5 turns per cm. If 0·5 ampere is passed through the solenoid, what is the magnetic moment of the rod? (C.)

8. A primary coil of 600 turns is wound uniformly on an iron ring of mean radius 8 cm. and cross-sectional area 4 sq.cm. A secondary coil of 5 turns is wound on the top of the primary but insulated from it, and is connected with a ballistic

galvanometer, the total resistance of the secondary circuit being 400 ohms. If the permeability of the iron is 500, what quantity of electricity will be discharged through the galvanometer, when a current of 3 amperes is reversed in the primary? (C. S.)

9. Explain the meaning of the term permeability. A soft iron ring of circumference 3 metres and diameter of cross-section 10 cm. has a permeability 2000. How many turns of wire carrying 1 ampere must be wound on the ring to produce in it a flux of 60,000 units? What would be the effect of introducing into the iron ring an air gap 1 mm. wide? (The value of the permeability may be supposed to remain unaltered.) (C. S.)

10. Discuss the analogy between electric and magnetic circuits. Use the idea of the magnetic circuit to calculate the effect of an air gap of 1 mm. in an iron ring of mean radius 10 cm. and area of cross-section π sq.cm. The iron has a permeability of 1000 and is wound with a coil of 600 turns carrying a current of 1 ampere. (C. S.)

11. Explain what is meant by a *ferromagnetic material.* Give a general account of the wide range of magnetic properties exhibited by different ferromagnetic materials and indicate the potential applications of these properties. (N.)

12. Iron, cobalt and nickel are said to be *ferromagnetic*, oxygen and green vitriol ($FeSO_4 . 7H_2O$) *paramagnetic*, bismuth *diamagnetic*. Describe what is meant by this statement and explain as far as you can one of these phenomena. (O. S.)

13. State and explain the effect of changing the concentration of ferric chloride solution, which is a paramagnetic liquid whose permeability increases with increasing concentration, if the following experiments are performed with apparatus immersed in the solution:

(i) a current is induced in a coil (*a*) by moving a bar magnet into the coil, (*b*) by varying an electric current in a nearby coil;

(ii) a torque is exerted on a small suspended magnet (*a*) by a nearby magnet, (*b*) by a coil carrying an electric current.

CHAPTER 7

THE ELECTRIC CURRENT

Ohm's law

The free electrons in a wire will begin to move and constitute an electric current if a potential difference is applied between the ends of the wire.

The distinction and relation between potential difference and current was discovered by Ohm, who was guided by the analogy of the flow of heat along a conductor due to a difference of temperature.

Ohm's law for metallic conductors

The current through a conductor is proportional to the potential difference between its ends, provided that physical conditions, such as the temperature, remain constant.

Thus

$$\frac{V}{i}=\text{constant},$$

where V=P.D., i=current.

V/i can be used to define numerically what we intuitively regard as the resistance R of a conductor and Ohm's law becomes

$$\frac{\mathbf{V}}{\mathbf{i}}=\mathbf{R}\,.$$

The practical unit of resistance is the *ohm*; it is *the resistance of a conductor through which a current of* 1 *ampere passes when a potential difference of* 1 *volt is maintained across its ends.* The ampere was defined on p. 85 and the volt on p. 6.

It should be noted that Ohm's law is empirical, i.e. based on experiment. It holds with remarkable accuracy for metallic conductors under limited physical conditions.

Resistances in series and in parallel

Two resistances R_1 and R_2, connected as in Fig. 124 (*a*), are said to be in series. Their combined resistance is equal to their sum:

$$R=R_1+R_2.$$

It is clear that the same current flows through both resistances.

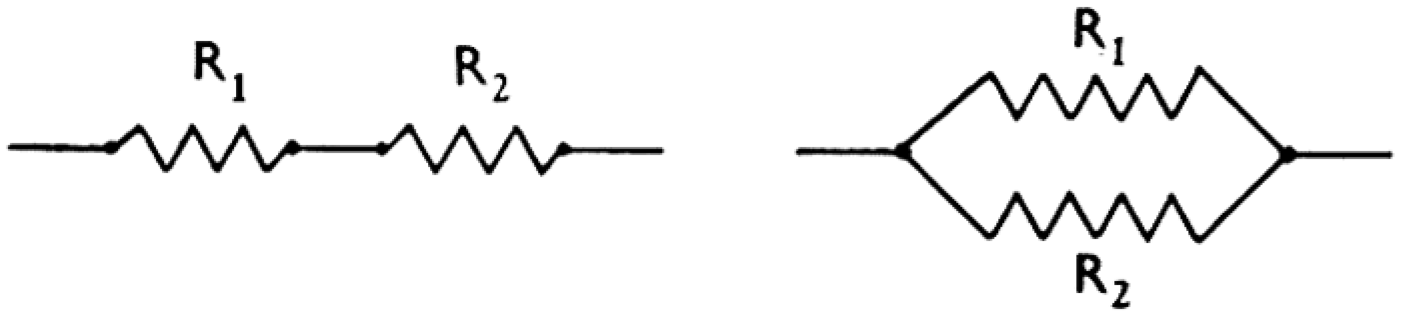
When the resistances are connected as in Fig. 124 (*b*) they are said to be in parallel. Their combined resistance R is given by the formula

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}.$$

In this case the current divides, the larger part going through the smaller resistance and the smaller part through the larger resistance.

If the conductance of a wire be regarded as the reciprocal of its resistance, $1/R$, the significance of the equation $1/R=1/R_1+1/R_2$ becomes apparent. The total conductance of two wires in parallel is equal to the sum of their conductances. As an alternative proof, if V is the P.D. across both resistances and i_1 and i_2 are the currents through each, by Ohm's law,

$$i_1+i_2=\frac{V}{R_1}+\frac{V}{R_2}, \quad \text{also } i_1+i_2=\frac{V}{R}. \quad \therefore \frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}.$$



(*a*) (*b*)

Fig. 124

Ohm's law applied to a complete circuit

A cell is said to produce an electromotive force (see p. 9). The E.M.F. of a cell is measured by the total P.D. that it can produce, i.e. the P.D. that is maintained between its terminals when on open circuit.

When delivering a current, a cell must overcome not only the resistance of the external circuit, but also its own internal resistance. Thus Ohm's law, applied to a complete circuit, is

$$\mathbf{\frac{E}{i}=R+r,}$$

where E volts = E.M.F. of the cell, i amperes = current, R ohms = external resistance, r ohms = internal resistance of the cell.

This equation may be written as follows:

E	$=$	iR	$+$	ir
E.M.F.		voltage to drive current through external resistance		'lost volts' driving current through internal resistance of cell

iR is the P.D. between the terminals of the cell; denoting this by V, $V=E-ir$. Thus the P.D. between the terminals of a cell drops when it delivers a current and 'the lost volts', ir, drive the current through the internal resistance of the cell.

EXAMPLE

A cell of E.M.F. 1·5 volts and internal resistance 1 ohm is connected to a wire of resistance 4 ohms. Find the current flowing and the drop in P.D. between the terminals of the cell.

Applying Ohm's law to the complete circuit

$$i=\frac{E}{R+r}=\frac{1\cdot 5}{4+1}=0\cdot 3 \text{ ampere.}$$

Applying Ohm's law to the 4-ohm resistance

$$V=iR=0\cdot 3\times 4=1\cdot 2 \text{ volts.}$$

Thus the P.D. across the 4-ohm resistance is 1·2 volts; this is also the P.D. between the terminals of the cell, since the ends of the 4-ohm resistance are connected to them. The drop in P.D. between the terminals of the cell when it delivers a current is therefore 1·5 − 1·2 = 0·3 volt. Alternatively,

$$\begin{aligned}\text{Lost volts} &= ir\\ &= 0\cdot 3\times 1 = 0\cdot 3 \text{ volt.}\end{aligned}$$

The simplest way of finding the internal resistance of a cell is to measure the drop in P.D. with a voltmeter when a known current, as measured by an ammeter, is taken from the cell (see Fig. 125).

The internal resistance is obtained from the equation

$$\text{Lost volts}=ir.$$

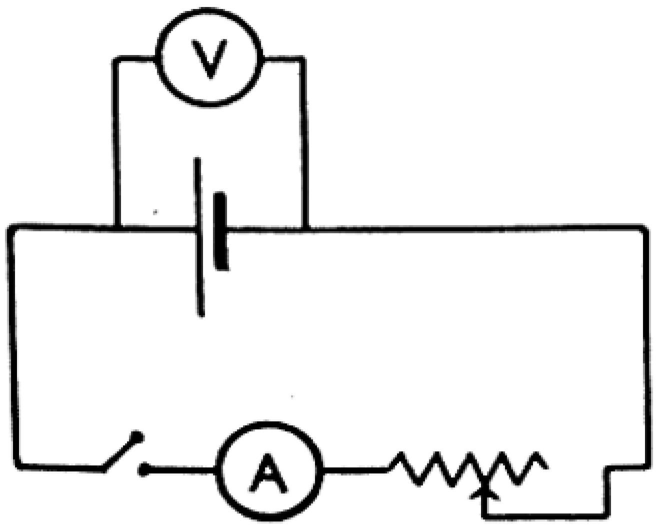
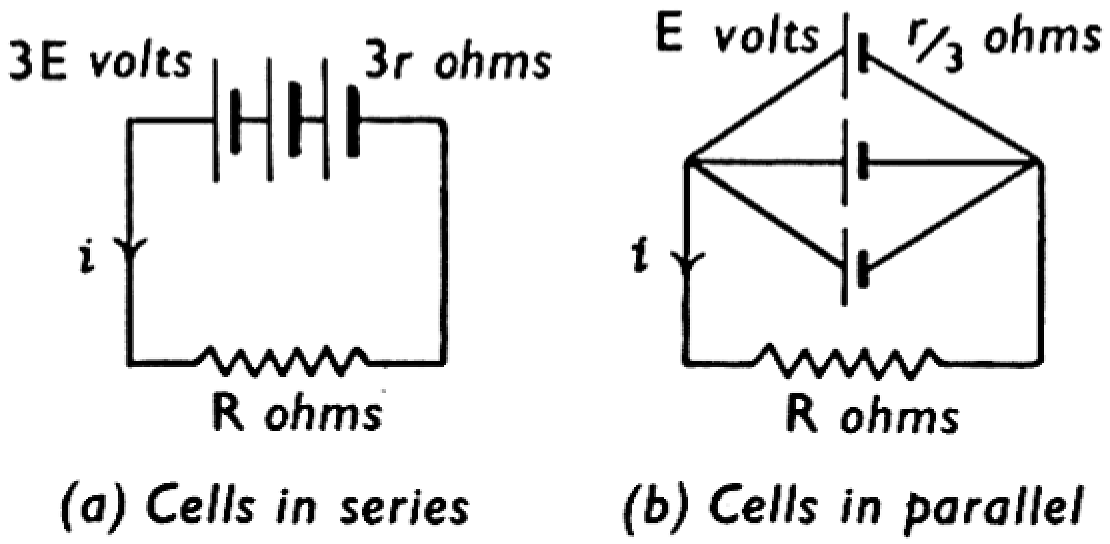


Fig. 125



(a) Cells in series (b) Cells in parallel

Fig. 126

Arrangement of cells

A number of cells is called a battery. Cells may be connected in series or in parallel.

If n cells are connected in series (Fig. 126 (a))

$$\frac{nE}{i}=R+nr.$$

If m cells are connected in parallel (Fig. 126 (b))

$$\frac{E}{i}=R+\frac{r}{m}.$$

Cells are seldom connected in parallel, but it is of interest to note that a battery gives its maximum current when its constituent cells are arranged so that its internal resistance is equal to the external resistance. This is not the most economical arrangement, however, since just as much energy is wasted inside the battery as is liberated in the external circuit.

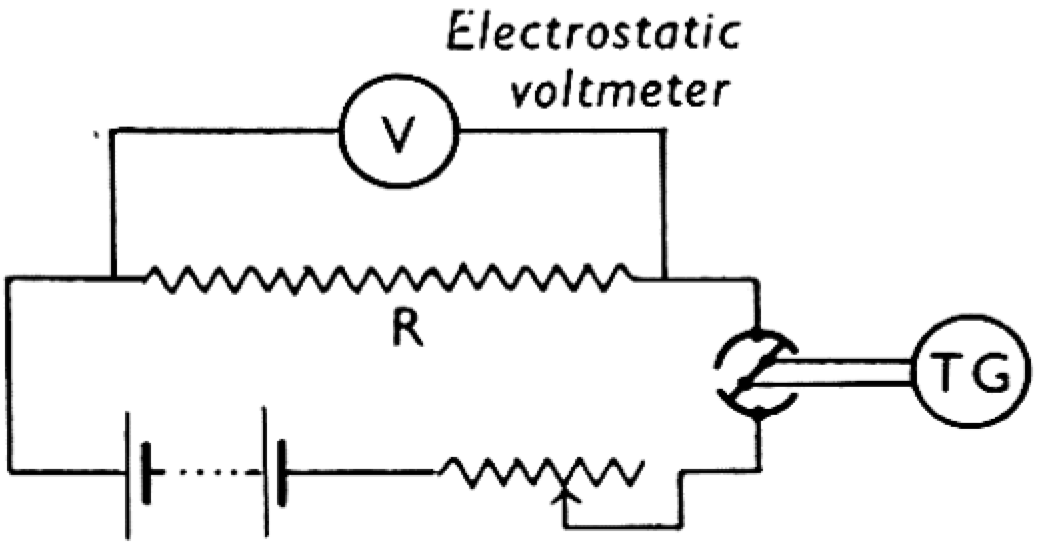


Fig. 127

Experimental verification of Ohm's law

To verify Ohm's law, the current i, through a fixed resistance R (see Fig. 127), is varied and measured by a tangent galvanometer or ammeter, while the corresponding potential differences V across R are measured with an electrostatic voltmeter. Then, if Ohm's law is true, $V/i=$constant.

A current voltmeter could not be employed, since it is really a high-resistance galvanometer and is calibrated assuming the truth of Ohm's law.

Ohm's own method

Fig. 128 represents Ohm's method of verifying his law. After experiencing great difficulty owing to the lack of constancy of the E.M.F. of his cells he used a copper-bismuth thermocouple as the source of E.M.F. In one set of experiments the junctions were kept at 0 and 100° C. The mercury cups were merely terminals between which were placed copper wires of the same cross-section, varying in length from 2 to 130 inches. The current was measured by the angle through which the torsion head had to be turned in order to bring the magnet back to its original position parallel to the wires. Thus Ohm obtained pairs of readings of length of wire, l, and angle, θ; he showed that they obeyed the relation $\theta \propto (l+l')^{-1}$, l' being a constant representing the length of wire which was equivalent to the remainder of the circuit. This corresponds to the expression $i = E/(R+r)$.

Ohm then changed the value of E by raising the temperature of the cold junction of the thermocouple to that of the room, and again found the same relation to hold. He also used different metals and found their relative resistances; he showed, too, that the resistance of a wire is inversely proportional to its area of cross-section.

Ohm's work was received with hostility in Germany; its value and fundamental nature were not recognised by his fellow-countrymen until he had been awarded the Copley medal of the Royal Society.

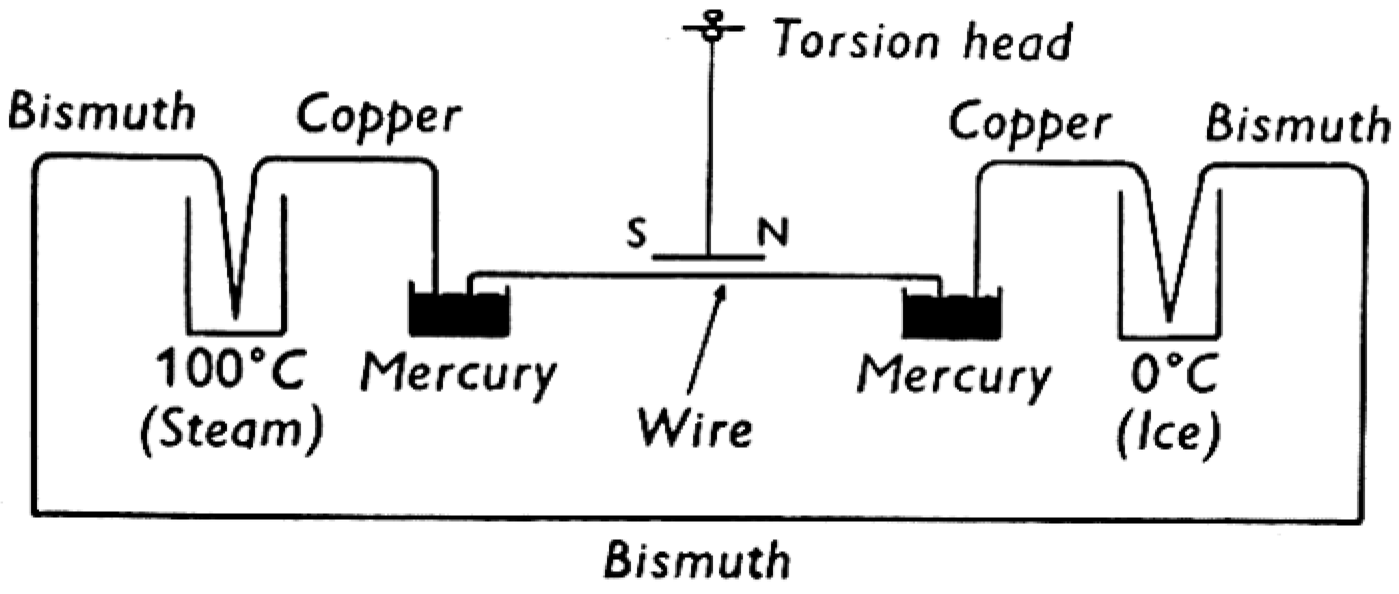


Fig. 128

Theory of metallic conduction

An electric current through a wire is a drift of free electrons. We can calculate very simply how quickly the electrons move. The current i is given by

$$i = ANev,$$

where A = area of cross-section of the wire, N = number of free electrons per c.c., e = charge on the electron, v = velocity of the electrons.

In the case of a copper wire it can be assumed that the number of free electrons per c.c. is equal to the number of atoms per c.c. = $8{\cdot}3 \times 10^{22}$. The charge of the electron is $1{\cdot}6 \times 10^{-19}$ coulomb. Suppose a current of 1 ampere flows through a copper wire of area of cross-section 0·01 sq.cm.

$$v = \frac{1}{0{\cdot}01 \times 8{\cdot}3 \times 10^{22} \times 1{\cdot}6 \times 10^{-19}} = 0{\cdot}0075 \text{ cm. per sec.}$$

The velocity of the electrons depends on the applied voltage. If the voltage per cm. required to drive the current through the wire is calculated it can be shown that the mobility of the electrons is 44 cm. per sec. per volt per cm.

Although the electrons drift comparatively slowly, the speed at which the disturbance travels through, say, the 2000 miles of Atlantic cable, when the current is switched on at one end, is nearly the velocity of light. Thus all the free electrons in the cable begin to drift almost simultaneously.

The drift of the electrons is superimposed upon their random thermal agitation. The electrons continually collide with the atoms of the wire, giving up energy and thereby increasing the thermal vibration of the atoms. In this way heat is generated in the wire.

Kirchhoff's laws

The following corollaries to Ohm's law, known as *Kirchhoff's laws,* are useful in calculating the currents in complicated circuits.

(1) *The algebraic sum of the currents at a junction in a circuit is zero, i.e.* $\Sigma i=0$.

This means that there is no accumulation of charge at any point in the circuit.

(2) *In any closed circuit, the algebraic sum of the* E.M.F.*'s is equal to the algebraic sum of the products of the current and resistance in each part of the circuit, i.e.* $\Sigma E=\Sigma iR$.

This is an application of Ohm's law.

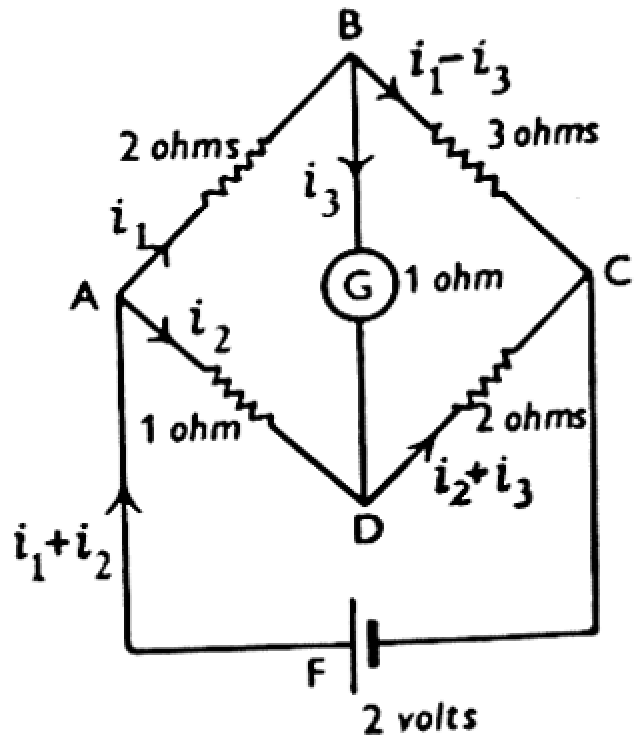


Fig. 129

EXAMPLE

Calculate the currents in Fig. 129. The cell has a negligible internal resistance.

Kirchhoff's first law has been applied in the figure by inserting the currents, i_1, i_2, i_3, etc.

Applying Kirchhoff's second law to

ABD, $2i_1+i_3-i_2=0$,

BCD, $3(i_1-i_3)-2(i_2+i_3)-i_3=0$,

FABC, $2i_1+3(i_1-i_3)=2$.

The law can be applied also to *FADC*, but only three equations are required; moreover, this fourth equation can be derived from the above so that it contributes nothing new.

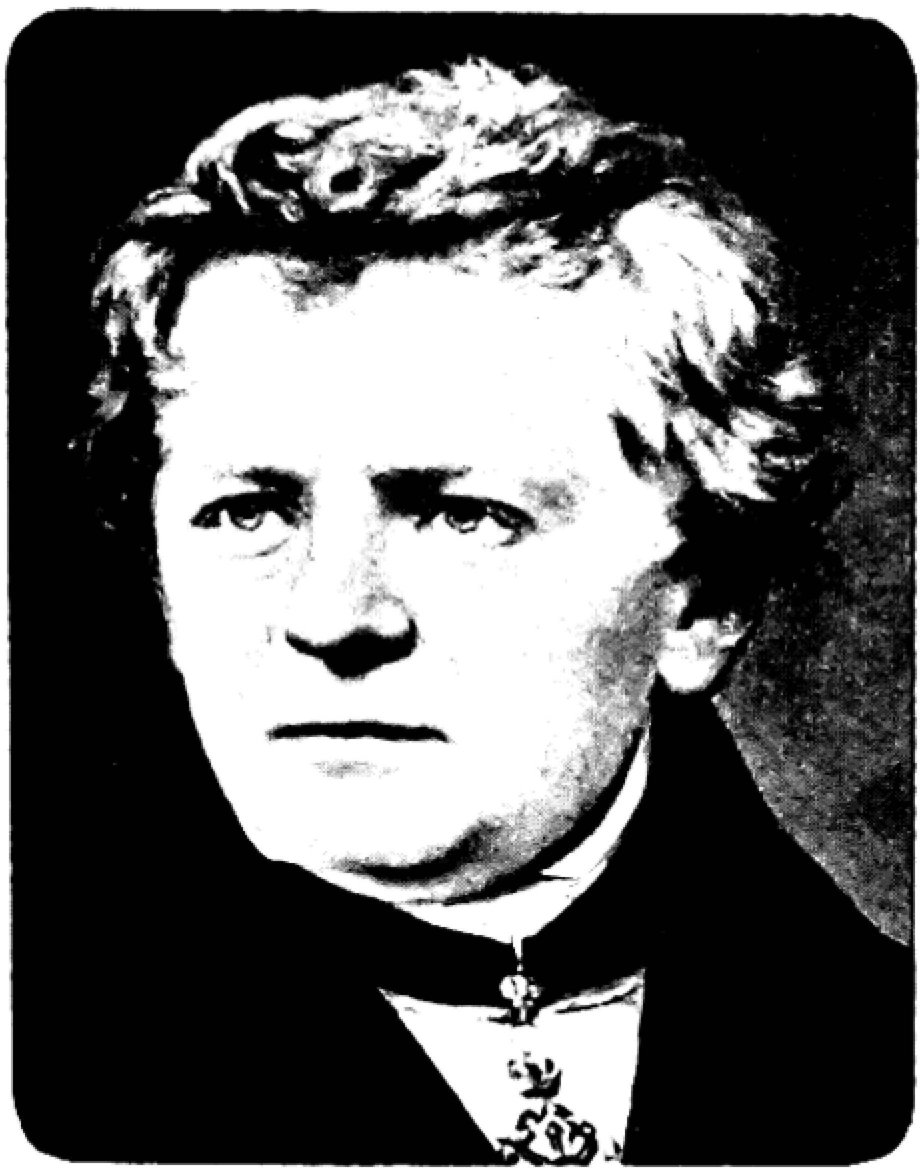
Solving the equations,

$$i_1=\frac{16}{43}\text{ ampere,}\quad i_2=\frac{30}{43}\text{ ampere,}\quad i_3=-\frac{2}{43}\text{ ampere.}$$

Since i_3 is negative it is clear that the arrow was marked in Fig. 114 in the wrong direction; such an error is always automatically corrected by a minus sign in the solution.

The heating effect of an electric current

In 1841 Joule showed experimentally that the heat generated by an electric current is proportional to the time and to the square of the current.



GEORGE SIMON OHM

JAMES PRESCOTT JOULE

GEORGE SIMON OHM (1789–1854), *discoverer of the famous law which bears his name, was a teacher of mathematics and physics in a school in Cologne, where, working completely alone, he pursued his experimental investigations, using the skill learnt from his father, a locksmith, to construct his own apparatus. In 1826 an account of his experiments was published and also, in 1827, a paper entitled 'The Galvanic Circuit Investigated Mathematically', from which the experimental basis was omitted. Unfortunately only the latter attracted attention and was denounced as fantasy. Ohm lived in poverty and obscurity until the importance of his work was recognised in England and he was awarded the Copley Medal of the Royal Society. He achieved his ambition, a university professorship, a few years before his death.*

JAMES PRESCOTT JOULE (1818–1889) *was the son of a brewer at Salford, near Manchester, and had the good fortune to be taught science by John Dalton. His father built him a laboratory and he devoted his whole life to his researches. He was one of the first to formulate clearly the principle of the conservation of energy. At a meeting of the British Association in 1847 his paper on this subject would have attracted little attention had not William Thomson initiated a discussion by his striking and enthusiastic comments. Joule spent about forty years making determinations of the mechanical equivalent of heat by a number of different methods. He made his own galvanometer and graduated it with a voltameter. Resistances were measured in terms of a copper resistance by means of the galvanometer using Ohm's law.*

This can be deduced from the definition of the volt given in Chapter 1, viz. a P.D. of 1 volt exists between two points if 1 joule of work is done when a charge of 1 coulomb passes from one point to the other.

Suppose a current i amperes flows for a time t sec. in a wire between whose ends the P.D. is V volts.

$$\text{Charge which has flowed} = it \text{ coulombs.}$$

$$\therefore\ \text{Work done} = Vit \text{ joules}$$

$$= \frac{Vit}{J} \text{ calories.}$$

J represents the *mechanical equivalent of heat*, which is the amount of mechanical work which will produce unit quantity of heat; its value is 4·2 joules per calorie.

Thus heat generated

$$= \frac{\mathbf{Vit}}{\mathbf{4{\cdot}2}} \text{ calories}$$

$$= \frac{\mathbf{i^2Rt}}{\mathbf{4{\cdot}2}} \text{ calories,} \quad \text{since } V = iR.$$

Although Ohm's law was discovered before Joule's law of the heating effect of an electric current, it is also logical to regard the latter as the more fundamental and to use it to define resistance and deduce Ohm's law as follows.

From experiment:

$$\text{Heat} \propto i^2t \text{ calories.}$$

$$\therefore\ \text{Work} \propto i^2t \text{ joules.}$$

$$\therefore\ \text{Work} = Ri^2t \text{ joules,}$$

where R is a constant for metals which we define as resistance.

$$\therefore\ \text{Work} = Ri \times Q, \quad \text{where } Q = it = \text{charge.}$$

$$\therefore\ \frac{\text{Work}}{Q} = Ri.$$

But Work/Q is, by definition, the P.D. V. Hence $V = Ri$, which is Ohm's law.

The advantage of this approach is that it reveals clearly the true nature of P.D. as work per unit charge.

Determination of the mechanical equivalent of heat

The mechanical equivalent of heat may be determined by measurement of the heating effect of an electric current.

A simple method is indicated in Fig. 130. The current through a heating coil is measured during a known time and also the P.D. across the coil.

$$\text{Work done} = Vit \text{ joules.}$$

The heat generated is measured by finding the product of the total water equivalent of the calorimeter plus contents and its rise in temperature.

$$J = \frac{\text{Total water equivalent} \times \text{rise in temperature}}{Vit}.$$

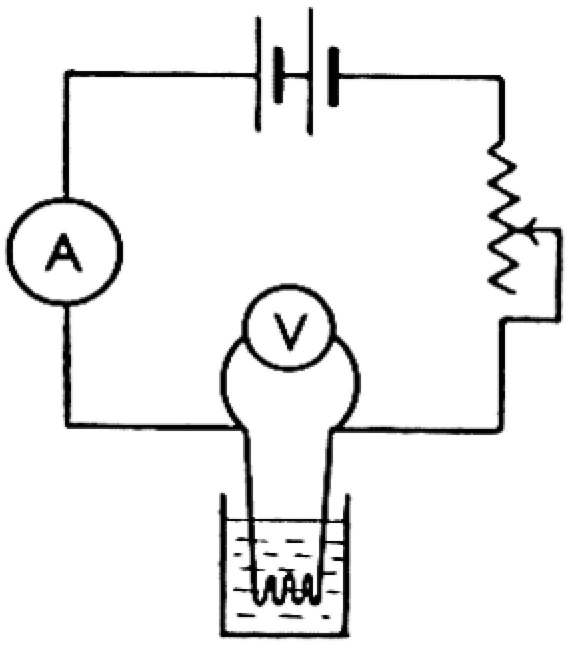


Fig. 130

The two most accurate electrical determinations of the mechanical equivalent of heat were those of Jaeger and Steinwehr made in 1921, and of Osborne, Stimson and Ginnings made in 1939. These have now supplanted the determination of Callendar and Barnes in 1902, employing a constant flow method. R. T. Birge, in his critical survey of the determination of physical constants, gives Jaeger and Steinwehr's result as 4·1832 ± 0·0009 international joules per 15° calorie and that of Osborne, Stimson and Ginnings at 4·1850 ± 0·0004 international joules per 15° calorie.

The method of Jaeger and Steinwehr was very similar in principle to that already described, but extreme care was taken with the necessary measurements. A large mass of water, 50 kg., was heated in a thin-walled calorimeter by a current in a coil. The rise in temperature, which was small, 1·4° C., was measured by a platinum resistance thermometer. The P.D. across the heating coil and the current through it were both measured by means of a potentiometer in conjunction with a Weston standard cell (see p. 151).

The water in the calorimeter was stirred by electrically driven paddles and the calorimeter was surrounded by an air space, enclosed by a constant-temperature water jacket.

Electric power

Power is a technical term in mechanics denoting rate of doing work. Its unit on the c.g.s. system is the watt, defined as 1 joule per second. Since 1 ampere is 1 coulomb per second, it follows from the definition of the volt that

$$\text{Amperes} \times \text{volts} = \text{watts.}$$

Electric lamps and heaters are rated in watts to denote the rate at which they absorb electrical energy.

Electrical energy is sold by the *kilowatt-hour*, which is the energy absorbed in 1 hour at a rate of 1000 watts, i.e. 1 kilowatt. The kilowatt-hour is often called the Board of Trade Unit (B.T.U.).

EXAMPLE

What factors determine the steady temperature of a wire through which an electric current is flowing?

Assuming that for small excesses of temperature, the heat lost per second from the surface of a wire is proportional to θ, the temperature difference between the wire and its surroundings, show that $\theta \propto i^2\rho/r^3$, where i is the current, ρ is the resistivity of the material of the wire, and r is the radius of the wire.

The temperature of the wire depends upon the rate at which heat is generated in the wire and also upon the rate at which heat is lost from the surface. When a certain temperature is reached the two rates are equal and no further rise in temperature occurs. The rate of loss of heat depends upon the area and nature of the surface of the wire, and, if Newton's law of cooling holds, upon the difference in temperature between the wire and its surroundings.

Let H=emissivity of the wire, i.e. the heat emitted per second per unit area per degree difference in temperature between the wire and its surroundings.

Resistance of wire per cm. length$=\rho/\pi r^2$ (see p. 155).

Surface area of wire per cm. length$=2\pi r$.

Heat generated in 1 cm. of wire per second=Heat lost per second, i.e.

$$\frac{i^2\rho/\pi r^2}{J}=H\,2\pi r\,\theta.$$

$$\therefore\ \theta=\frac{1}{2\pi^2 JH}\frac{i^2\rho}{r^3},$$

$$\theta\propto\frac{i^2\rho}{r^3}.$$

The transmission of electric power

It is preferable to transmit electric power along cables in the form of a small current at a high potential rather than a large current at a low potential, since the heat loss in the cable is smaller in the former case. The transmission voltage in the main lines of the Grid is 132,000 volts.

The power loss in the cables is i^2R watts, where i is the current and R the resistance. Clearly this is small when i is small.

EXAMPLE

A D.C. dynamo, of output 20 kilowatts, supplies power through two cables each of resistance 1 ohm. Calculate the P.D. at the receiving end of the cables, and also the power loss in the cables, if the P.D. at the generating end is (*a*) 500 volts, (*b*) 10,000 volts.

(*a*) When generated at 500 volts, current $=\frac{20{,}000}{500}$

$=40$ amperes.

$\therefore$ Potential drop in the cables $=iR$

$=40\times 2=80$ volts.

$\therefore$ Potential difference at receiving end $=500-80$

$=420$ volts.

Loss in cables $=i^2R$

$=40^2\times 2$

$=3200$ watts.

(*b*) When generated at 10,000 volts, current $=\frac{20{,}000}{10{,}000}$

$=2$ amperes.

$\therefore$ Potential drop in the cables $=iR$

$=2\times 2$

$=4$ volts.

$\therefore$ Potential difference at receiving end $=10{,}000-4$

$=9996$ volts.

Loss in cables $=i^2R$

$=2^2\times 2$

$=8$ watts.

Edison and electric lighting

One of the most important practical applications of the heating effect of an electric current is electric lighting. Some insight into the way in which the study of electricity was given practical significance can be gained from the work of Thomas Alva Edison (1847–1931).

Edison had no academic training in science and his knowledge of its theory was not very profound, but he was an inventive genius. 'I do not search for the laws of Nature', he said, 'and I have made no great discoveries of such laws. I am only a professional inventor. My studies and experiments have been conducted entirely with the object of inventing that which will have commercial utility.'

It may be argued that the work of the academic scientist is in a higher category than that of the inventor, being inspired by pure curiosity and free from commercial taint. But as J. G. Crowther* has written, 'The full importance of the discoveries of Faraday and Clerk Maxwell was not clear until inventors, such as Edison and Marconi, had proved that they were of profound value, and were far more than ingenious intellectual tricks.'

---

* J. G. Crowther, *American Men of Science.*

Edison, greatest of American inventors, was encouraged and aided by a society peculiarly receptive to inventions, due to its independent spirit and its rapidly increasing surplus capital which sought an outlet in new industries.

Edison was not first in the field of electric lighting. An arc lamp supplied by a dynamo was installed in the Dungeness lighthouse, England, in 1862, and twelve years later arc lighting was tried in London in Holborn Viaduct, and on the Thames Embankment. This form of street lighting was not satisfactory; the carbons hissed and burnt away rapidly, the light was dazzling and cast harsh shadows. An Act of Parliament stipulated that after twenty years all electric light plants were to be bought by the Government. This effectively strangled private enterprise, and England, after being the pioneer in the adoption of electric light, subsequently lagged behind. Not until eight years later was this law repealed.

In 1875 Edison first saw the arc lamp. 'It was easy to see', he said, 'what the thing needed; it wanted to be subdivided. The light was too bright and too big. What we wished for was little lights and a distribution of them to people's houses in a manner similar to gas.' He realised that the lamps must be in parallel, and threw himself with unremitting industry into the task of producing a commercial lamp.

By this time, on the proceeds of large earnings from his telegraphic inventions, he had established a laboratory at Menlo Park, a small group of houses round a tiny railway station in open country. Here he had gathered a group which grew eventually to a hundred men, mainly mechanics, but some academic scientists and mathematicians, whom he drove and worked with ruthless energy. His life as a telegraph operator had inured him to working throughout the night, and he would work for days and nights on end with only a few odd hours of sleep, lying on the floor or bench of the laboratory. He expected his men to do the same. It was during this period that he made his most brilliant inventions, including the gramophone, and he became known throughout the world as 'The Wizard of Menlo Park'.

For thirteen months he and his staff experimented with metal filaments in an evacuated glass bulb. He tried platinum filaments and platinum alloyed with iridium, but could not find a material whose melting-point was sufficiently high to permit bright incandescence to be maintained. He then turned his attention to carbon and tried a carbonised rolled-out mixture of lampblack and tar; this was too brittle to last. He learnt the importance of continuing to pump out the air after the filament was hot, particularly in the case of carbon with its marked property of occluding gases. In October 1879 he acquired one of the new mercury vacuum pumps invented by Sprengel, which could reduce the pressure to one-millionth of an atmosphere and which was to play an indispensable part in the researches leading to the discovery of the electron.

Edison decided to try to carbonise cotton thread for a filament. A piece of thread in the shape of a hairpin was placed in a nickel mould and baked for five hours in a muffle furnace. The patience and persistence of Edison and his workmen are revealed in his own words: 'All night Bachelor, my assistant, worked beside me. The next day and the next night again, and at the end of that time we had produced one carbon out of an entire spool of Clark's thread. Having made it, it was necessary to take it to the glass-blower's house. With the utmost precaution Bachelor took up the precious carbon, and I marched after him, as if guarding a mighty treasure. To our consternation, just as

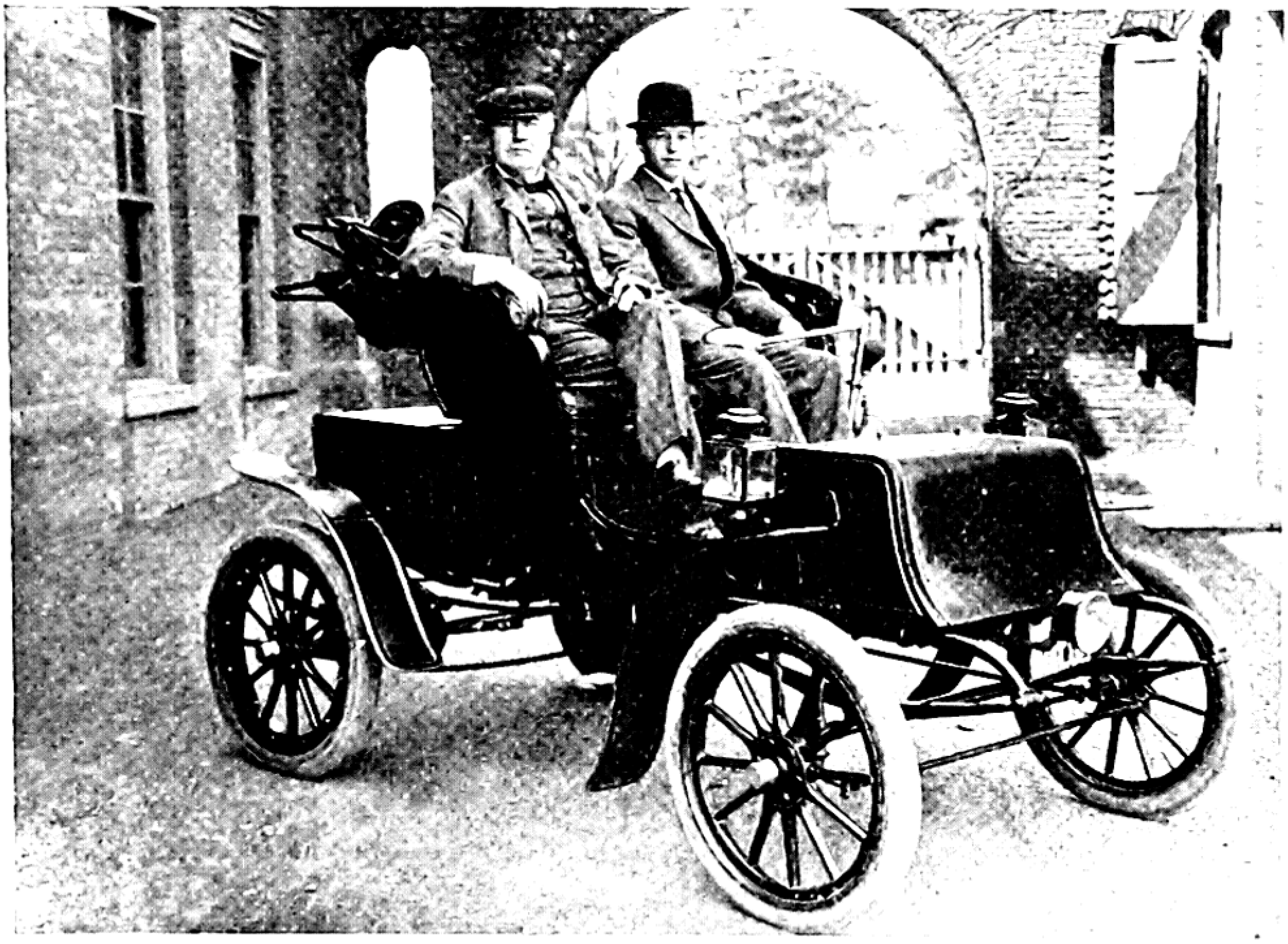


Fig. 131. Edison setting out in a specially made electric car to test the storage battery he invented. With him is one of his sons.

we reached the glass-blower's bench the wretched carbon broke. We turned back to the main laboratory and set to work again. It was late in the afternoon before we had produced another carbon, which was again broken by a jeweller's screw-driver falling against it. But we turned back again, and before night the carbon was completed and inserted in the lamp. The bulb was exhausted of air and sealed, the current turned on, and the sight we had so long desired to see met our eyes.

'Then we sat down and looked at that lamp. We wanted to see how long it would burn. The problem was solved if the filament would last. The day was—let me see—October 21st 1879. We sat and looked and the lamp continued to burn, and the longer it burned the more fascinated we were. None of us could go to bed, and there was no sleep for any of us for forty hours. We sat and just watched it, with anxiety growing into elation.'

The lamp burned for 45 hours before the filament broke. Edison was confident that it was now only a matter of time before he produced a commercial lamp. He began to carbonise all manner of things from paper and wood shavings to the red hairs from the beard of a friend. One day he picked up in his laboratory a palm-leaf fan along the edge of which was a thin flexible strip of bamboo. He carbonised the bamboo and found that it gave the best results so far.

At once he searched all the literature on the bamboo, and discovered that there were twelve hundred different varieties. He sent out men to Burma, China, Japan, India, Ceylon, Mexico and the Amazon to bring back specimens. One of his men died of yellow fever in Havana, and in all the search cost 100,000 dollars. The best bamboo for filaments came from Japan, and as late as 1908 was used in certain types of lamps.

In the meantime, however, Edison had begun to use a 'squirted' filament. A solution of cellulose was forced through a die and when dry looked like catgut. This was carbonised at very high temperature, and proved strong and durable.

The carbon filament has now given place to the tungsten filament which, although originally produced by the 'squirted process', is now made from drawn wire.

Other men besides Edison had endeavoured to produce a filament lamp, and Sir Joseph Swan made a lamp with a filament of carbonised paper. The two inventors came to an agreement and formed the Ediswan Company to manufacture lamps in England.

Having invented a lamp, despite numerous predictions that the feat was impossible, Edison turned his attention to the problems of the generation and distribution of electrical power. Only 40% of the energy produced by the Gramme dynamo was available in the arc. There was a mistaken notion prevalent that the internal resistance of the dynamo ought to be equal to the external resistance. Edison wanted energy outside the dynamo where it brought profits. Besides reducing the internal resistance, Edison made the armature core of laminations insulated by paper and later by mica, to reduce eddy currents. He investigated the magnetic properties of iron, particularly the field required for saturation. In consequence he greatly increased the iron in the dynamo and raised the efficiency to 90%.

Owing to the voltage drop in the cables, the lamps at a distance from the dynamo were dimmer than those near. To minimise this, Edison introduced the feeder system. The main conductors for the various areas were connected to the dynamo by feeders, and the potential drop in the feeders compensated by an extra voltage at the dynamos.

Edison invented an electrolytic meter consisting of two zinc plates in a solution of zinc sulphate. The anode lost weight and the cathode gained; the plates were taken out, dried and weighed to calculate the service charge. Consumers, however, did not like a meter on which they had no check, and the electrolytic meter was subsequently replaced by a motor type.

A generating station was opened in Pearl Street, New York, in September 1882, and numerous difficulties were surmounted successfully as they arose. 'Finally we got our feeders all down', said Edison, 'and started to put on an engine and turn over one of the machines to see how things were....Then we started another engine and threw them in parallel. Of all the circuses since Adam was born we had the worst then. One engine would stop and the other would run up to about a thousand revolutions and then they would see-saw.' One of the dynamos was driving the other as a motor and then their roles would reverse.

On a later occasion a young man burst into the dynamo room and shouted, 'Your electricity has got into the pavement up in Fulton St. and all the horses are dancing.'

By the end of 1882 Edison had wired 900 buildings in New York and fixed 14,000 lamps.

Edison made mistakes. He believed he could prevent magnetic leakage in his dynamos by encasing them in zinc and that electromagnets with multiple legs, separately wound, were more efficient than those with single legs.

Edison left Menlo Park in 1882 and on the site has been erected a bronze tablet, inset in a boulder of granite, bearing the following inscription:

ON THIS SITE
1876–1882

THOMAS ALVA EDISON

BEGAN HIS WORK
OF SERVICE FOR THE WORLD
TO ILLUMINE THE PATH OF PROGRESS
AND
LIGHTEN LABOUR FOR MANKIND

THIS TABLET IS PLACED BY THE
EDISON PIONEERS TO ATTEST THE
GRATITUDE OF THE INDUSTRIES
HE DID SO MUCH TO CREATE

DEDICATED MENLO PARK N.J.
MAY 16 1925

QUESTIONS

1. State Ohm's law and discuss its experimental verification.

2. Why is it that, in general, a voltmeter placed across the terminals of a cell gives a lower reading when the cell is giving current than when it is not?

Two identical cells are connected first in parallel and then in series to a resistance of 20 ohms. If the readings of a voltmeter connected across the resistance are 1·28 and 1·60 in the two cases, find the internal resistance of the cell and its E.M.F. on open circuit. (O. & C.)

3. Two coils, one of 100 ohms and the other of 200 ohms resistance, are connected in series with a 4-volt battery of negligible resistance. A voltmeter, of resistance 200 ohms, is connected in parallel with each of the coils in turn. What voltage will it show in each case? (L.)

4. The resistance of a telegraph wire between two stations is 55 ohms, the receiving instrument has a resistance of 60 ohms and needs a minimum operating current of $\frac{1}{20}$ ampere. Find the smallest number of cells, each of E.M.F. 2·0 volts and internal resistance 0·2 ohm, which, when joined in series, will enable a signal to be sent from one station to the other. (N.)

5. How would you arrange twelve similar cells, each of internal resistance 1 ohm so as to send a maximum current through a wire of 0·70 ohm resistance? (C. S.)

6. State and explain Kirchhoff's laws for the distribution of currents in a network of conductors.

Two batteries of E.M.F.'s 3 and 1 volt respectively, and each of internal resistance 2 ohms, are connected in parallel and their terminals are joined by a resistance of 5 ohms. Determine the current in each branch of the circuit. (L.)

7. The positive poles *A* and *C* of two cells are connected by a uniform wire of resistance 4 ohms, and their negative poles *B* and *D* by a uniform wire of resistance 6 ohms. The middle point of *BD* is connected to earth. The E.M.F.'s of the cells *AB* and *CD* are 2 volts and 1 volt respectively, and their resistances 1 ohm and 2 ohms respectively. Find the potential at the middle point of *AC*. (O. & C.)

8. Two cells whose E.M.F.'s are 1·6 and 2·4 volts have internal resistances of 2 and 4 ohms respectively. The two positive poles are joined by a wire of 6 ohms and the two negative poles by a wire of 8 ohms. If another wire of resistance 10 ohms is placed between the midpoints of these two wires, what is the P.D. between its ends? (O. & C.)

9. State the laws which govern the distribution of current in a network of conductors.

Four wires, *AB*, *BC*, *CD* and *DA*, of resistance 2, 4, 2 and 4 ohms respectively, are connected in series. The positive pole of a cell of E.M.F. 2 volts and internal resistance 1 ohm is connected to *A* and its negative pole to *C*. The positive pole of a cell of E.M.F. 1 volt and internal resistance 2 ohms is connected to *D* and its negative pole to *C*. Find the current in each branch of the network, assuming that the resistances of the connecting wires may be neglected. (C. S.)

10. State and explain Kirchhoff's laws giving the division of current in a network.

A tetrahedron *ABCD* is built up of wires each of resistance 2 ohms. The wires *AB*, *BC*, *BD* are of substance *p* and *AD*, *AC*, *DC* are of substance *q*, such that, if a circuit is made of *p* and *q* in series, and the junctions are kept at a temperature difference of 100° C., then an E.M.F. *E* is produced, tending to drive current from *p* to *q* across the hot junction. Calculate the currents in the wires of the tetrahedron if the points *A* and *C* are placed in ice, and *B* and *D* in boiling water. (C. S.)

11. How many of the following quantities must be known in order to determine the rest: current in amperes, P.D. in volts, resistance in ohms, energy in joules, power in watts, charge in coulombs? Write down the relations between these quantities.

12. State the laws of the heating effect of an electric current. Describe in detail an experiment to show how the rate of production of heat in a conductor during the passage of an electric current depends upon the strength of the current.

A 240-volt electric kettle which has a heating element of resistance 80 ohms takes 14 minutes to heat $1\frac{1}{2}$ litres of water from 20° C. to boiling-point. What fraction of the heat generated is used to heat the water? (N.)

13. A recently marketed steam generator uses bare electrodes immersed in water and is served by the A.C. mains. Why would a D.C. supply be unsuitable? Calculate the mass of water at 100° C. vaporised per kilowatt hour assuming that the latent heat of steam at this temperature is 540 calories per g. and that 1 calorie is equivalent to 4·2 joules. (N.)

14. How is electrical energy measured?

An undergraduate left his electric fire (230 volts, 5 amperes) switched on all night (9 hours). The college fined him 5*s*. With electricity at 1*d*. per B.T.U. did this cover the loss to the college? (O. S.)

15. Making plausible assumptions about the quantities involved, estimate how much electrical energy is required to heat your morning bath.

Suggest a suitable power for an electric heater for this purpose and calculate for how long it would have to be switched on. (C. S.)

16. A condenser of capacity 100 microfarads is charged to a potential of 1000 volts. It is discharged through a coil placed in a calorimeter of total mass 5 g. and average specific heat 0·5. Calculate the rise of temperature in the calorimeter. (C. S.)

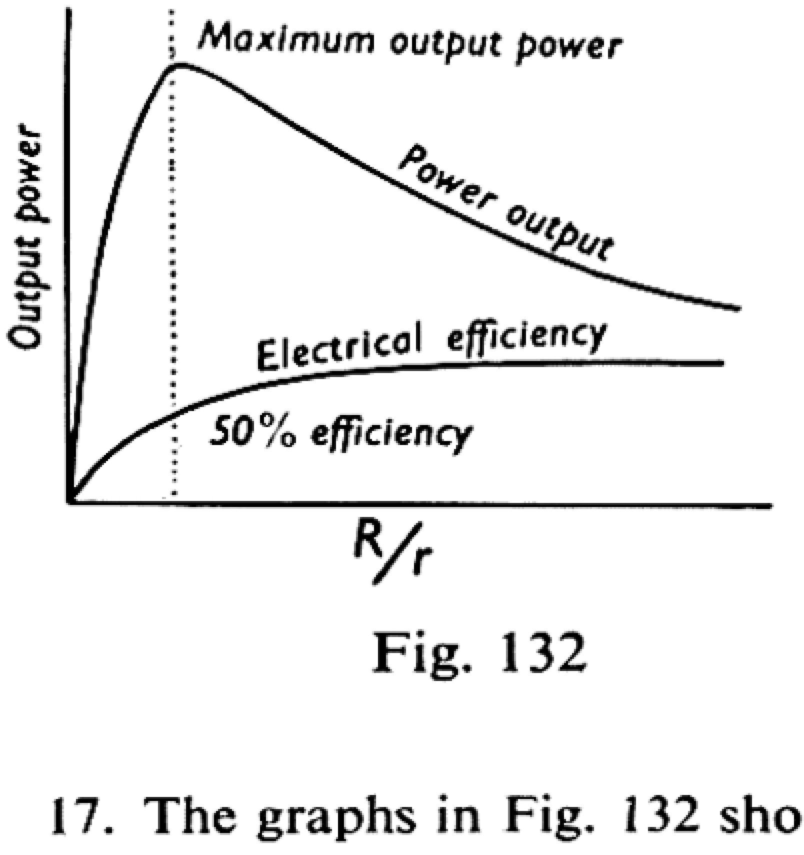


Fig. 132

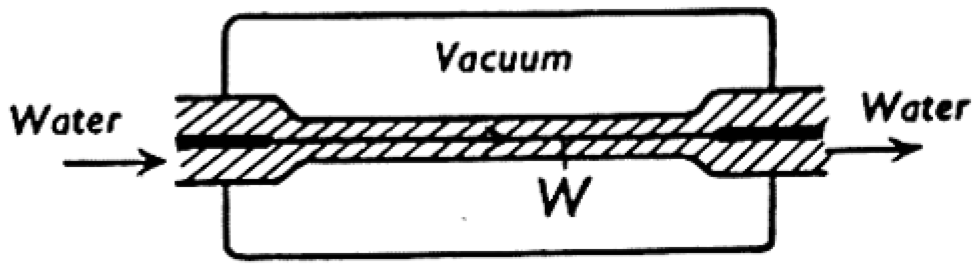


Fig. 133

17. The graphs in Fig. 132 show how (1) the power output in an external circuit varies with R/r, where R is the external resistance and r the resistance of the source of supply; (2) the electrical efficiency, which is the ratio of the power expended in the external circuit to that in the complete circuit, varies with R/r.

Explain the general shape of the graphs and why the maximum power output coincides with an electrical efficiency of 50%.

18. A wire of resistance R is connected in series with a battery of E.M.F. V volts whose internal resistance is r ohms. Find the rate of energy dissipation in the resistance R, and the condition for this to be a maximum.

Find the wattage and voltage of the brightest lamp which can be used with a $4\frac{1}{2}$-volt battery of resistance 2 ohms. (O. & C.)

19. Give a physical explanation of the heating of a wire through which an electric current is flowing.

A current i is divided between two wires of resistance r_1 and r_2. Find what the currents in each arm must be if the heat produced is to be a minimum. (O. S.)

20. In their determination of the mechanical equivalent of heat Callendar and Barnes used the apparatus in Fig. 133. A steady stream of water was passed through a tube and heated by the wire W carrying a current. Heat losses were reduced by surrounding the tube with a vacuum but, in order to eliminate losses, two experiments were performed. After performing one experiment the flow of water and the current were altered in such a way that the temperature of the outflowing water was unchanged. The rate of heat loss in the two experiments was then the same.

Suppose that, in the first experiment, the current in the wire is 2 amperes and the P.D. between its ends is 10 volts, the rise in temperature of the water, flowing at 38·4 g. per min., is 5·85° C. In the second experiment the P.D. is 12 volts, the rate of flow 60 g. per min. and the rise in temperature is unchanged. Calculate the mechanical equivalent of heat.

21. A current of 10 amperes flows through a copper wire 0·166 cm. diameter in a room whose temperature is 15° C. Calculate the steady temperature which the wire will finally attain. Assume that the emissivity of the surface of the wire is 0·00025 calorie per sq.cm. per deg. C., that the emission of heat follows Newton's Law of Cooling and that the resistivity of copper is 2×10^{-6} ohm-cm. (D.)

22. Derive an expression for the rate of generation of heat in a wire of resistance R traversed by a current i.

Calculate the heat radiated per square cm. per sec. from the surface of the tungsten filament of an electric lamp in terms of the current, specific resistance, and radius. Hence, compare the filament diameters of a 60-watt, 100-volt lamp and a 100-watt, 200-volt lamp, assuming that the filaments are run at the same temperature. (O. & C.)

23. A straight tungsten wire, 100 cm. long, is heated *in vacuo* to a temperature of 2200° C. by a current from a 200-volt D.C. supply and consumes power at the rate of 100 watts. Assuming the specific resistance of tungsten to be proportional to the absolute temperature, calculate (*a*) the diameter of the wire, (*b*) the emissivity of the tungsten surface, (*c*) the voltage at which the power consumed would be 30 watts.

Assume the emissivity to be independent of temperature and explain clearly what other assumptions you make. Indicate briefly what other considerations would enter into the calculation if an alternating instead of a direct current were used. (Stefan's constant $= 5{\cdot}7 \times 10^{-5}$ erg cm.$^{-2}$, degree^{-4}, sec.$^{-1}$; specific resistance of tungsten at 727° C. is $2{\cdot}5 \times 10^{-5}$ ohm-cm.) (C. S.)

(By emissivity is here meant the ratio of the heat emitted by the surface to that emitted by a black surface under the same conditions. Thus the heat emitted by the wire per sq.cm. per sec. at absolute temperature θ is $e \times 5{\cdot}7 \times 10^{-5}\, \theta^4$ ergs, where e is the emissivity.)

24. In the 'Grid' system of electrical distribution, electrical energy is transmitted at 132 kilovolts. Why is it advantageous to transmit energy at this high voltage and by alternating current, although the usual supply is 240 volts?

The potential in a thundercloud is estimated to be about a million kilovolts, but the quantity of electricity discharged in one lightning flash is only about 20 coulombs. If the energy of the flash could be spread out over 24 hours, how many 5 H.P. machines could be kept working by it for this period? (1 H.P. = 746 watts.) (C.)

25. A battery of accumulators, each of E.M.F. 2·0 volts and internal resistance 0·002 ohm, supplies energy to a factory through cables of total resistance 0·2 ohm. The supply voltage at the factory is 100 volts and the power consumed there is 5 kilowatts. Find the least number of cells which is necessary. (N.)

26. Discuss the advantages and disadvantages of the transmission of electrical power at high voltages.

A certain works consumes electrical energy at the average rate of 200 kilowatts, and the voltage of the supply (at the works) is 230 volts. The resistance of the transmission line from the power station is 3×10^{-2} ohms. Calculate the loss in the cables. If the transmission were carried out at 2000 volts in the same cables, being stepped up at the power station and down at the works by transformers, each having 99% efficiency, what would be the total energy loss in transmission? (O. & C.)

27. If the safe current-carrying capacity of copper cable of specific resistance $1{\cdot}6 \times 10^{-6}$ ohm per cm. cube is 1000 amperes per sq. in. of cross-section, calculate the length in metres in which the voltage drop is equal to 1 volt, the current rating being at the above value.

A generator supplies 500 amperes to a factory 1 km. away, through cables of cross-section corresponding to the above current density. What is the cost per 8-hour day of the power wasted in the leads? (1 inch = 2·54 cm.; 1 B.T.U. costs 1*d.*) (B.)

28. D.C. electric power is supplied to a factory by a pair of insulated copper cables 5 km. long. The voltage at the generating station is 555, the maximum steady demand at the factory is 200 kilowatts, and the volt drop in the cable must not exceed 10%. Calculate (*a*) the minimum cross-section of the cable, (*b*) the rate at which energy is dissipated in the cable in H.P. and in calories per second, (*c*) the cost to the generating authority of this wasted power, assuming it to be valued at ½*d.* per kilowatt-hour.

What would be the ratio of the costs of the copper required for a supply at 550 volts and at 220 volts? (Specific resistance of copper 1·72 microhm-cm.; 1 H.P. = 746 watts.) (B.)

CHAPTER 8

ELECTRICAL MEASUREMENTS

METHODS of measuring current, potential difference and resistance will be considered in this chapter.

MEASUREMENT OF CURRENT

To determine the magnitude of a current, its magnetic effect, heating effect or chemical effect may be measured. The instruments used may be classified as follows:

(1) *Magnetic effect*, moving-magnet, moving-coil, moving-iron galvanometers.*

(2) *Heating effect*, hot-wire galvanometer, thermocouple combined with moving-coil galvanometer.

(3) *Chemical effect*, voltameter.

A typical moving-magnet instrument, the tangent galvanometer, has already been described (p. 85). The voltameter will be described in Chapter 10.

Moving-coil galvanometer

The moving-coil galvanometer consists of a coil of fine wire suspended, or pivoted on jewelled bearings, between the poles of a permanent magnet (Fig. 134). When a current passes through it the coil tends to set with its plane at right angles to the magnetic field of the permanent magnet. A controlling torque, exerted by the control springs (Fig. 135) which serve also to lead the current in and out of the coil, counterbalances the deflecting torque due to the current. Since the deflecting torque can be made proportional to the current, while the controlling torque is proportional to the angle of deflection, the deflection is a measure of the current. The angle of deflection may be measured by a pointer or by reflecting a beam of light from a small mirror attached to the coil.

The coil swings round a fixed cylinder of soft iron, the purpose of which is to make the lines of magnetic force converge towards the axis of revolution of the coil; the deflection is then directly proportional to the current (p. 146).

---

* The term galvanometer is sometimes confined to instruments measuring small currents only; in this chapter it includes the movements of ammeters and voltmeters.

The coil is wound on a light frame of copper or aluminium. Eddy currents are induced in this metal frame while it is in motion causing 'damping' or resistance to motion, so making the instrument dead-beat; that is to say the pointer does not oscillate like a pendulum but, on reaching its deflected position, remains at rest.

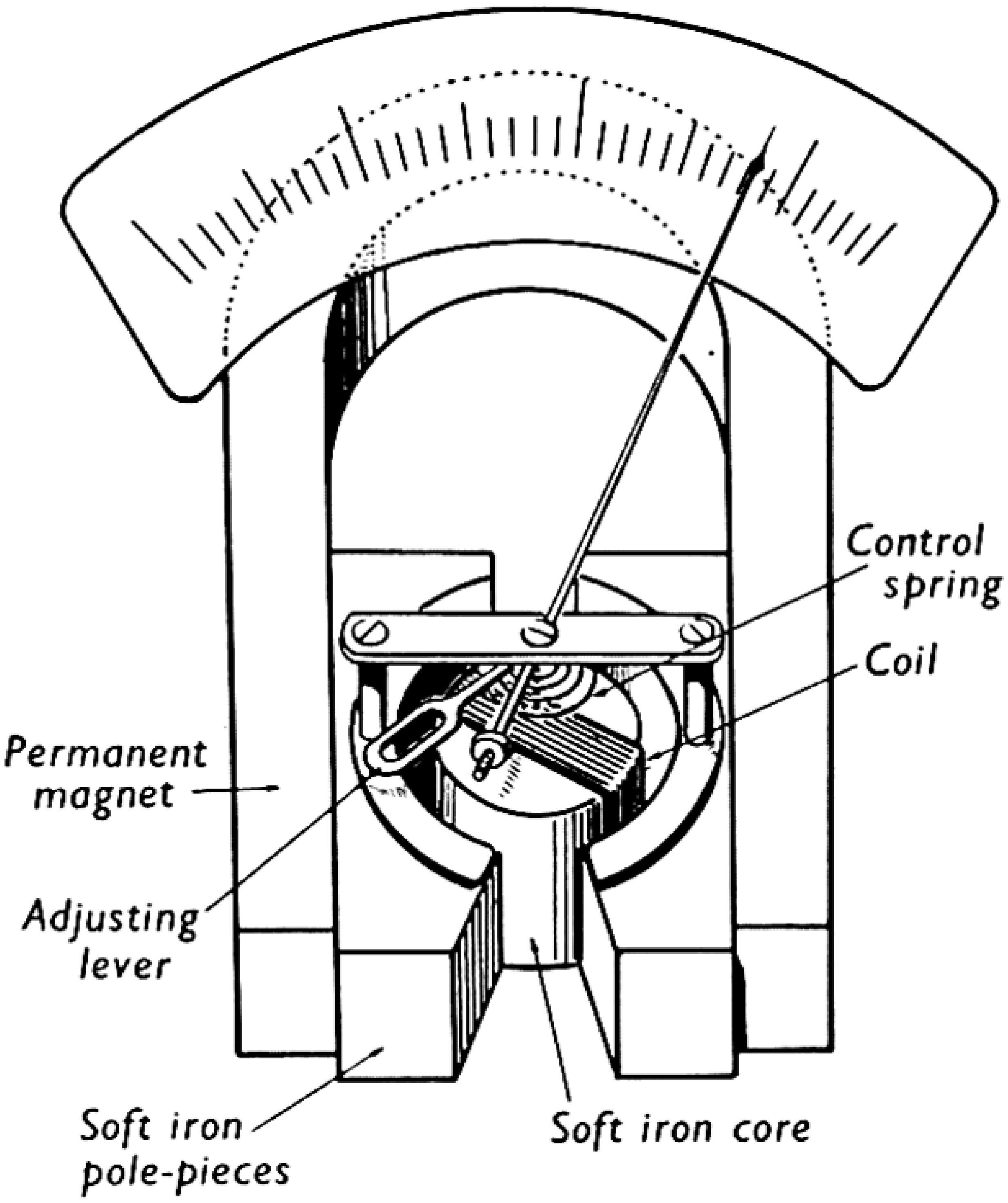


Fig. 134. Moving-coil galvanometer.

The moving-coil galvanometer is the most accurate and most widely used type of galvanometer; it can measure only direct (and not alternating) current.

Moving-iron galvanometer

There are two types of moving-iron galvanometer, the attraction type and the repulsion type.

The attraction type (Fig. 136) consists of a piece of soft iron mounted eccentrically on a spindle at the side of a coil through which the current to be measured is passed. The soft iron is attracted into the coil towards the region of stronger magnetic field just as a piece of iron is attracted towards a magnet. The controlling force in Fig. 136 is provided by gravity; when the pointer moves to the right the control weight moves to the left and the moment of the restoring force increases with the deflection. The disadvantage of gravity control is that the instrument cannot be tilted since this causes a change in the zero position of the pointer; the advantages are cheapness, independence of temperature and freedom from deterioration with age.

Damping in Fig. 136 is provided by air friction. A light aluminium piston moves in a chamber closed at one end. There is a small clearance (a few thousands of an inch) between the piston and the walls of the chamber. When the piston moves towards the open end of the chamber the air pressure behind it is less than atmospheric, causing resistance to motion, until the air has had time to pass through the clearance gap to equalise the pressures.

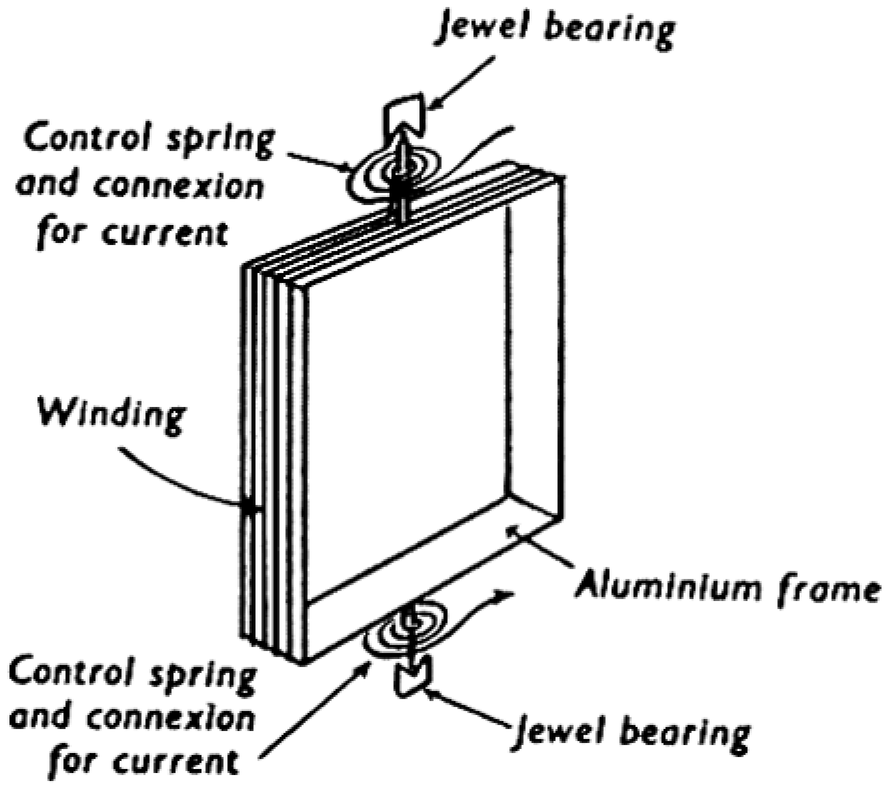

Jewel bearing
Control spring and connexion for current
Winding
Aluminium frame
Control spring and connexion for current
Jewel bearing

Fig. 135. Coil and control springs of moving-coil galvanometer.

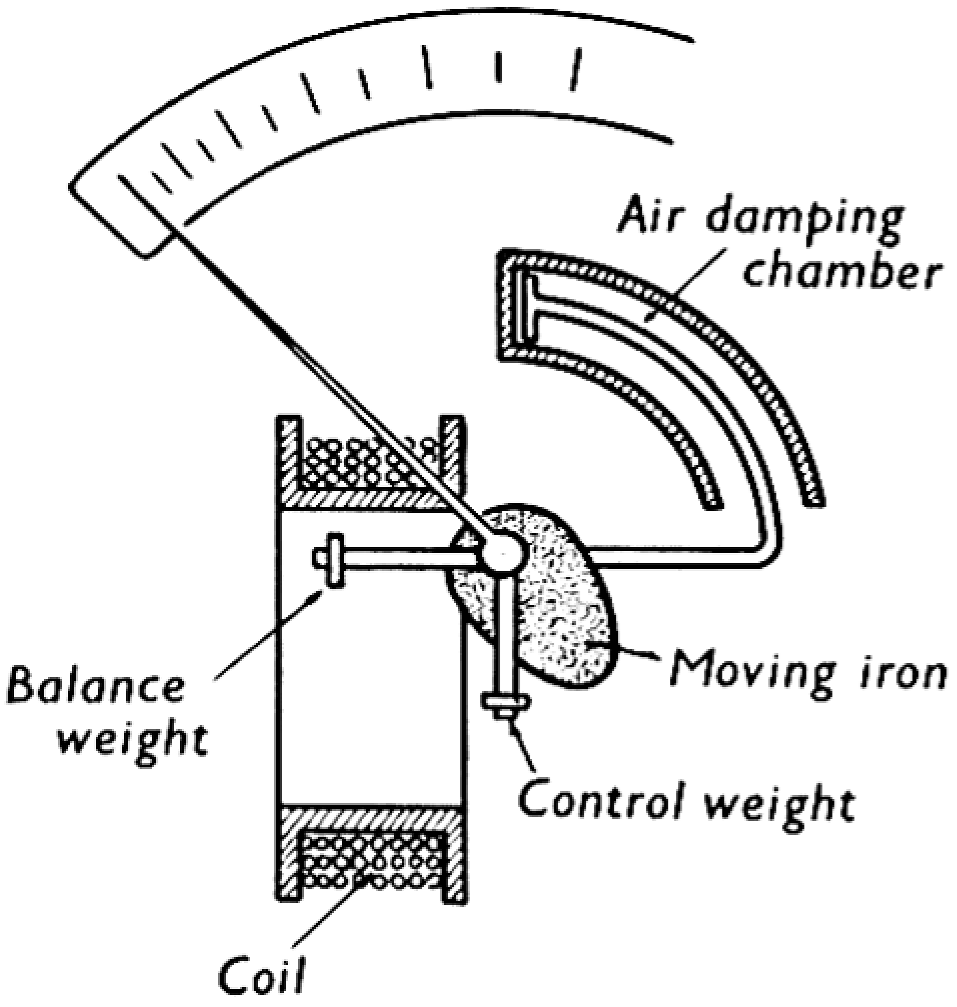


Fig. 136. Moving-iron galvanometer, attraction types.

The repulsion type of moving-iron galvanometer (Fig. 137) consists of two rods of iron, one fixed rigidly in position and the other attached to a pivoted pointer, inside a coil carrying the current to be measured. Both rods become magnetised in the same direction and their mutual repulsion causes a deflection of the pointer. In Fig. 137 spring control is shown, but in some instruments gravity control is used.

Moving-iron instruments have two great advantages, they are cheap and they can be used to measure A.C. as well as D.C. The iron in Fig. 136 is attracted into the coil whatever the direction of the current; both rods in Fig. 137 are always magnetised in the same direction and they continue to repel each other even though the current is alternating. A marked disadvantage of the moving-iron instrument is the unevenness of its scale. It is liable to be affected by stray magnetic fields unless shielded by a soft-iron case. It is not as sensitive as the moving-coil galvanometer but its design has been greatly improved in recent years.

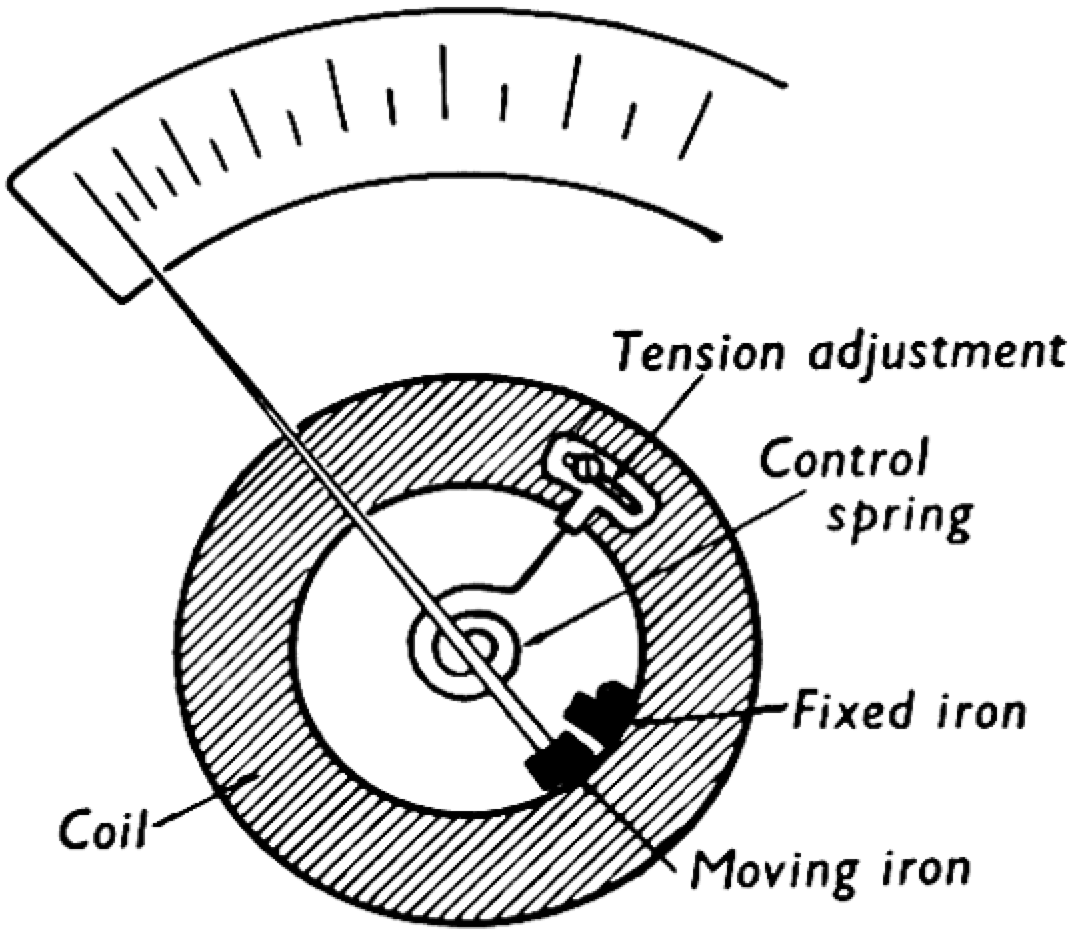


Fig. 137. Moving-iron galvanometer, repulsion types.

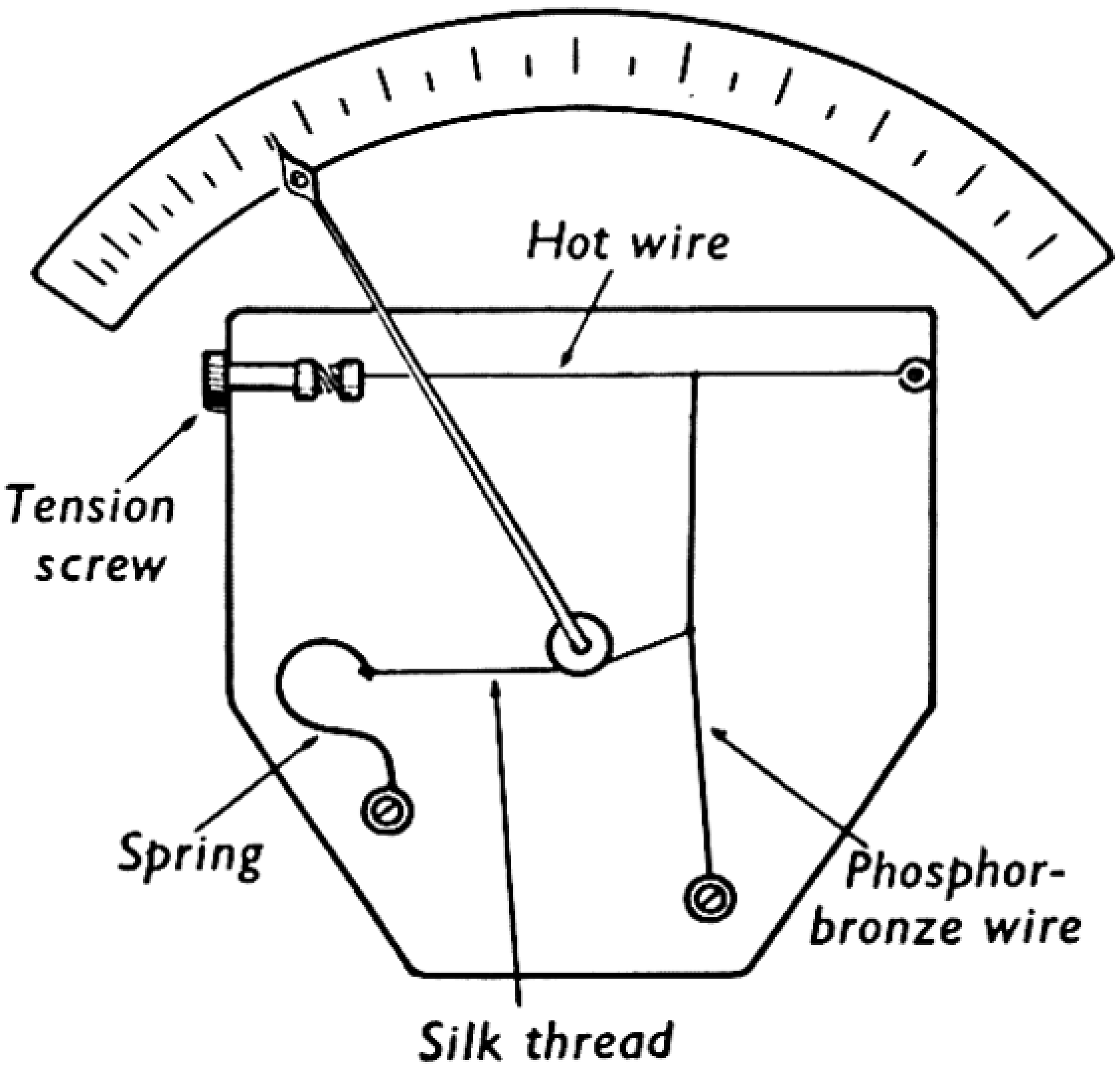


Fig. 138. Hot-wire galvanometer.

The hot-wire galvanometer

The hot-wire galvanometer (Fig. 138) consists of a fine wire, which is heated by the current to be measured, and a mechanism to enable the expansion or sag of the wire to cause movement of a pointer. The 'hot-wire' is made of platinum-iridium, which will withstand high temperatures without oxidation. To it is attached a phosphor-bronze wire; this in turn is attached to a silk thread, kept taut by a spring, and passing round a pulley carrying the pointer.

The scale is uneven; the wandering of the zero, due to changes in room temperature, may be corrected by the tension adjustment. The power consumption is about four times that of the moving-coil galvanometer. The instrument is often used for radio-frequency, alternating currents since it is independent of change in frequency or wave form.

Thermocouple instrument

Fig. 139 represents one type of thermocouple instrument. The current passes through a heater, the radiation from which warms the junction of a thermocouple connected in series with the coil of a moving-coil galvanometer. When the junction of the thermocouple is warmed a small current flows (see p. 251). The thermocouple may be in actual contact with the heater; the design of the instrument must then obviously be modified. The instrument is superior to the hot-wire galvanometer for radio-frequencies.

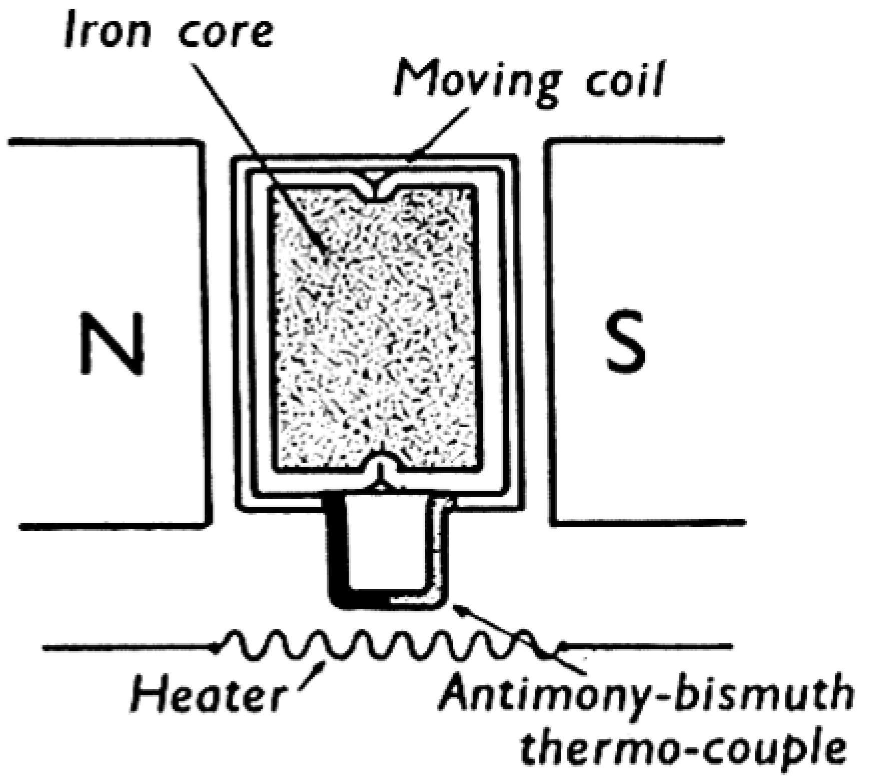
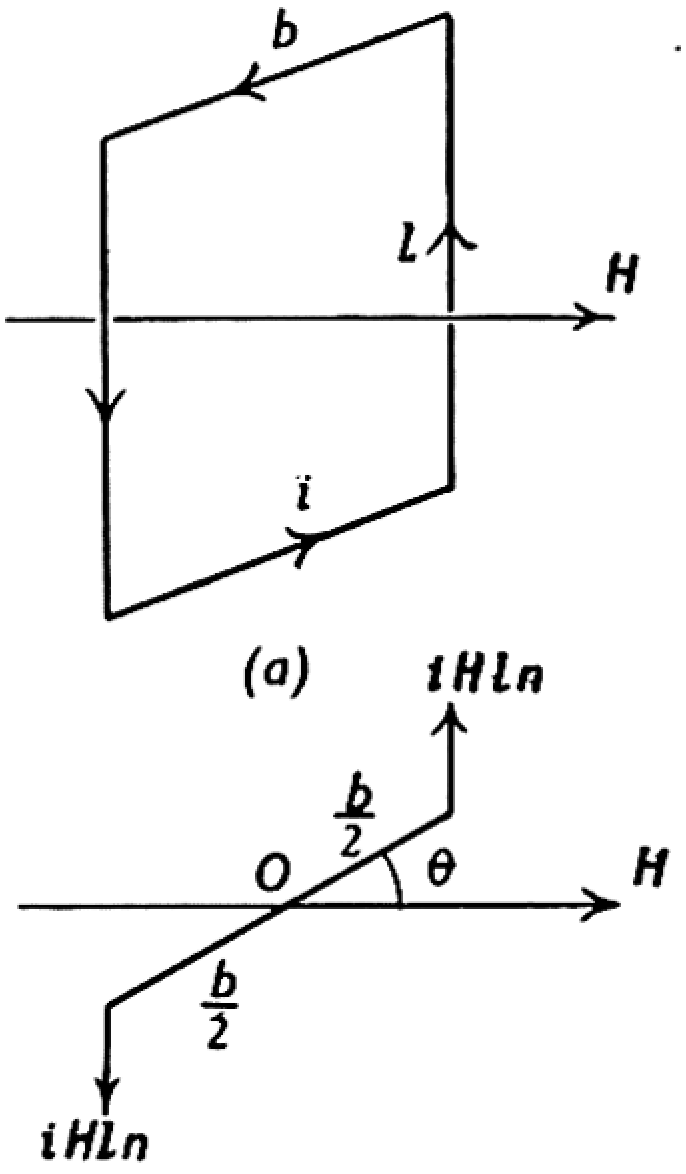


Fig. 139. Thermocouple instrument.



(b) Plan

Fig. 140

Theory of the moving-coil galvanometer

Fig. 140 (*a*) and (*b*) represents a vertical rectangular coil of length *l* and breadth *b*, carrying a current *i* e.m.u. with its plane at an angle *θ* to a horizontal magnetic field of strength *H* oersted. Applying Fleming's left-hand rule to Fig. 140 (*a*) it will be seen that the left-hand vertical side is urged out of the paper, the right-hand vertical side into the paper and the top and bottom are urged up and down respectively. If the coil is free to turn about a vertical axis, only the forces on the vertical sides will have a turning effect. Assuming that the coil has n turns, the forces on these sides are each $Hiln$ (p. 88); they are shown in Fig. 140 (*b*).

$$\begin{aligned}\text{Moment of the forces about } 0 &= Hilnb\cos\theta\\ &= iHAn\cos\theta,\end{aligned}$$

where A = area of coil $= bl$.

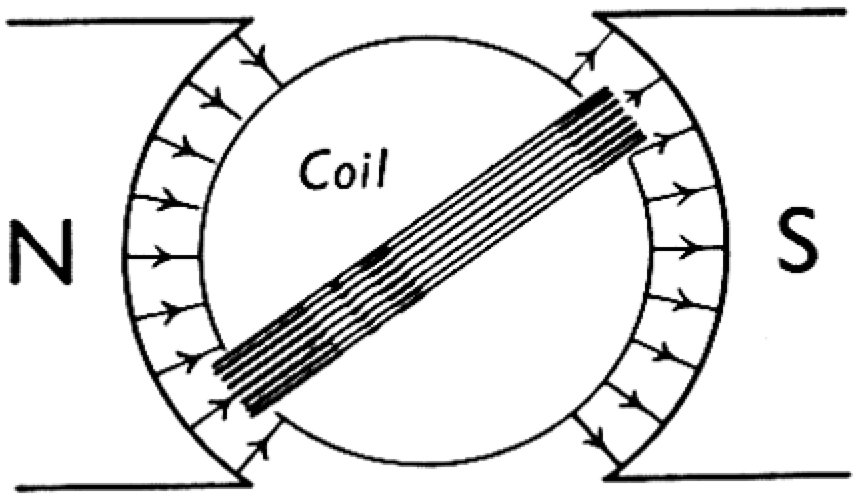


Fig. 141

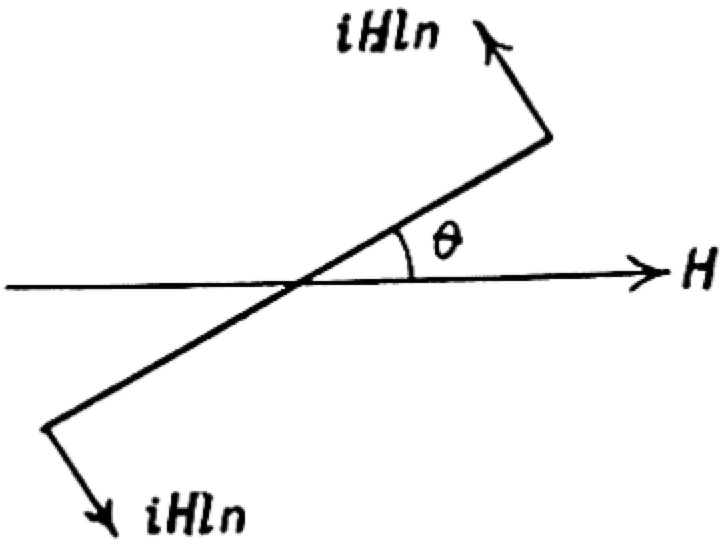


Fig. 142

In a moving-coil instrument the coil is in equilibrium when the above moment is equal to the moment of the controlling forces, provided by the torsion of a suspending phosphor-bronze fibre or by hair springs. The controlling couple is proportional to the angle turned through. If we assume that, when no current flowed, the coil was lying with its plane in the direction of the magnetic field, and that the controlling couple per unit angle is c,

$$\text{Controlling couple} = c\theta.$$

$$\therefore\ HiAn\cos\theta = c\theta,$$

$$i = \frac{c}{HAn}\,\frac{\theta}{\cos\theta}.$$

In order that the current i may be proportional to the deflection, θ, the field is made radial by means of a soft-iron cylinder (Fig. 141). The vertical sides of the coil are then always in a magnetic field which is in the direction of the plane of the coil, whatever the position of the coil, and hence the forces on the sides of the coil are always at right angles to its plane (Fig. 142). The turning couple is now $iHAn$ instead of $iHAn\cos\theta$.

Thus

$$i = \frac{c}{HAn}\,\theta.$$

The sensitivity of a galvanometer is the deflection per unit current, θ/i,

$$\frac{\theta}{i}=\frac{HAn}{c}.$$

Determination of the sensitivity of a galvanometer

The most interesting and useful information about a galvanometer is its sensitivity, which may be expressed in divisions per microampere. To determine the sensitivity it is necessary to know the resistance of the galvanometer coil; this may be measured by the method given on p. 158.

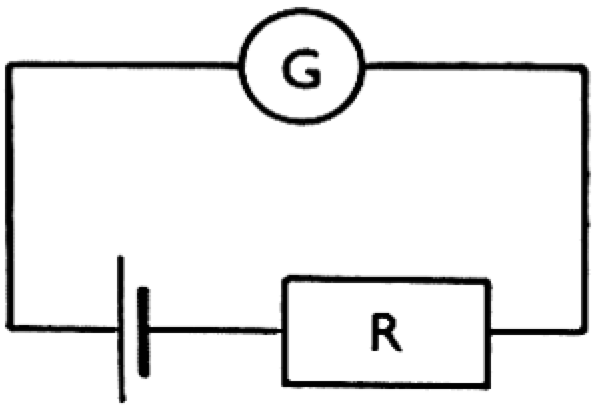


Fig. 143

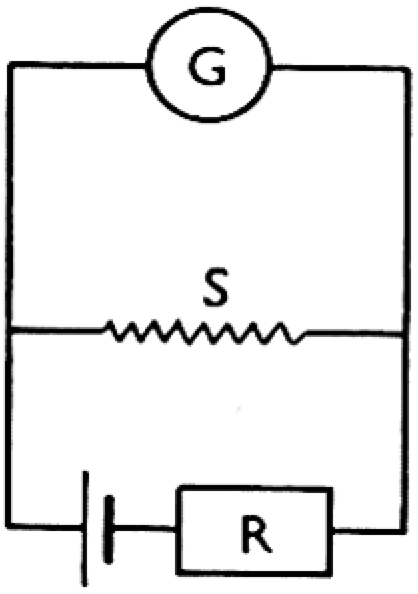


Fig. 144

The galvanometer is connected (Fig. 143) in series with a resistance box and an accumulator, whose E.M.F., E, has been determined by means of an accurate voltmeter. The deflection, θ, is measured with different, suitable resistances R in the box.

Then $i=E/(R+G)$, where G is the resistance of the galvanometer; i is plotted against θ, whence θ/i may be obtained.

If the galvanometer is very sensitive it may not be possible to obtain a sufficiently high resistance from the box. It is then necessary to shunt the galvanometer with a known resistance, S (perhaps $\frac{1}{10}$ ohm). Fig. 144 shows the circuit; a commutator also may be included to enable deflections to be obtained in both directions in the galvanometer. The reader should verify that the current in the galvanometer is

$$\frac{SE}{R(G+S)+SG}.$$

Ammeters

The sensitivity of a galvanometer may be reduced by connecting in parallel with it a low resistance called a shunt. Part of the current then passes through the shunt instead of through the galvanometer. In this way a galvanometer can be converted into an ammeter. By means of a set of shunts of different resistances its range may be varied (Fig. 145), for example, from 0 to 1·5 amperes, 0 to 5 amperes, 0 to 10 amperes, and so on.

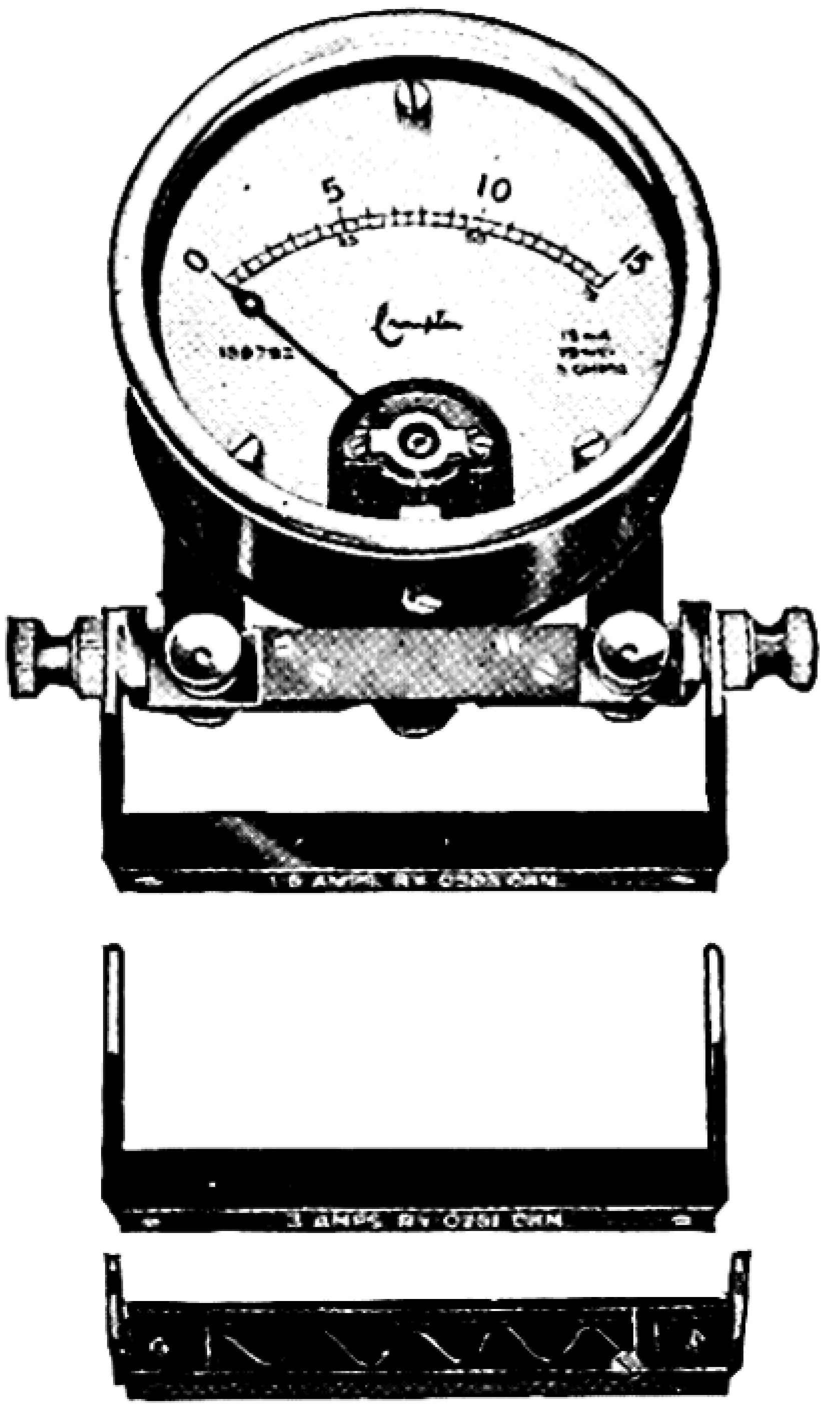


Fig. 145

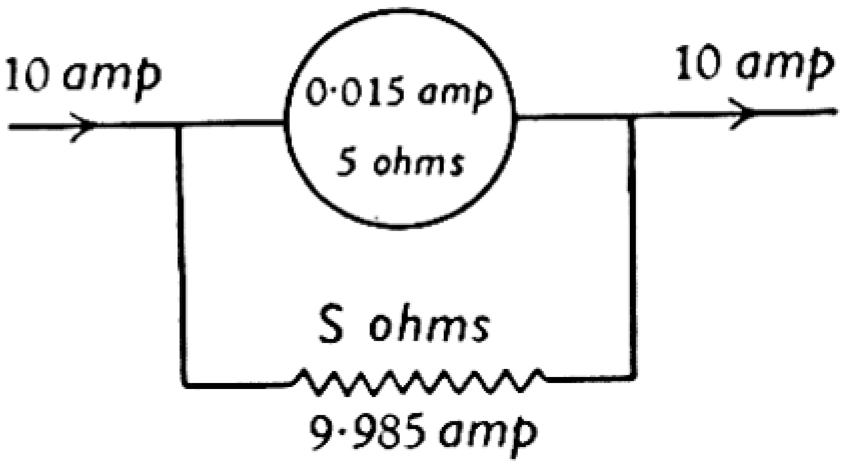


Fig. 146

Example

A moving-coil galvanometer of resistance 5 ohms requires a current of 0·015 ampere to produce a full-scale deflection. Calculate the resistance of the shunt necessary to convert it into an ammeter reading from 0 to 10 amperes.

In Fig. 146

P.D. across moving coil = P.D. across shunt, i.e.

$$0{\cdot}015\times 5=9{\cdot}985\,S,$$

$$S=\frac{0{\cdot}075}{9{\cdot}985}=0{\cdot}00751 \text{ ohm.}$$

The total resistance of the ammeter is less than that of the shunt; in this case it is less than 0·00751 ohm. The resistance of an ammeter should always be low since otherwise, when placed in a circuit to measure the current, it might reduce the current considerably. Moving-iron, as well as moving-coil, galvanometers are often converted into ammeters.

The ballistic galvanometer

A galvanometer specially designed to measure the discharge of a condenser, or a quantity of electricity induced momentarily, is known as a ballistic galvanometer. Normally the instrument is of the moving-coil type, but galvanometers of other types could be designed to be used ballistically. The discharge gives an impulse to the coil, which has a large inertia so that it does not move appreciably during the passage of the discharge. The swing or throw of the coil is then proportional to the discharge. Damping is reduced to a minimum by winding the coil on an insulating, instead of a metal frame. Even so damping does occur owing to the eddy currents induced in the coil. When comparing two discharges, however, it is necessary only to note the ratio of the throws of the coil since the damping corrections cancel out.

MEASUREMENT OF POTENTIAL DIFFERENCE

The voltmeter

The obvious method of measuring the P.D. between two points is to connect them to a suitable galvanometer. Then, by Ohm's law, the P.D. is the product of the current in the galvanometer and the resistance of the galvanometer. The galvanometer must have a high resistance, since otherwise it will take a large current and lower appreciably the P.D. it is required to measure.

A voltmeter is a sensitive galvanometer connected in series with a high resistance.

EXAMPLE

A moving-coil galvanometer, of resistance 5 ohms, requires a current of 15 milliamperes to produce a full-scale deflection. What resistance must be placed in series with it to convert it into a voltmeter reading 0–10 volts?

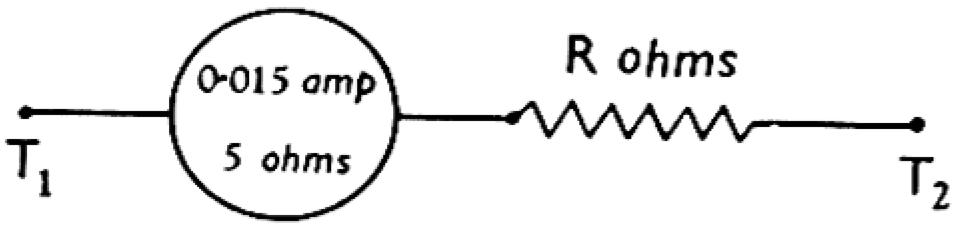


Fig. 147

Let the resistance in series with the galvanometer be R ohms (Fig. 147).

When a P.D. of 10 volts is applied across the terminals, T_1 and T_2, a current of 0·015 ampere must flow to provide a full-scale deflection.

Applying Ohm's law

$$10 = 0{\cdot}015\ (5 + R).$$

$$\therefore\ R = 661{\cdot}7 \text{ ohms.}$$

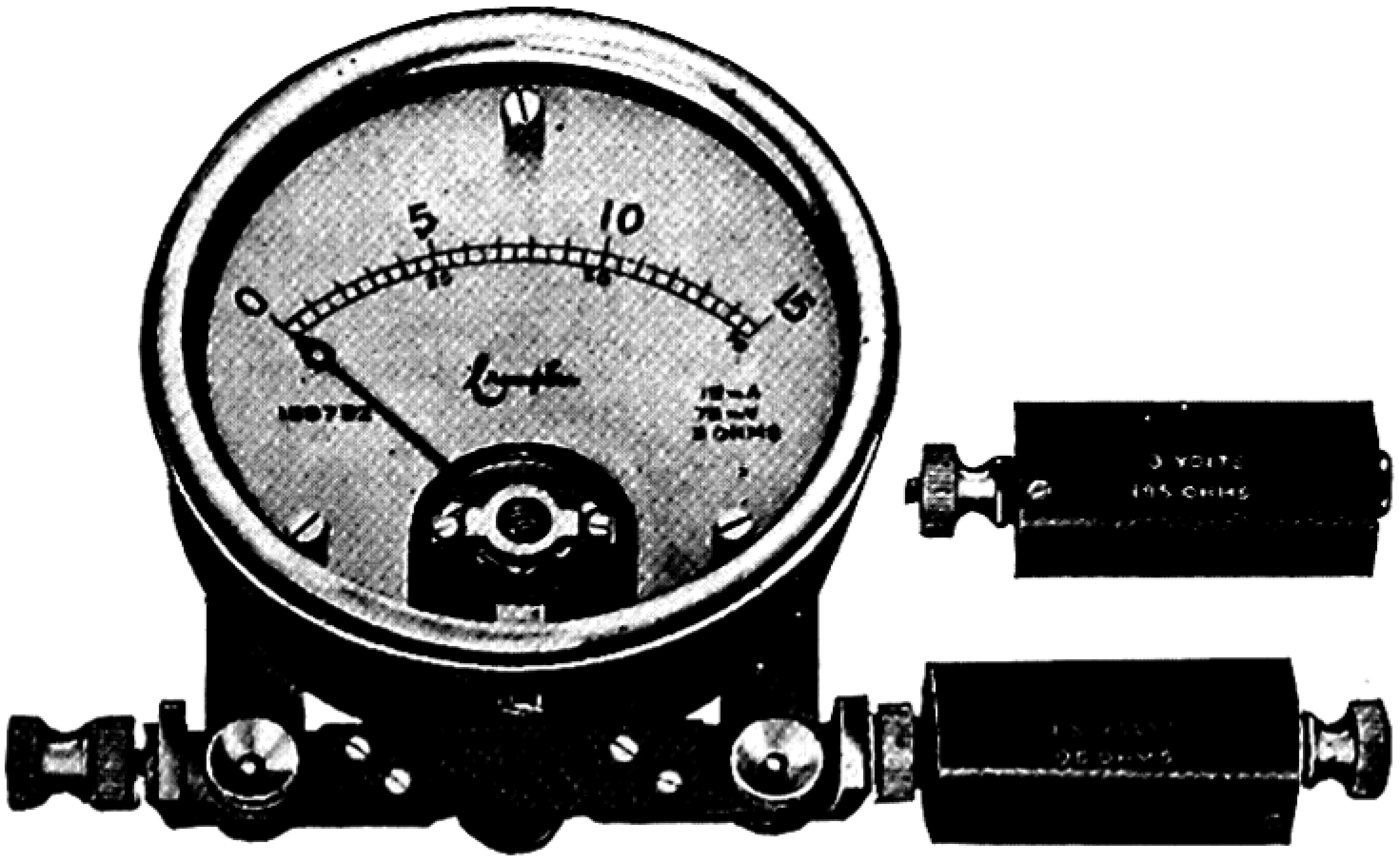


Fig. 148

The series resistance of a voltmeter is usually fitted inside the case of the instrument; but sometimes a galvanometer is provided with a set of resistances which can be screwed to one of its terminals, thus enabling it to be converted into a voltmeter with different ranges (Fig. 148). Both moving-coil and moving-iron voltmeters are in common use.

The potentiometer

Since the voltmeter just described must take some current, however small, it is bound to lower slightly the P.D. it is required to measure. There are two methods of measuring a P.D. without taking a current, the electrostatic voltmeter (see p. 41), and the potentiometer.

In its simplest form the potentiometer consists of a resistance wire of uniform cross-section through which a steady current is passed. In Fig. 149 *AB* represents the potentiometer wire and *C* the cell (usually an accumulator) supplying the steady current. There is a drop of potential down the wire from *A* to *B*; the P.D. between two points on the wire is proportional to their distance apart, and can be used to counterbalance an unknown P.D.

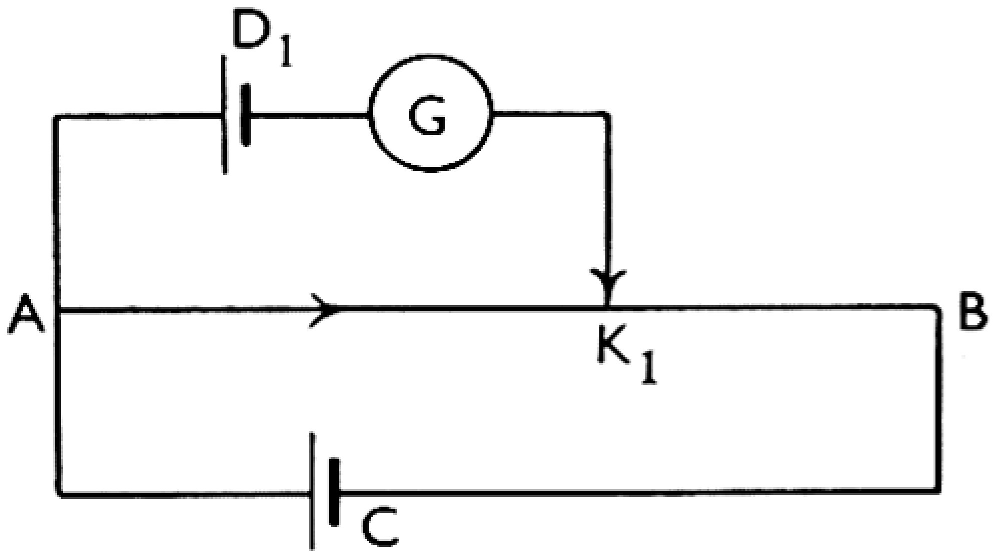


Fig. 149

Thus, to compare the E.M.F.'s of two cells, one of the cells D_1 is connected as in Fig. 149 and the sliding contact is moved along *AB* until no current flows in the galvanometer. The P.D. between *A* and K_1 is then equal to the E.M.F. of the cell D_1. It is essential that the same poles of *C* and D_1 should be connected to *A* and that the E.M.F. of *C* should be greater than that of D_1.

The cell D_1 is now replaced by the second cell D_2 and the new position of the sliding contact, K_2, found.

$$\frac{E_1}{E_2}=\frac{AK_1}{AK_2}, \quad \begin{array}{l} E_1=\text{E.M.F. of cell } D_1, \\ E_2=\text{E.M.F. of cell } D_2. \end{array}$$

The Weston standard cadmium cell

A standard of P.D. must be employed with the potentiometer if the actual value of an unknown P.D. is to be determined. The standard used is the Weston standard cadmium cell shown in Fig. 150; when constructed to certain specifications it yields accurately 1·0183 volts at

20° C. No appreciable current should be taken from a standard cell or it will be ruined; it should therefore be connected in series with a high resistance when making preliminary adjustments with a potentiometer.

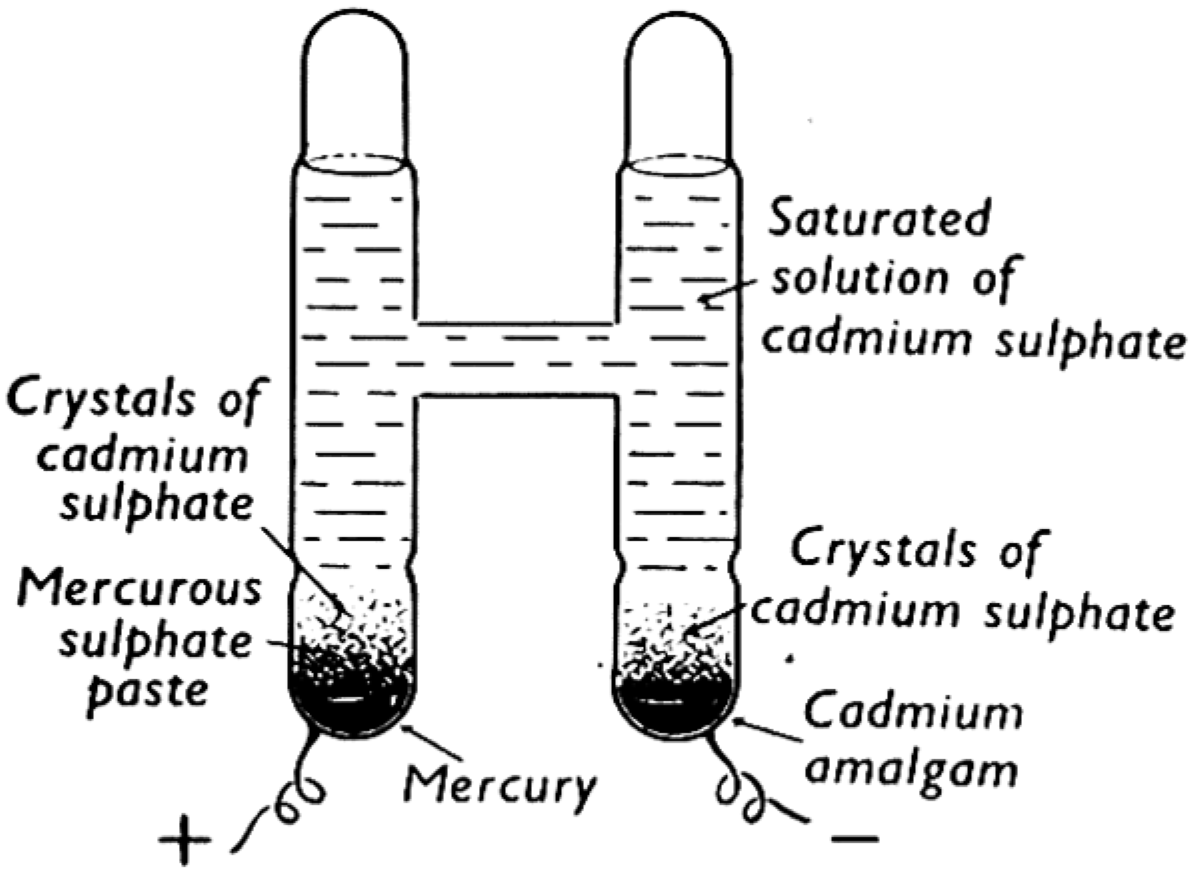

Saturated solution of cadmium sulphate
Crystals of cadmium sulphate
Mercurous sulphate paste
Crystals of cadmium sulphate
Cadmium amalgam
Mercury
+
−

Fig. 150. Weston standard cadmium cell.

Calibration of a voltmeter by a potentiometer

The potentiometer may be used to calibrate a voltmeter (Fig. 151). A suitable P.D. is put across the voltmeter by means of a cell and variable resistance—not shown in the figure. The sliding contact on the potentiometer wire is adjusted until no current flows in the galvanometer. Then the P.D. across the voltmeter is equal to the P.D. across AK_1. The voltmeter is now replaced by a standard cell of E.M.F., E, and the new position, K_2, of the sliding contact found,

$$\frac{\text{P.D. across voltmeter}}{E} = \frac{AK_1}{AK_2}.$$

The reading of the voltmeter is compared with this calculated value.

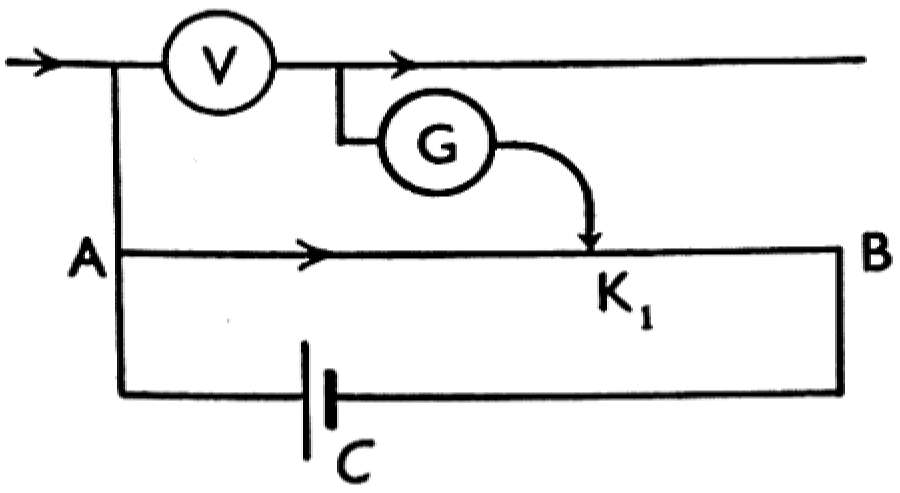


Fig. 151

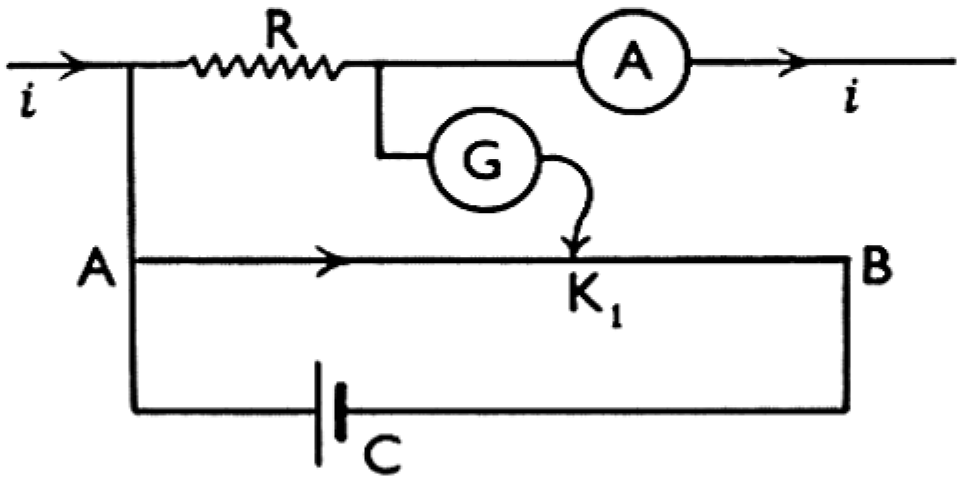


Fig. 152

Calibration of an ammeter by a potentiometer

The potentiometer is very sensitive and easy to use. It is therefore employed in conjunction with a standard resistance and a standard cell, to calibrate ammeters.

In Fig. 152 R is the standard resistance, which is connected in series with the ammeter to be calibrated, so that a suitable current, i, can be passed through both. The sliding contact is adjusted until the galvanometer is undeflected; then the P.D. across R is equal to the P.D. across AK_1. The length, AK_2, of the potentiometer wire whose P.D. balances the E.M.F., E, of a standard cell is then found.

By Ohm's law, P.D. across $R = iR$.

$$\therefore \frac{iR}{E} = \frac{AK_1}{AK_2},$$

$$i = \frac{AK_1}{AK_2}\frac{E}{R}.$$

Knowing E and R and measuring AK_1 and AK_2 the current may be determined; its value is compared with the reading of the ammeter.

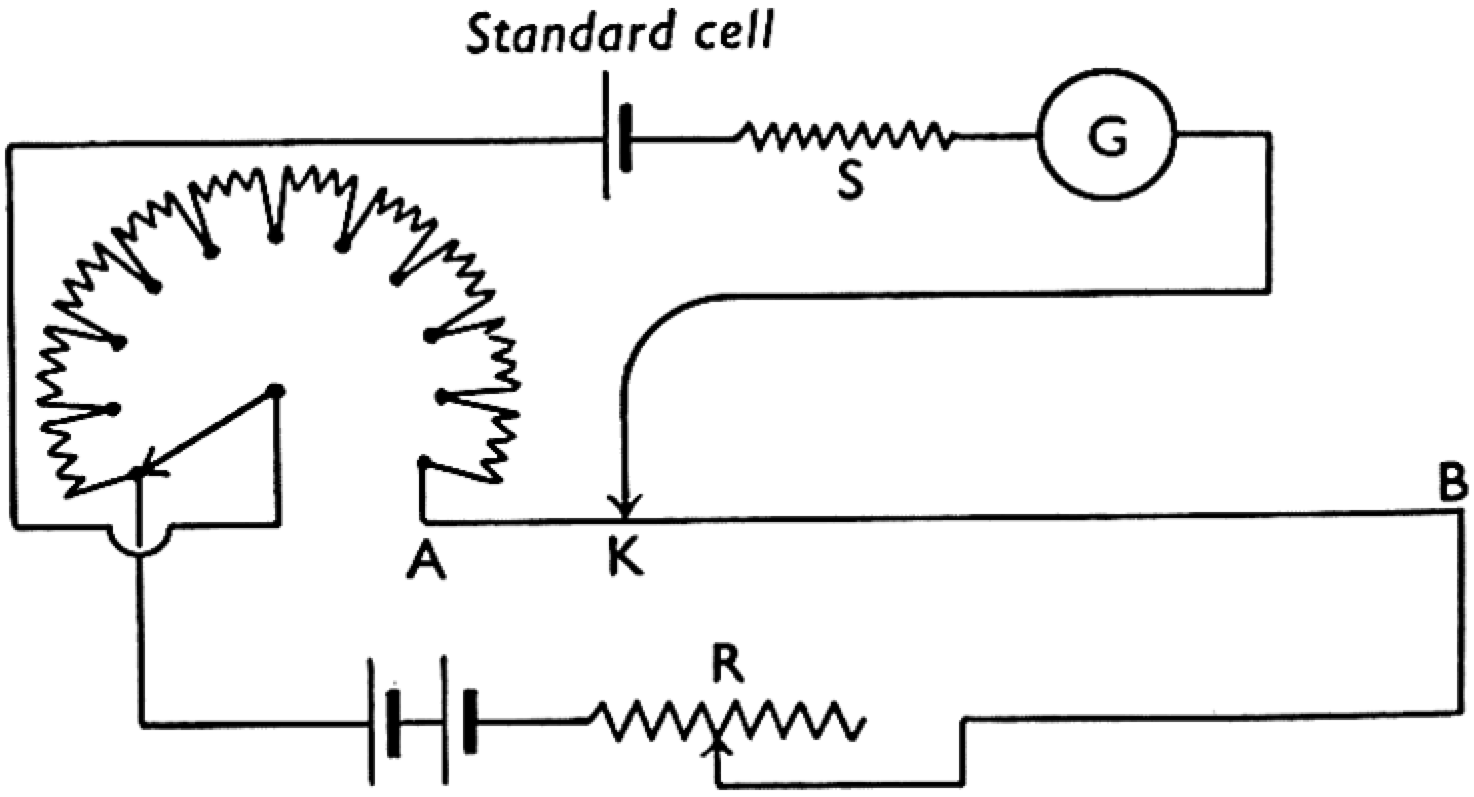


Fig. 153. Crompton potentiometer.

The Crompton potentiometer

A potentiometer, in order to be sensitive, must have a very long wire. This is obviated, however, in the Crompton potentiometer, by having in series with a short wire, AB (50 cm. long), a number of coils each of resistance equal to that of the wire. Fig. 153 shows the principle of the instrument. Suppose ten coils and a length equal to 0·18 of AB are made to balance a standard Weston cell of E.M.F. 1·018 volts by varying the current through the instrument by means of R (actually incorporated in the instrument). This means that the voltage across each coil or AB is exactly 0·1 volt and, if the wire is divided into 100 divisions, each division represents a millivolt. The current through the instrument is then left unaltered and unknown P.D.'s can be balanced against the requisite number of coils and part of AB. The resistance S is to protect the standard cell. This type of instrument is employed in many laboratories for calibrating voltmeters and ammeters.

MEASUREMENT OF RESISTANCE

The Wheatstone Bridge

The most used method of measuring resistance is the Wheatstone Bridge. This has the great advantage of being a null method; that is to say adjustments are made until a galvanometer is undeflected and hence the result does not depend on the accuracy of an instrument. Reliable standard resistances are, however, required.

Four resistances P, Q, R, S are arranged as in Fig. 154. One of these, say P, is the unknown resistance; then Q must be a known standard resistance and the values of R and S or their ratio must also be known.

A sensitive galvanometer and a cell are connected as shown. If the resistances R and S are adjusted so that no current flows in the galvanometer (when the bridge is said to be balanced), it can be proved that

$$P/Q=R/S,$$

whence P can be calculated.

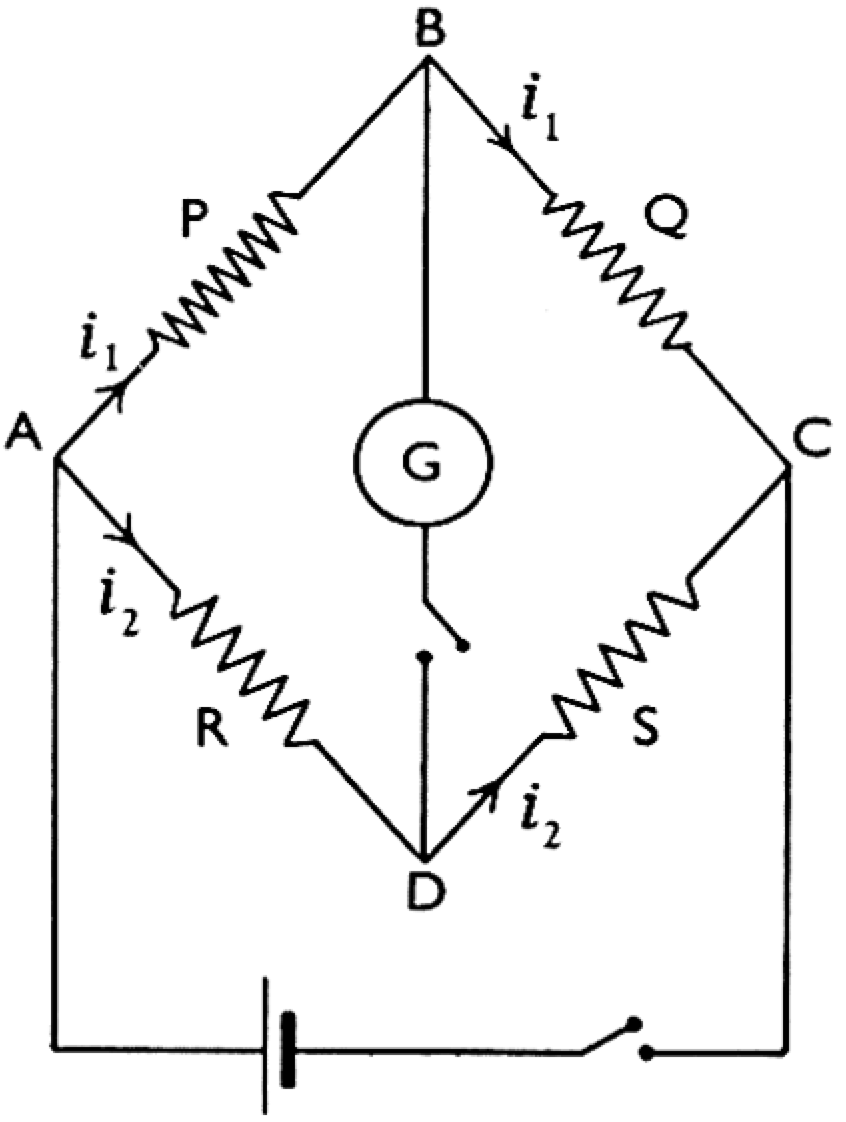


Fig. 154

Since no current flows through the galvanometer,

$$\text{current through } P=\text{current through } Q=i_1,$$

and $$\text{current through } R=\text{current through } S=i_2.$$

Also $$\text{Potential at } B=\text{Potential at } D.$$

$$\therefore \text{ P.D. between } A \text{ and } B=\text{P.D. between } A \text{ and } D,$$

$$\text{i.e.} \quad i_1P=i_2R.$$

Similarly $$\text{P.D. between } B \text{ and } C=\text{P.D. between } D \text{ and } C,$$

$$\text{i.e.} \quad i_1Q=i_2S,$$

dividing $$P/Q=R/S.$$

The metre bridge

The simplest experimental arrangement of the Wheatstone Bridge is the metre bridge (Fig. 155). A wire AC of uniform cross-section and 1 metre long, made of some alloy such as constantan so that its resistance is of the order of 1 ohm, lies stretched between two thick brass or copper strips bearing terminals, above a metre ruler. There is another thick brass strip bearing three terminals to facilitate connections and also a sliding contact *D* which can move along the metre wire.

Fig. 155 has been lettered identically with Fig. 154 to show that the two circuits are similar.

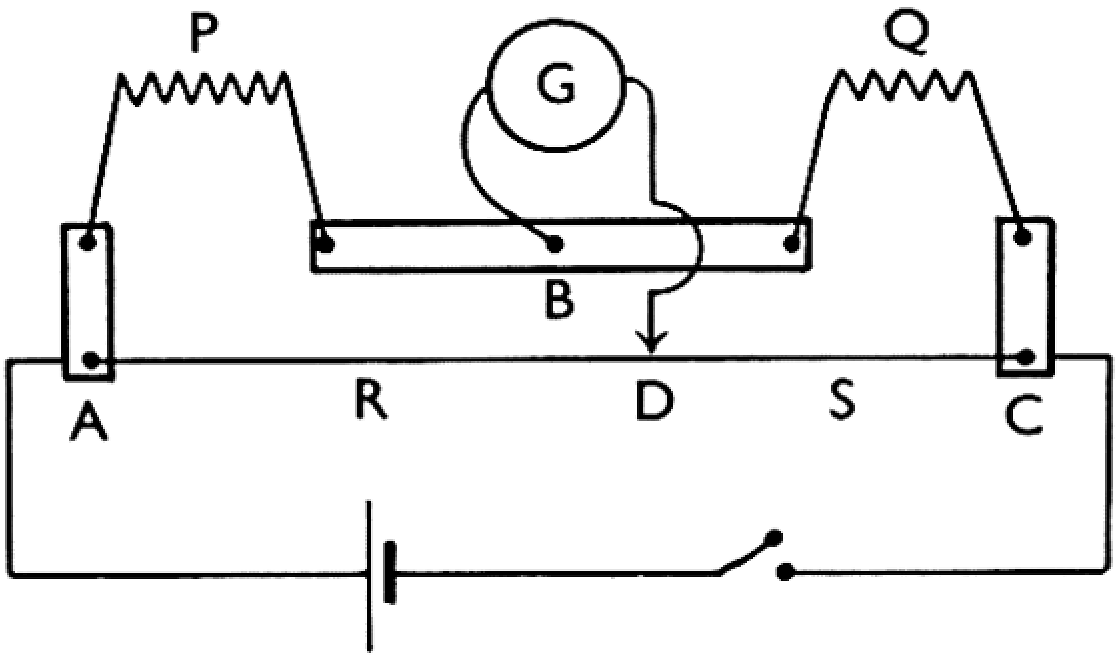


Fig. 155

The position of the sliding contact is adjusted until no current flows in the galvanometer. Then

$$\frac{P}{Q}=\frac{R}{S}=\frac{\text{Length } AD}{\text{Length } CD}.$$

$$\therefore\ P=Q\,\frac{AD}{CD}.$$

The Post Office Box

Another practical form of the Wheatstone Bridge is the Post Office box (Fig. 156), originally designed for locating faults in telephone cables.

The box consists of three sets of resistances which can be brought into circuit by removing plugs. *P* and *Q* are known as the ratio arms and each contains resistances of 10, 100 and 1000 ohms; *R* will give any resistance up to about 6000 ohms (in this particular box). Two switches are connected inside the box to the terminals indicated by the dotted lines. The circuit, connected as shown, is identical with that in Fig. 154, but in this case *S* is the unknown resistance to be measured.

When the circuit has been connected up the method of procedure is as follows:

Take out the 10-ohm plug from both *P* and *Q*. Suppose the galvanometer deflects to the left when $R=1$ ohm and to the right when $R=2$ ohms. Then the resistance of *S* lies between 1 and 2 ohms. Now make *P* 100 ohms and *Q* 10 ohms. *R* will lie between 10 and 20 ohms, say between 16 and 17. Finally, make *P* 1000 ohms and *Q* 10 ohms. *R* will lie between 160 and 170 ohms. Suppose 163 ohms produces a deflection of $\frac{1}{2}$ division to the left in the galvanometer and 164 ohms a deflection of $\frac{1}{2}$ division to the right. Then the resistance of S is 1·635 ohms. See that all the plugs in the box are tight while performing the experiment; otherwise unexpected resistances are introduced into the circuit.

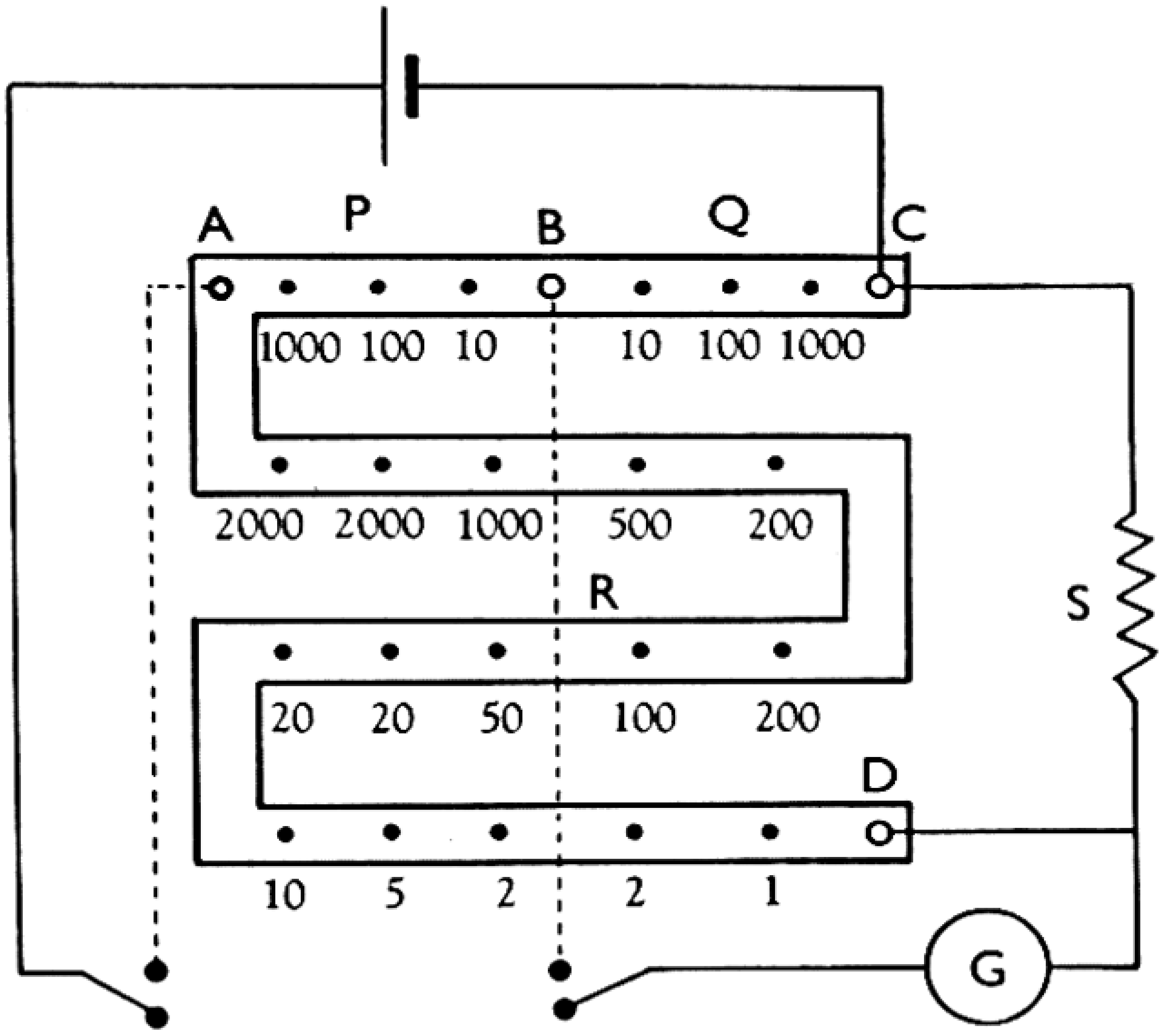


Fig. 156

Resistivity

To compare the resisting powers of different substances a term *resistivity* or specific resistance is used.

The resistivity of a substance is the resistance of a wire of the substance per cm. length and per sq.cm. cross-section.

If the length of a wire is doubled this is equivalent to two of the original lengths in series; hence the resistance is doubled. Thus the resistance R of a wire is proportional to its length, l. Again, if the area of cross-section of a wire is doubled, this is equivalent to two wires in parallel; hence the resistance is halved. Thus the resistance is inversely proportional to the area of cross-section A, i.e.

$$R \propto l/A,$$

If ρ = resistivity of the substance of which the wire is made,

$$R = \rho(l/A).$$

The resistivity of a substance is found by measuring the resistance of a known length of wire and also its diameter by means of a micrometer screw gauge.

The effect of temperature on resistance

The resistance of most substances increases when the temperature rises. The *temperature coefficient of resistance* is defined by the equation

$$R_t = R_0(1+\alpha t),$$

where R_t = resistance at $t°$ C., R_0 = resistance at 0° C., α = temperature coefficient of resistance. The temperature coefficient of resistance of a substance in the form of a wire may be found by measuring its resistance at different temperatures. If resistance is plotted against temperature the graph will be found to be straight only over small ranges of temperature showing that the above equation is only approximately true. The value of α, which is $(R_t - R_0)/R_0 t$, may be obtained from the graph (see Fig. 157).

The temperature coefficients of resistance of most pure metals do not differ widely and are of the order of 0·004 per °C., which is roughly equal to $\frac{1}{273}$, the coefficient of expansion of gases.

Experiments performed in 1911 and subsequently have shown that when rings of mercury, lead, tin and thallium are cooled to the temperature of liquid helium their resistances disappear almost entirely. A current once started in the ring will continue for days. The phenomenon is called *supraconductivity*.

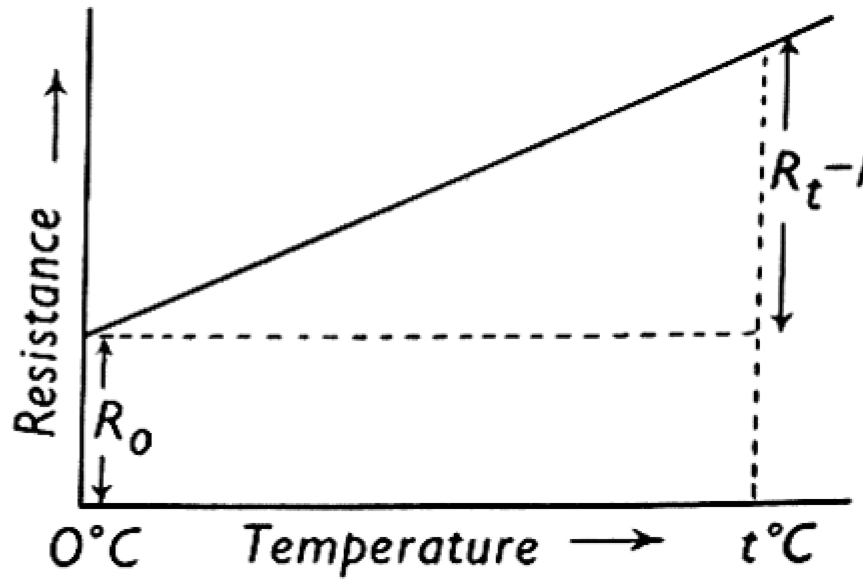


Fig. 157

When the temperature of a pure metal is raised from 0° C. its resistance increases by approximately $\frac{1}{273}$ of its resistance at 0° C. for each 1° C. rise. Since the temperature of the filament of a gas-filled tungsten lamp when lit is over 2730° C., it follows that the resistance of the lamp when lit is more than ten times its resistance when cold.

The temperature coefficient of resistance of most alloys is considerably less than that of pure metals. The value for manganin is of the order of 0·00005 and for eureka 0·00001. These two alloys are used for making resistance coils, since a slight change of temperature has no appreciable effect on their resistance.

Carbon, boron, glass, porcelain and electrolytes show a decrease in resistance with rise in temperature.

Resistivity and temperature coefficient of resistance

Substance	Resistivity ohm-cm.		Temp. coefficient per °C.	
	Value $\times 10^{-6}$	Temp. °C.	Value $\times 10^{-4}$	Temp. Range
Copper, drawn	1·78	18	42·8	18
Aluminium	3·21	18	38	18–100
Iron, pure	11·5	50	62	18
Platinum	11·0	18	38	0–100
Eureka or constantan	49·0	18	−0·4 to +0·1	18
Manganin	44·5	18	0·02–0·5	20
German silver	16–40	18	2·3–6	18
Platinoid	34·4	18	2·5	18

The platinum resistance thermometer

The variation in the resistance of platinum with temperature is utilised in the platinum resistance thermometer.

A fine platinum wire is wound in a double spiral on a mica former enclosed in a tube of fused silica and connected by relatively thick copper leads to one arm of a Wheatstone Bridge (see Fig. 158). If the resistance of the platinum wire is of the order of 2 or 3 ohms at 0° C., it increases by about 1 ohm for a rise of 100° C. Thus to measure temperature to $\frac{1}{100}$ °C. it is necessary to measure a change of resistance of $\frac{1}{10,000}$ ohm. This fine adjustment is achieved by means of a sliding contact and a wire, *AB*, of suitable uniform resistance per unit length, *r* ohms per cm. The resistances of the arms *P* and *Q* are made equal and *R* is made approximately equal to the resistance of the platinum wire *S*.

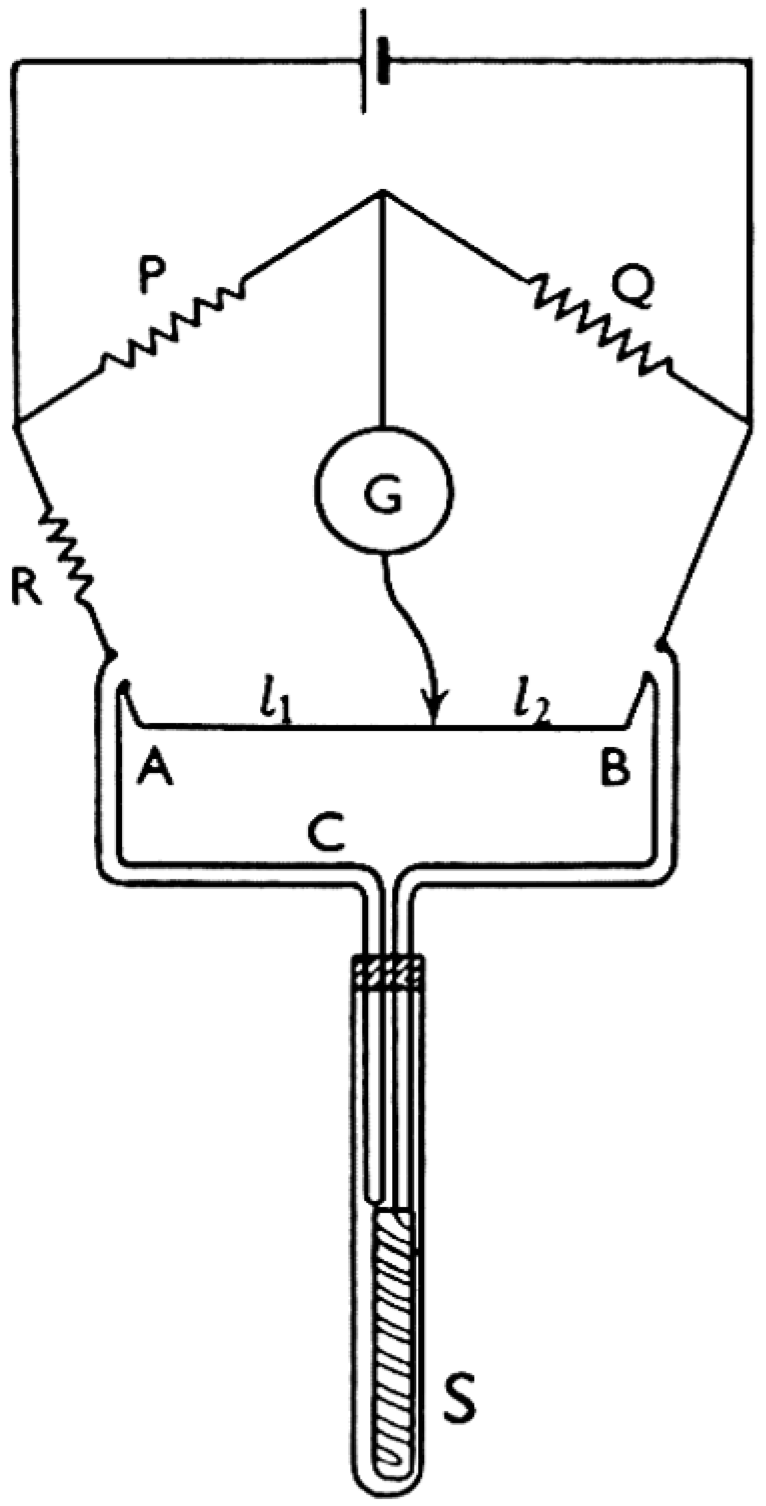


Fig. 158

When the bridge is balanced, since $P = Q$,

$$R + rl_1 = S + rl_2.$$

$$\therefore\ S = R + r(l_1 - l_2).$$

The effect of a change in resistance with temperature of the copper leads to the platinum wire is eliminated by means of similar, compensating leads C.

The resistance of platinum varies with temperature, as measured on the perfect gas scale, in accordance with the equation

$$R_t = R_0(1+\alpha t+\beta t^2),$$

where R_t and R_0 are the resistances at t and 0° C. respectively, and α and β are constants. To find the three unknown constants R_0, α and β it is necessary to determine the resistance of the platinum wire at three fixed points; the boiling-point of sulphur under standard atmospheric pressure (444·60° C.) is used in addition to the usual ice and steam.

The advantages of the platinum resistance thermometer are its accuracy and its wide range, −200 to 1100° C.

The resistance of a galvanometer

The most accurate method of finding the resistance of a galvanometer is to clamp the moving part and insert the galvanometer in the arm of a Wheatstone Bridge. It is not always possible, however, to do this, in which case the method of Fig. 159, due to Lord Kelvin, may be employed. The variable resistance T is adjusted until a suitable deflection is obtained in the galvanometer. The resistances R, S or Q are adjusted until, on depressing the switch K, there is no change in the deflection of the galvanometer. The potentials of the corners of the bridge connected through K must then be at the same potential, and the bridge is balanced.

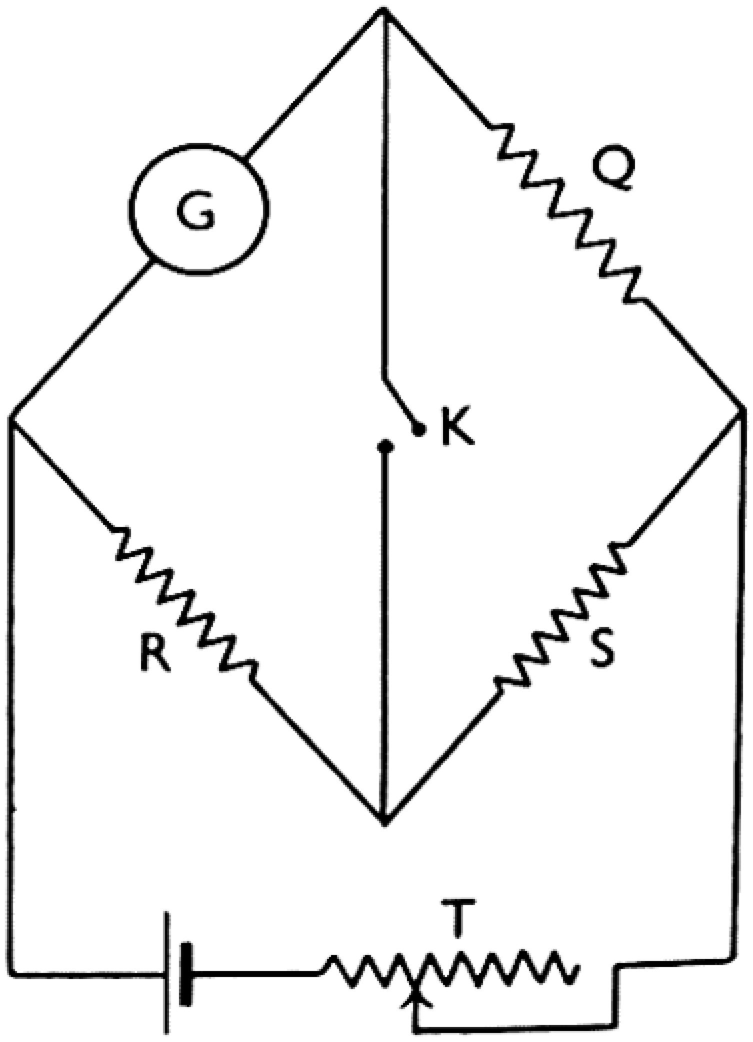


Fig. 159

Hence

$$G/Q = R/S.$$

Measurement of low resistance by a potentiometer

The Wheatstone Bridge is not suitable for measuring resistances of less than $\frac{1}{10}$ ohm, since the resistances of the connecting wires and at the terminal contacts are appreciable. Low resistances are most conveniently measured by means of a potentiometer. A suitable current, i, is passed through the unknown low resistance, R_1, and a standard low resistance, R_2, in series (see Fig. 160). The P.D.'s across the two resistances, iR_1 and iR_2, are compared by means of a potentiometer. When the balance point, K_1, has been found, the leads from A and from the sliding contact are connected to the ends of R_2 (instead of R_1) and a new balance point, K_2, is found.

$$\frac{iR_1}{iR_2}=\frac{AK_1}{AK_2}.$$

$$\therefore\ \frac{R_1}{R_2}=\frac{AK_1}{AK_2}.$$

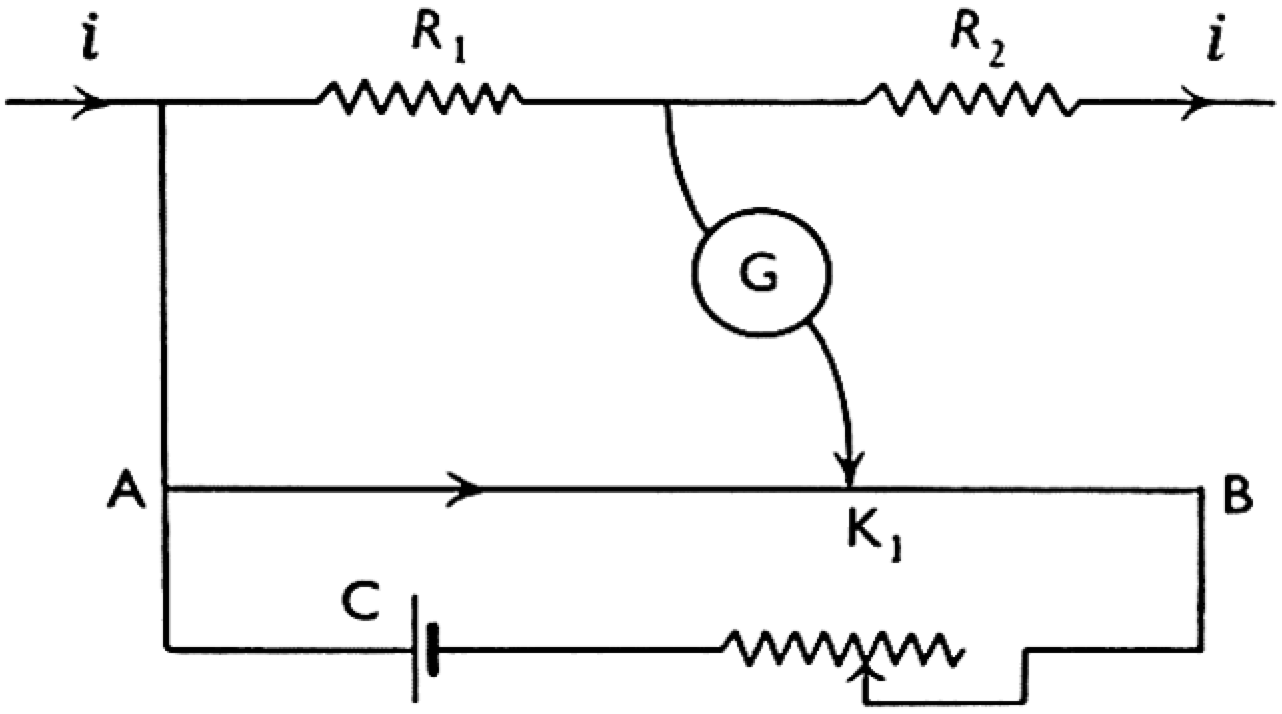


Fig. 160

Suppose the resistances of R_1 and R_2 are of the order of 0·1 ohm and i is 1 ampere; then the P.D.'s across R_1 and R_2 are about 0·1 volt. If the lead accumulator, C, were connected directly across AB the P.D. across AB would be about 2 volts and K_1 would be very near to A. The variable resistance is adjusted until the balance point, K_1, is a reasonable distance from A; the P.D. across AB is then probably of the order of 0·2 volt and the rest of the P.D., 1·8 volts, is across the variable resistance.

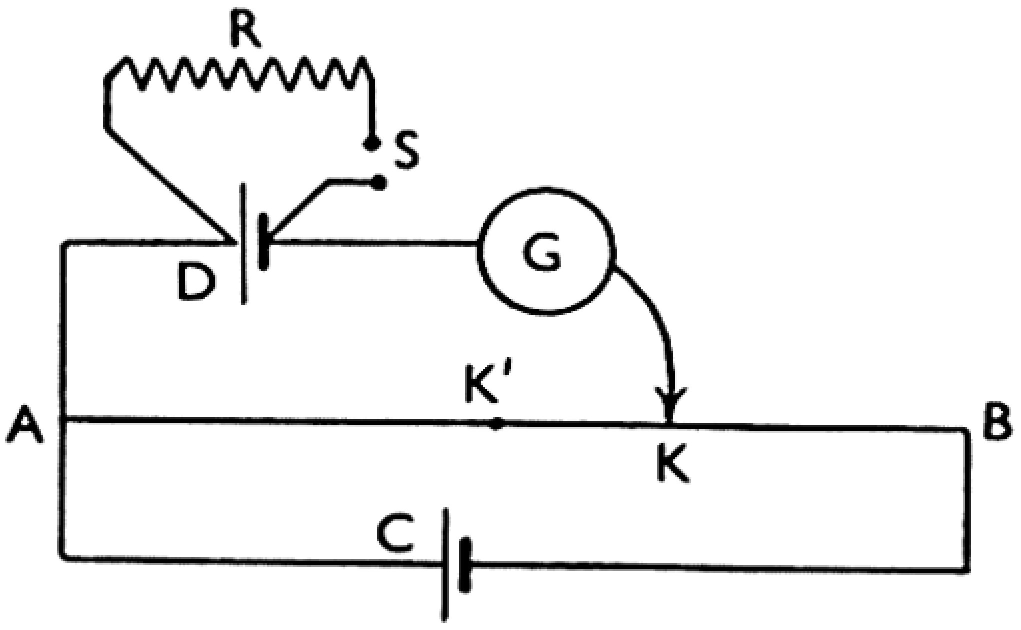


Fig. 161

Measurement of internal resistance of a cell by a potentiometer

The internal resistance of a cell may be measured by comparing the P.D.'s between the terminals of the cell when on open circuit and when bridged by a known resistance, R (Fig. 161). The balance point K on the potentiometer wire is determined with the switch, S, open, and the balance point, K', with the switch closed.

Let E=E.M.F. of cell, i.e. the P.D. on open circuit.

V=P.D. of cell when connected to R.

i=current flowing through R.

r=internal resistance of cell.

By Ohm's law

$$E=i(R+r) \quad \text{and} \quad V=iR.$$

$$\therefore \quad \frac{R+r}{R}=\frac{E}{V}=\frac{AK}{AK'}.$$

Knowing R, AK and AK' it is possible to calculate r.

Measurement of a high resistance

The Wheatstone Bridge method is unsuitable for the measurement of high resistances—of the order of 1 megohm (1,000,000 ohms).

To measure a high resistance the method of substitution can be employed. The resistance is placed in series with a battery and sensitive galvanometer whose deflection is noted. The unknown resistance is replaced by a standard high resistance which is adjusted until the deflection is the same as before. The value of the standard high resistance is equal to that of the unknown resistance.

An alternative method is a direct application of Ohm's law; the high resistance is placed in series with a galvanometer of known sensitivity and a suitable known P.D. is applied.

The ohmmeter

A galvanometer can be converted into an ammeter by means of a shunt and into a voltmeter by means of a high series resistance. It is possible also to use a galvanometer as an ohmmeter; its scale may be graduated in ohms.

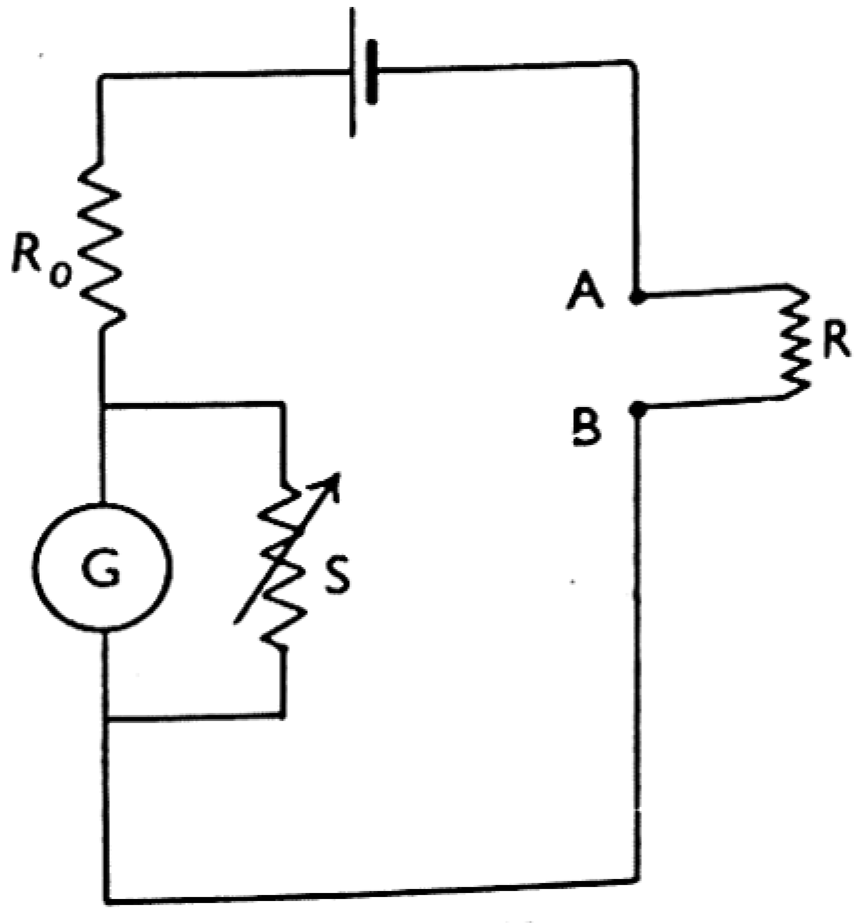


Fig. 162

The galvanometer is shunted with a variable resistance S and is connected in series with a cell, usually of E.M.F. 1·5 volts, and a standard resistance, R_0 (Fig. 162). The scale is graduated 'backwards'; a full-scale deflection reads zero ohms.

Before using the instrument the terminals AB are short-circuited and the shunt resistance, S, is adjusted to give a full-scale deflection, i.e. a zero reading. S is necessary to compensate for the change of the E.M.F. of the battery with age.

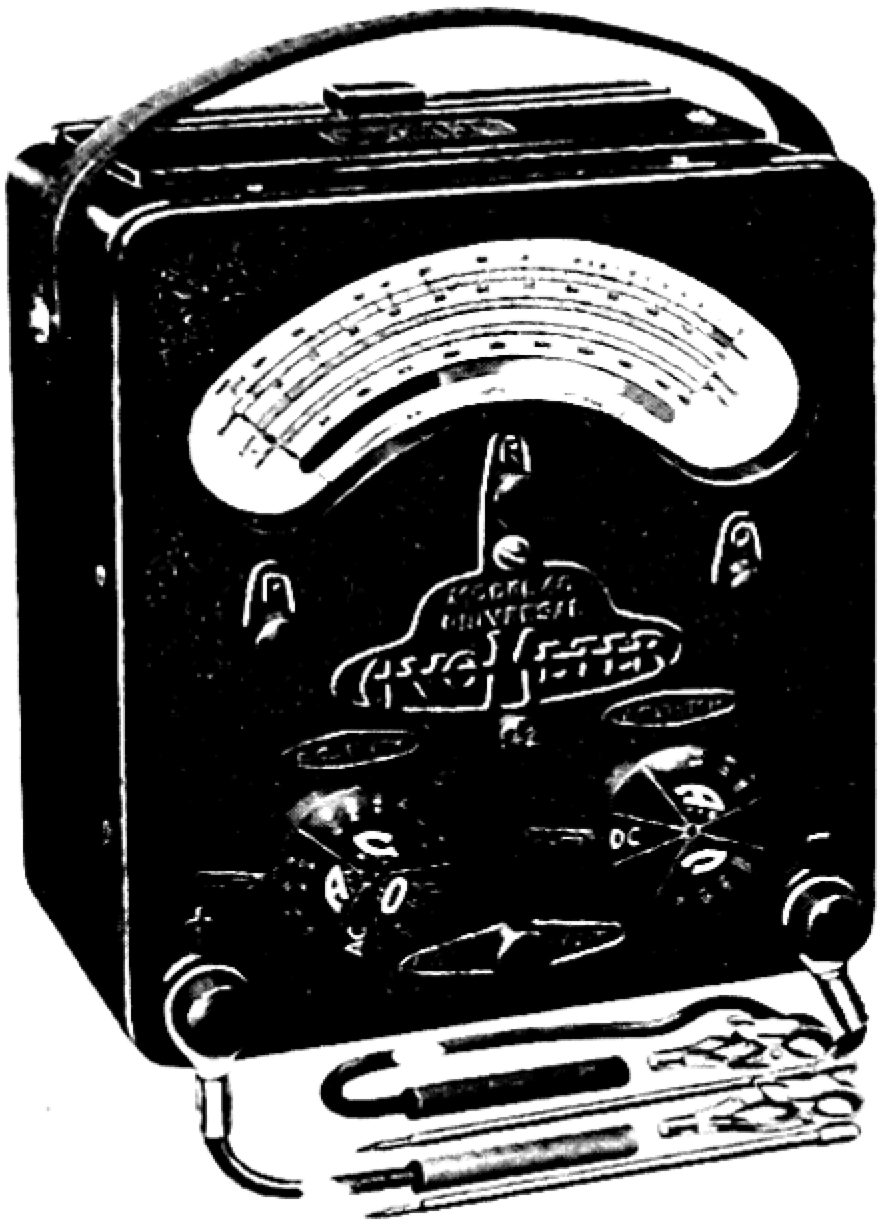


Fig. 163. Universal avometer. The three scales are graduated in ohms, amperes and volts, and different ranges are possible in each case.

Suppose the current is i_0 when the terminals are short-circuited, and suppose it is i when an unknown resistance R is put across the terminals. If E is the E.M.F. of the battery, ignoring its internal resistance,

$$E = i_0 R_0 = i(R + R_0).$$

$$\therefore \quad R = R_0\left(\frac{i_0}{i} - 1\right).$$

Since R_0 and i_0 are constants, the value of R depends on the current i and hence the scale can be graduated in ohms. The mid-scale reading, when $i = \frac{1}{2}i_0$, is clearly equal to R_0.

Fig. 163 shows a universal meter with scales graduated in amperes, volts and ohms; it consists of a moving-coil galvanometer, a set of shunts, a set of series resistances and a battery, all enclosed in the case. A complicated switch enables the appropriate connections to be made.

QUESTIONS

1. (*a*) Explain the principle of the moving-coil galvanometer.

(*b*) Describe how you would find the current sensitivity of a galvanometer of this type, giving a diagram of your electrical circuit.

(*c*) A milliammeter of resistance 6 ohms reads up to 10 milliamperes. How would you convert it into an ammeter reading to 3 amperes? (N.)

2. A certain galvanometer gives a full-scale deflection for $\frac{1}{2}$ milliampere, and has a resistance of 20 ohms. How can it be adapted for use as (*a*) an ammeter to measure currents up to 5 amperes, (*b*) a voltmeter to measure potentials up to 5 volts? (O. & C.)

3. A galvanometer whose resistance is 12 ohms gives full-scale deflection for $\frac{1}{100}$ ampere. What modifications would be necessary to make it read (*a*) voltages up to 100 volts, (*b*) currents up to 100 amperes? (O. S.)

4. How may a milliammeter of resistance 20 ohms be adapted to act as a voltmeter, so that the scale formerly reading milliamperes now reads the same number of volts? (O. S.)

5. In order to find the current sensitivity of a galvanometer it is shunted with a resistance of 10 ohms and connected in series with a cell and a resistance of 1 megohm. If the galvanometer has a resistance of 90 ohms, the cell an E.M.F. of 1·5 volts, and the deflection obtained is 150 scale divisions, find an approximate value for the sensitivity of the galvanometer in amperes per scale division. (N.)

6. Describe the construction of a radial-field, moving-coil galvanometer and discuss the factors which affect its sensitivity. Why does the coil of such a galvanometer rapidly cease to oscillate when its terminals are short-circuited?

If the coil is a rectangle of length 3 cm. and breadth 1 cm. and has 100 turns and the strength of the magnetic field is 4000 oersteds, calculate the deflecting couple on the coil when a current of 1 milliampere flows through it. (B.)

7. The coil of a galvanometer has 300 turns of mean area 1 cm.^{2} A couple of 1 dyne-cm. applied to the coil causes it to twist through 1 revolution against the torsional control of the suspension. If there is a radial field of 2000 oersteds what current, when passed through the galvanometer, will cause the spot of light on a scale 1 metre away to be deflected through 1 mm.? (C.)

8. A galvanometer coil is 2 cm. square and contains 100 turns. Its period is 20 sec. and its moment of inertia is 2 g.cm.^{2} The field of the magnet is 2000 gauss. Determine the current sensitivity of the galvanometer in terms of the deflection of a reflected beam of light on a screen 1 metre away. (C. S.)

9. Describe in detail the construction of a moving-coil voltmeter suitable for measuring potential differences of the order of 100 volts. What steps are taken in a high-class instrument of this type:

(*a*) to provide adequate damping of the pointer;

(*b*) to enable the user to avoid errors of parallax?

An instrument maker is constructing a moving-coil galvanometer, which is required to give as large a deflection as possible when connected to a cell of E.M.F. E and resistance R. The maker has settled all the features of his design except the gauge of wire with which he is to wind the moving coil. Show that, if he now uses the best possible gauge of copper wire, the resistance of his meter will be equal to R. (It may be assumed: (i) that the number of turns which can be wound on the moving coil is inversely proportional to the area of cross-section of the wire chosen; (ii) that the resistance of the moving coil constitutes the whole resistance of the meter.) (C. S.)

10. Two moving-coil galvanometers with similar coils, but of different sizes, are wound with wire of the same diameter and are suspended by means of similar wires of the same size. If the magnetic fields are the same in the two cases, show how the current sensitivities are related to the times of swing. (C. S.)

11. A condenser is alternately charged 50 times per sec. to 100 volts and discharged through a galvanometer of resistance 50 ohms shunted by a resistance of 10 ohms. Each division of the galvanometer scale represents a current of 6×10^{-6} ampere through the galvanometer. If a steady deflection of 30 divisions is produced, calculate the capacity of the condenser in microfarads. (N.)

12. What standard differences exist between the dead-beat and ballistic types of moving-coil galvanometers, and what are the reasons for such differences?

Describe how (*a*) the resistance, (*b*) the current sensitivity, (*c*) the quantity sensitivity of a moving-coil ballistic galvanometer may be determined experimentally no other galvanometer being available. (N.)

13. Give the theory of the potentiometer method of comparing the electromotive forces of two cells.

A student who has put together such a circuit fails to find a dead-point. State the possible reasons for this.

A Daniell cell on open circuit gives a dead-point when the terminals are connected across 90·0 cm. of potentiometer wire. When the cell terminals are connected through a 4-ohm coil, the length of wire necessary for a balance is reduced to 80·0 cm. Calculate the internal resistance of the cell. (C.)

14. A 2-volt accumulator of negligible internal resistance is connected to the ends of a potentiometer wire 1 metre long and 5 ohms resistance. A cell of E.M.F. 1·5 volts and internal resistance 0·9 ohm is connected through a galvanometer in the usual way; what length of potentiometer wire will be required to produce a balance?

What length of potentiometer wire will be required to balance in the following cases:

(*a*) When 1 ohm is placed in series with the accumulator?

(*b*) When 1 ohm is placed in parallel with the cell of E.M.F. 1·5 volts? (L.)

15. Explain the principle of the potentiometer.

You are supplied with the following apparatus: an accumulator of E.M.F. about 2 volts; a standard cell of E.M.F. 1·018 volts; a metre wire of resistance 1 ohm; two variable resistance boxes each providing resistances up to 2000 ohms; a sensitive galvanometer and connecting wire. Explain with the help of a labelled diagram how, using this apparatus, you would arrange and standardise a potentiometer which would measure potential differences up to 1 millivolt along the potentiometer wire. (N.)

16. In an experiment to determine the resistivity of copper, a length of copper wire of resistance r_1 is connected in series with a comparable standard resistance r_2 of 0·01 ohm, an accumulator of just over 2·00 volts E.M.F. and a resistance r of 20 ohms. The potential differences across r_1 and r_2 are compared by a potentiometer arrangement consisting of a uniform wire of 30 ohms resistance, a resistance R in series, a 2-volt accumulator, and the usual accessories. Draw the circuit diagram. What is the purpose of including the resistance r in the circuit? Estimate the approximate value of R if the balance points are near the middle of the potentiometer wire. (N.)

17. An accumulator, a resistance box, a coil of resistance 400 ohms, and a potentiometer wire of length 10 metres and resistance 40 ohms, are connected in series. It is found that for a certain value of the resistance in the box, 685 cm. of potentiometer wire are required to balance the E.M.F. of a concentration cell. For the same value of the box resistance the potential drop across the 400-ohm resistance plus 320 cm. of potentiometer wire just balances a standard cell of E.M.F. 1·018 volts. Calculate the E.M.F. of the concentration cell. (O. & C.)

18. Describe the Wheatstone Bridge method of measuring resistances, and prove the formula connecting the resistances when a balance is obtained.

In a Wheatstone Bridge the four resistances in the arms of the bridge are AB 2 ohms, BC 4 ohms, AD 1 ohm, and DC 3 ohms. The terminals of a cell of E.M.F. 2 volts and negligible resistance are connected by wires of negligible resistance to A and C. If a galvanometer of resistance 10 ohms is connected between B and D, find the current in the galvanometer. (O. & C.)

19. Discuss in general terms the difficulties which present themselves in the measurement of low and of high resistance by the simple Wheatstone Bridge method.

Describe methods, one for each case, which are suitable for measuring a resistance of order 0·1 ohm, and one of order 100 megohms. (N.)

20. A telegraph line BCD, which is 50 miles long, has a fault due to earthing at an unknown point C. The end B of the line is joined to the end D through resistances P, Q, R in series and in that order. A battery is connected from the end B to the junction of the resistances Q and R and a galvanometer is connected from the junction of P and Q to earth. The resistance of R is equal to that of 12·5 miles of the telegraph line. When $P=1500$ ohms and $Q=1425$ ohms the galvanometer is not deflected. Where is the fault in the line? (D.)

21. Given that the resistivity of aluminium is twice that of copper, and that the density of aluminium is one-third that of copper, find the ratio of the masses of aluminium and copper conductors of equal length and equal resistance. (N.)

22. State Kirchhoff's laws for an electrical network and show how they may be used to obtain the balance condition in a Wheatstone Bridge.

A Wheatstone Bridge is used to measure a resistance and at the balance point the resistances of all four arms are equal. Calculate how the accuracy of the measurement depends upon the resistance of the instrument used in detecting the balance point. (C. S.)

23. Describe an experiment to determine the temperature coefficient of resistance of copper.

A metal wire, 100 cm. long and of 1 sq.mm. cross-section, has a resistance of 0·20 ohm at 0° C. Calculate the specific resistance of metal at 500° C. given that its temperature coefficient of resistance is 62×10^{-4} per °C., and neglecting its thermal expansion. (N.)

24. The resistance of an electric lamp is found to be 55 ohms when measured by a Post Office box at 15° C. When it is connected to the 230-volt supply it uses 100 watts. Assuming that the resistance of the lamp is proportional to the absolute temperature, find the working filament temperature. (B.)

25. The resistance of an iron wire was measured as 16·00 ohms at 20° C. The standard coil used was correct at 15° C. and it consisted of wire whose temperature coefficient was 0·00027 per °C. If the temperature coefficient of the iron wire was 0·007 per °C., find the true resistance of the iron wire at 0° C. (D.)

26. Give an account of the variation of the electric resistance of metals with temperature.

Describe how the variation of the resistance of a platinum wire may be applied to determine the temperature of a bath of hot oil. (O. S.)

27. The resistance of the platinum wire of a platinum resistance thermometer increases by 4·0 ohms for a rise in temperature of 100° C. If the bridge wire (*AB* in Fig. 158) is 50·0 cm. long, find what its resistance must be in order that a movement of 0·5 mm. of the sliding contact may correspond to a change of temperature of 0·01° C.

CHAPTER 9

ABSOLUTE MEASUREMENTS UNITS AND DIMENSIONS

IN the last chapter various methods of making electrical measurements were discussed but these involved the use of standard resistances and standard cells. It must now be asked how these standards are calibrated and what are the ultimate, experimental determinations on which the ampere, the volt and the ohm are based.

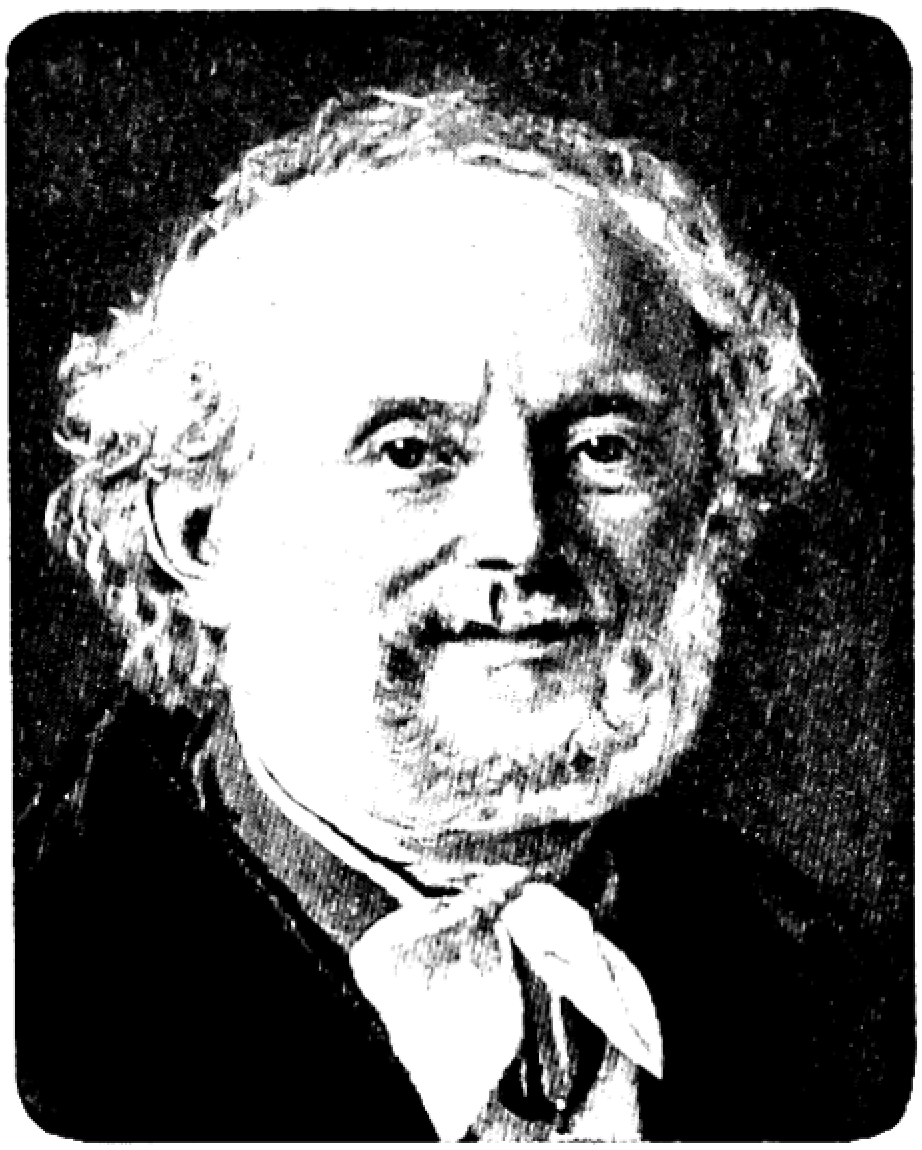
The electrical units of the earliest investigators were necessarily purely arbitrary; for example, the capacitance of a Leyden jar was measured by the volume of liquid it would hold and the standard of resistance was the resistance of a length of iron wire of certain thickness. All electrical units could have been defined in some such conveniently practical manner, but there would have been no logical or simple correlation between them.

In 1832 Gauss developed a system of magnetic units based on three fundamental units, those of mass, length and time. His method of deriving magnetic units from mechanical units was the same as that described in Chapter 4, although he chose the millimetre, milligramme and second instead of the centimetre, the gramme and second as employed in the c.g.s. system. It should be noted that the units of mass, length and time are purely arbitrary—the mass of a certain lump of platinum, the length of a certain bar, and $\frac{1}{86,400}$ of a mean solar day. But once these arbitrary standards have been chosen, all the units of magnetism and electricity can be derived from them.

A measurement which is made in terms of these derived units is called an absolute measurement; this may be contrasted with the comparison of two quantities of the same kind, such as the determination of resistance by the Wheatstone Bridge.

In 1840 Weber made the first absolute determination of electric current with a tangent galvanometer, finding the value of H in the formula $i=\frac{Hr}{2\pi n}\tan\theta$ by the method described on p. 64. By 1851 he had worked out a complete system of electrical units based on the magnetic units of Gauss, starting from the definition of the unit magnetic pole.

KARL FRIEDRICH GAUSS



WILHELM WEBER

KARL FRIEDRICH GAUSS (1777–1855), *called the prince of mathematicians, was the son of a gardener. When he was fourteen his exceptional intellectual gifts became known to the Duke of Brunswick, who took him under his protection; at thirty he became professor of mathematics at the University of Göttingen. He was particularly interested in terrestrial magnetism and was one of the founders of the German Magnetic Union, whose object was to make continuous determinations of the magnetic elements at different places. In 1832 he proposed the system of electromagnetic units, and it was his interest in magnetism that induced him to take the unit magnetic pole, rather than the unit electrostatic charge, as the starting-point. The difficulty of measuring magnetism accurately was overcome by his concept of magnetic moment.*

WILHELM WEBER (1804–1890) *was called to the chair of physics at Göttingen in 1831, where he worked with Gauss, and developed the electromagnetic system of units. In the course of his investigations he invented several well-known instruments such as the earth inductor, he made the earliest absolute determinations of current and electromotive force, and discovered that the ratio between the electromagnetic and electrostatic units is equal to the velocity of light. Resentment was aroused in Germany when, at the congress in Paris in 1881 at which the names of the electrical units were selected, Gauss and Weber, the founders of the system adopted, were overlooked, and some writers in Germany have used weber instead of ampère to denote the unit of electric current. The term gauss is now employed as the unit of flux density and the term weber as the practical unit of flux, equal to* 10^8 *lines.*

In 1861 the British Association for the Advancement of Science set up a committee on Standards of Electrical Resistance. The Committee, which included such famous men as Clerk Maxwell, Lord Kelvin and Joule, adopted the c.g.s. electromagnetic system which has been used ever since. The definitions of the units of current, potential difference and resistance on this system have already been described, but they are given also in the second column in the table opposite.

The practical units

The practical units are defined in terms of the electromagnetic units:

$$\begin{aligned} 1 \text{ ampere} &= 10^{-1} \text{ e.m.u.} \\ 1 \text{ volt} &= 10^{8} \text{ e.m.u.} \\ 1 \text{ ohm} &= 10^{9} \text{ e.m.u.} \end{aligned}$$

These are the so-called absolute, practical units to distinguish them from the 'international' practical units which are slightly different. The international units have recently been discarded but a brief account of them must be given.

The committee of the British Association in 1861 set in hand research for the devising of an easily reproducible standard ohm equivalent to the defined value of 10^9 e.m.u. In those days there was not the same need for practical standards of current and potential difference since the tangent galvanometer was regarded as satisfactory and the Daniell cell was in use as a standard cell.

The standard resistance took the form of a column of mercury of cross-section 1 sq.mm. measured at 0° C., and in 1893 the Chicago Conference adopted 106·3 cm. as its length. The London Electrical Conference in 1908 decided to make the figure 106·300 cm., not because the length equivalent to the absolute ohm was known to this degree of accuracy, but to make the legal 'international ohm', which was defined for practical purposes in this way, more definite. The slight discrepancy between the international ohm and the absolute ohm was recognised and it was planned to adjust the former when more accurate determinations of its value were made in terms of the latter.

Now it is possible to construct the international ohm to 1 part in 1000 quite easily and to 1 part in 10,000 with considerable difficulty, but no one has succeeded in doing so to 1 part in 100,000. The main difficulty is the lack of uniformity of the bore of the glass tube. Owing to the greatly increased accuracy of absolute determinations the labour of setting up reproducible standards of resistance is as great as making the absolute measurement itself. The International Committee of Weights

The three systems of c.g.s. electrical units

	Electrostatic system (based on unit charge)	Electromagnetic system (based on unit pole)	Practical system	
			Absolute units	International units, now discarded
Current	1 e.s.u. of charge per sec.	1 e.m.u. of current, flowing in an arc 1 cm. long, of a circle of radius 1 cm., creates a field of 1 oersted at the centre	1 ampere $=10^{-1}$ e.m.u.	A current of 1 international ampere, when passed through an aqueous solution of silver nitrate, deposits silver at the rate of 0·00111800 g. per sec.
Potential difference	A P.D. of 1 e.s.u. exists between two points if 1 erg of work is done when 1 e.s.u. of charge moves from one point to the other	A P.D. of 1 e.m.u. exists between two points if 1 erg of work is done when 1 e.m.u. of charge moves from one point to the other	1 volt $=10^{8}$ e.m.u. [1 joule of work is done when 1 coulomb moves through a P.D. of 1 volt]	A P.D. of 1 international volt applied to 1 international ohm causes a current of 1 international ampere to flow
Resistance	A resistance of 1 e.s.u. is such that a P.D. of 1 e.s.u. across it causes a current of 1 e.s.u.	A resistance of 1 e.m.u. is such that a P.D. of 1 e.m.u. across it causes a current of 1 e.m.u.	1 ohm $=10^{9}$ e.m.u. [1 ampere flows when 1 volt is applied across 1 ohm]	A resistance of 1 international ohm is that of a column of mercury of mass 14·4521 g. and length 106·300 cm. at 0° C.

and Measures therefore decided that from 1 January 1940 the international ohm should be discarded. This did not take effect at once owing to the war.

It was decided also to discard the international ampere and the international volt. The international ampere had previously been defined as the unvarying electric current which, when passed through a solution of silver nitrate in water, deposits silver at the rate of 0·00111800 g. per sec. This is not exactly equal to the absolute ampere, i.e. 10^{-1} e.m.u. The international volt had been defined in terms of the international ohm and ampere.

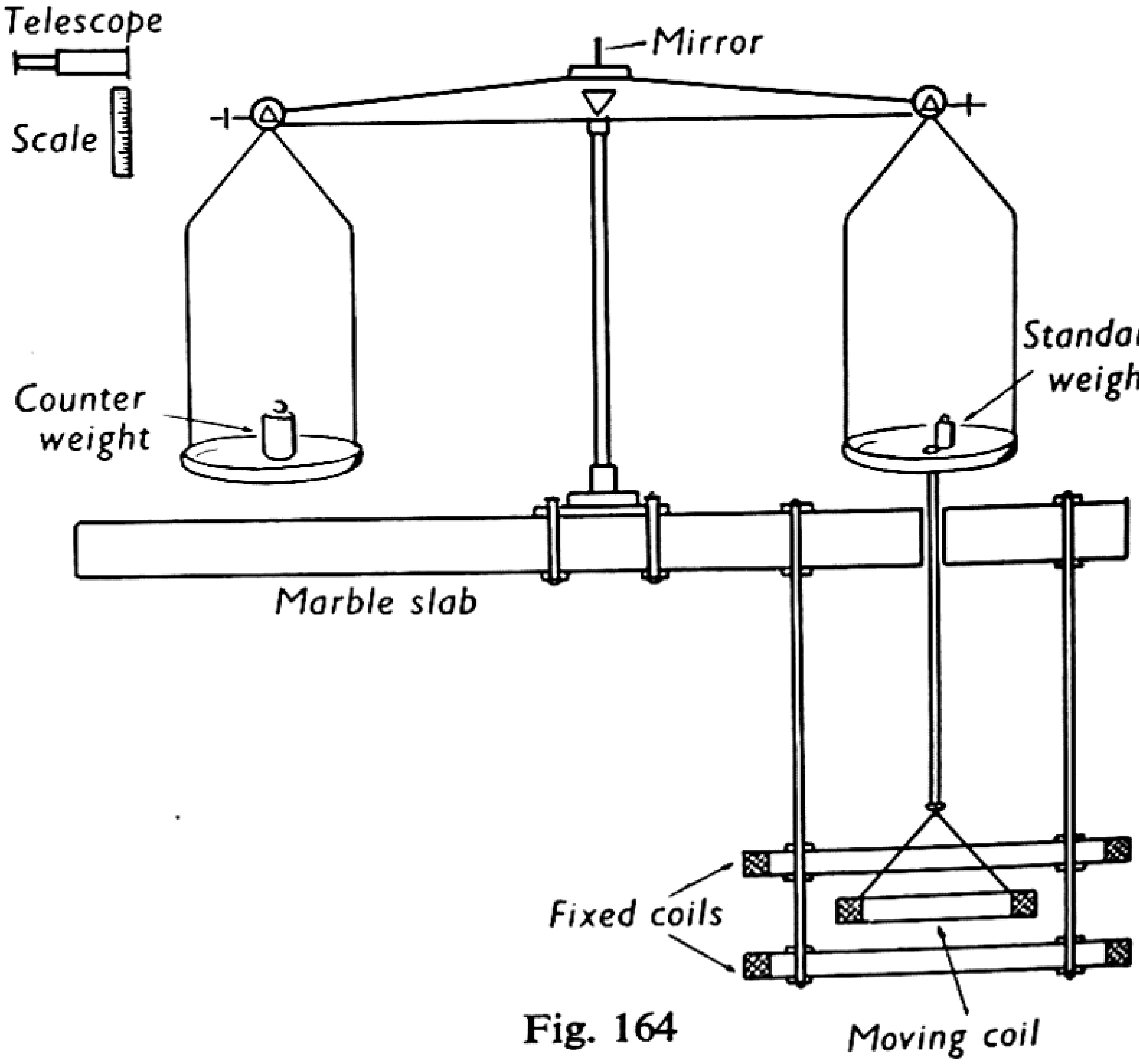


Fig. 164

Absolute measurement of the ampere

For half a century the tangent galvanometer was used for the absolute measurement of current, and between 1875 and 1890 several 'galvanometer houses' were specially built of non-magnetic material. With the advent of electric light, tramcars and the generation of electricity on a commercial scale, the disturbances to the earth's magnetic field made the tangent galvanometer so unreliable that its use was abandoned. It has now been replaced by the current balance.

The most recent accurate determination of the absolute ampere, expressed in terms of the weight of silver it will deposit in electrolysis, was made in 1934 by Curtis and Curtis at the Bureau of Standards, Washington, using a Rayleigh current balance (see Fig. 164).

The instrument consists of a movable coil arranged between two larger fixed coils, the force between the fixed and moving coils being measured by a balance capable of weighing to 1 part in 100 million.

The standard weight in Fig. 164 is first removed and a current sent through the coils connected in series in such a way that the moving coil is forced downwards. The counterweight is adjusted so that the balance swings about its position of rest, the swings being observed by a telescope and scale.

The current in the fixed coils is then reversed causing the moving coil to be forced upwards and the standard weight replaced. The standard weight is of such a value that, for the particular current being measured, it is equal to twice the force between the coils, thus causing the balance once more to swing about its position of rest. Then

$$2Fi^2 = mg,$$

where F = the force in dynes per unit current exerted on the moving coil, calculated from the dimensions of the coils by a method similar in principle to that on p. 94,

i = current in e.m.u.,
m = mass of the standard weight in grams,
g = acceleration due to gravity at the pan of the balance.

The current, measured by the current balance, was found to deposit silver in a silver voltameter at the rate of 0·00111804 g. per ampere per sec. Elaborate refinements and precautions are necessary to measure to this high degree of accuracy.

Absolute measurement of the ohm

Two phenomena can be utilised to make an absolute measurement of resistance, either the heating effect of an electric current or electromagnetic induction.

The heat generated in a wire is i^2Rt/J calories (see p. 128). Since the absolute value of i can be found with a current balance the formula enables the absolute value of R to be calculated. The value of J, however, is not easy to obtain accurately by a mechanical method, and the heating effect of an electric current is utilised to determine J. Thus R must be measured by the electromagnetic method.

The best known determination of the absolute ohm by the electromagnetic method is that of Lorentz, later improved by Smith at the

National Physical Laboratory. A copper disc has its axle coincident with that of a long solenoid carrying a current (see Fig. 165). The disc is rotated at speed and the induced E.M.F. between the rim and the axle is balanced against the potential difference between the ends of the resistance to be determined, R, through which the same current flows as through the solenoid.

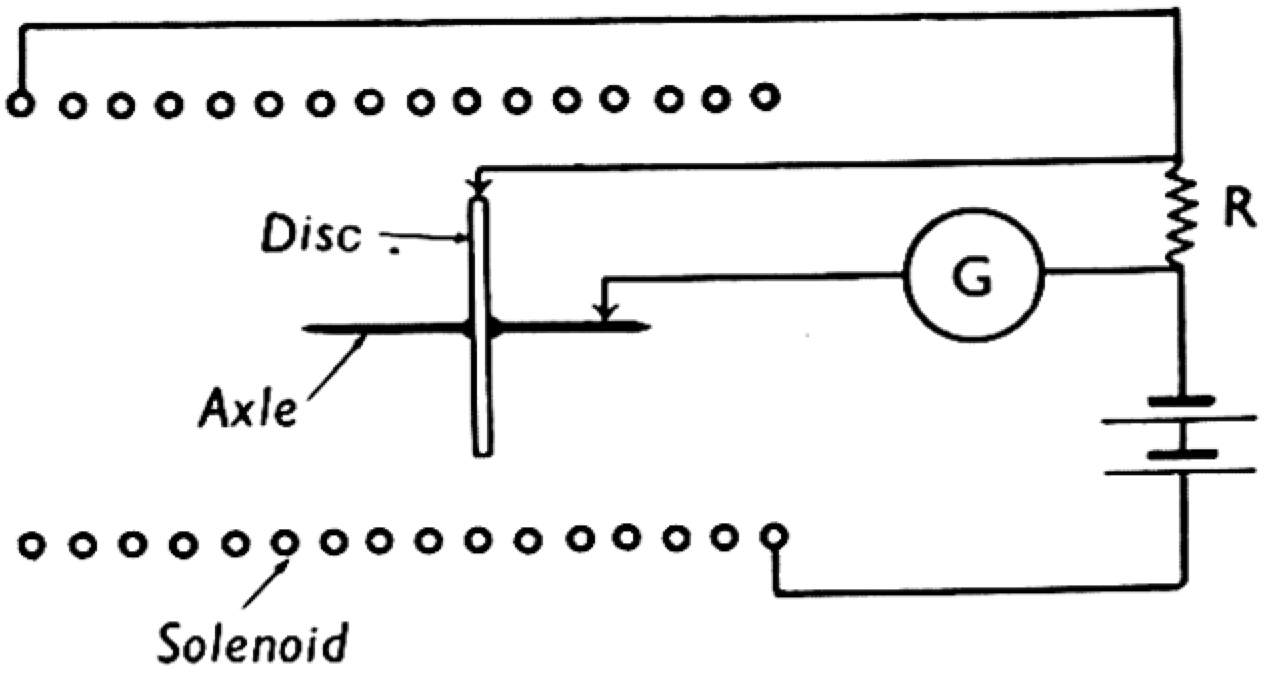


Fig. 165

The induced E.M.F. is proportional to the rate at which the lines of magnetic flux are cut (see p. 90). Each radius of the disc cuts $\pi a^2 H/T$ lines of magnetic flux per second, where a is the radius of the disc, H the magnetic field inside the solenoid, and T the time for 1 revolution of the disc.

But $H=4\pi ni$ (see p. 94),

$$\therefore \text{ Induced E.M.F.}=\frac{\pi a^2 . 4\pi ni}{T}.$$

$$\text{P.D. across resistance}=iR.$$

$$\therefore \; iR=\frac{\pi a^2 . 4\pi ni}{T},$$

$$R=\frac{4\overset{2}{\pi} a^2 n}{T}.$$

Thus R is measured in terms of the radius of the disc a, the number of turns per unit length of the solenoid, n, and the time of revolution of the disc T.

There are difficulties to be overcome such as the induced E.M.F. in the disc due to the earth's magnetic field; the smallness of the induced E.M.F. renders thermoelectric E.M.F.'s, due to heating at the rubbing contacts, important, and makes it necessary to send only a fraction of the current i through R. Moreover, the above simple calculation must be elaborated to allow for the lack of uniformity of the magnetic field inside the solenoid.

When the absolute value of one resistance has been found in this way it can be used to find the absolute values of other resistances by the Wheatstone Bridge method. Resistances can be compared with an accuracy of 1 in 10^6 fairly easily; the absolute ohm has been established to an accuracy of 1 in 10^5.

Absolute determination of the volt

The E.M.F. of a standard cell can be found in absolute volts by balancing it against the P.D. set up in a standard resistance by a current measured in absolute units.

Concrete standards of the units

The national laboratories of the world maintain concrete standards of resistance and E.M.F. which have been measured very accurately in absolute units.

Standards of resistance always take the form of manganin coils immersed in oil to keep the temperature constant. Since 1910 the concrete standard of resistance at the National Physical Laboratory has been four 1-ohm coils.

The concrete standard of E.M.F. in universal use is the Weston standard cadmium cell (see p. 151), of E.M.F. 1·0183 volts at 20° C. At the National Physical Laboratory three sets of Weston cells are maintained; in normal times, a new set of cells is constructed every year and after proper ageing, they are introduced into the group, the oldest cells being discarded. At the last extensive international comparison of standard Weston cells, in 1930, it was found that the standards of the different countries, England, Germany, France, U.S.A., U.S.S.R. and Japan, differed by not more than 1 part in 10,000.

The relation between the electromagnetic and electrostatic systems

Side by side with the electromagnetic system of units there developed the electrostatic system, based on the unit electric charge as described in Chapters 1 and 2. The unit of electric charge on the electromagnetic system is of course defined differently. It is the quantity of electricity obtained when unit current flows for 1 second, unit current being defined by its magnetic effect.

In 1856 Weber made an experimental determination of the ratio of the e.m.u. and e.s.u. of charge and obtained approximately 3×10^{10} which is the velocity of light in cm. per sec. This was regarded as more than an extraordinary coincidence by Maxwell and led him to develop his electromagnetic theory of light.

Repeated determinations of the ratio between electromagnetic and electrostatic units have been made; we shall outline two methods.

Maxwell calculated the capacitance of a condenser in e.s.u. from its dimensions and measured its capacitance in e.m.u. by the apparatus shown in Fig. 166. The condenser is charged and discharged f times per second by the rocker A, which may take the form of a rotating commutator or a tuning fork of known frequency. If f is sufficiently large the galvanometer G gives a steady deflection. Each charge on the condenser is EC, where E is the E.M.F. of the cell and C the capacitance of the condenser; the current is therefore fEC.

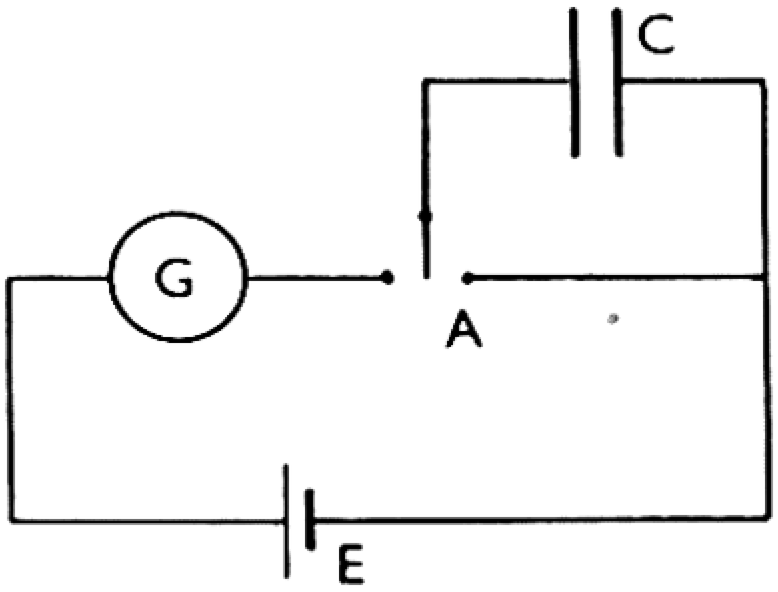


Fig. 166

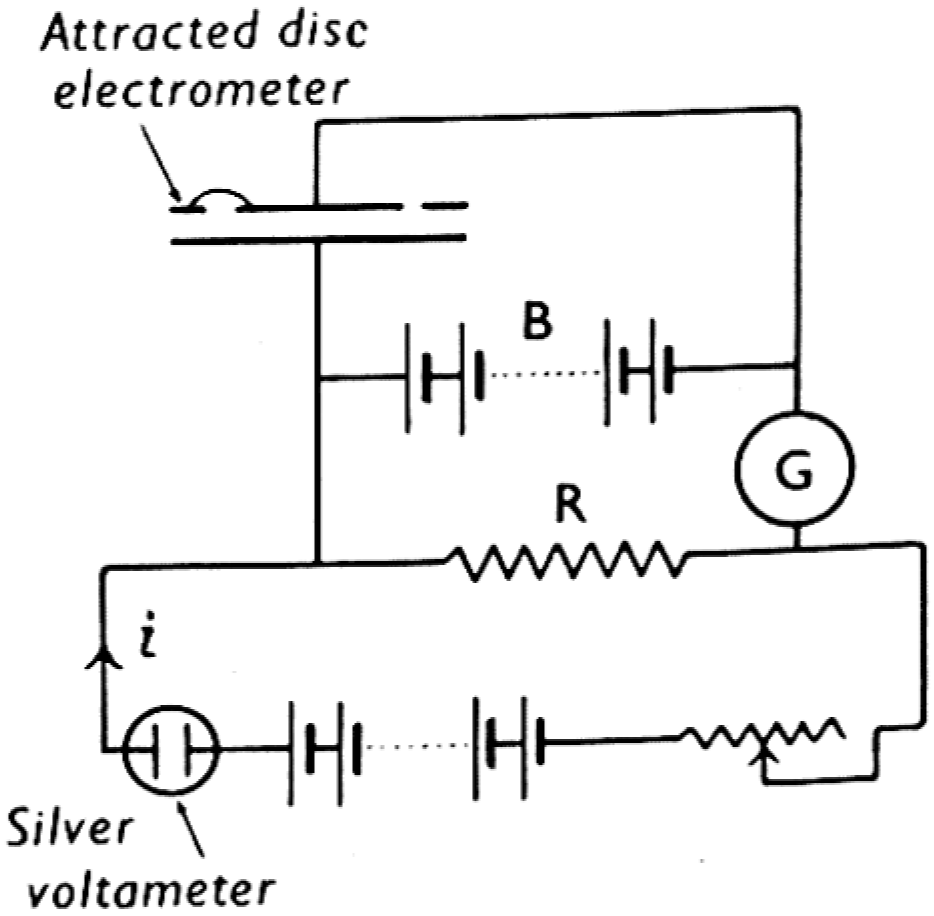


Fig. 167

The condenser and rocker are replaced by a resistance of such value, R, that the current through the galvanometer is the same.

Then
$$i=\frac{E}{R}=fEC.$$
$$\therefore\ C=\frac{1}{fR}.$$

Fabry and Perot compared the e.s.u. and e.m.u. of P.D., using the apparatus in Fig. 167. The E.M.F. of a battery B is measured in e.s.u. by means of an attracted disc electrometer and in e.m.u. by balancing it against the P.D. across a known resistance, R, through which flows a current, i, measured by a silver voltameter or current balance.

It is found that
$$\frac{\text{e.m.u. of current (or charge)}}{\text{e.s.u. of current (or charge)}}=3\times 10^{10}$$
and
$$\frac{\text{e.m.u. of potential}}{\text{e.s.u. of potential}}=\frac{1}{3\times 10^{10}}.$$

Relation between practical and electrostatic units

Since $$1 \text{ coulomb} = 10^{-1} \text{ e.m.u.}$$

and $$1 \text{ e.m.u} = 3 \times 10^{10} \text{ e.s.u. of charge.}$$

$$\therefore\ 1 \text{ coulomb} = 3 \times 10^{9} \text{ e.s.u. or statcoulombs.}$$

Since $$1 \text{ volt} = 10^{8} \text{ e.m.u.}$$

and $$1 \text{ e.m.u.} = \frac{1}{3 \times 10^{10}} \text{ e.s.u. of potential.}$$

$$\therefore\ 1 \text{ volt} = \frac{10^{8}}{3 \times 10^{10}} = \frac{1}{300} \text{ e.s.u. or statvolts.}$$

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

$$= \frac{3 \times 10^{9}}{\frac{1}{300}} = 9 \times 10^{11} \text{ e.s.u. or statfarads.}$$

The relations between other units on the two systems may be worked out similarly.

Dimensions

All magnetic and electrical units and, indeed, all physical units are derived ultimately from the fundamental units of mass, length and time. It is instructive to form what are known as dimensional equations. An area is a product of two lengths and we can write

$$[\text{Area}] = L^2.$$

Similarly $$[\text{Velocity}] = [\text{Distance}] \div [\text{Time}] = LT^{-1},$$

$$[\text{Acceleration}] = [\text{Velocity}] \div [\text{Time}] = LT^{-2},$$

$$[\text{Force}] = [\text{Mass}] \times [\text{Acceleration}] = MLT^{-2},$$

$$[\text{Work}] = [\text{Force}] \times [\text{Distance}] = ML^2T^{-2}.$$

The powers of M, L and T are called the dimensions of a quantity. From them it is possible to check or even to derive an equation, since each term must have the same dimensions. If the terms had different dimensions they would be affected differently by a change in the units of mass, length and time and the equation would not hold for all systems of units.

It is sometimes necessary, in order to make an equation dimensionally correct, to introduce a dimensional constant. Thus Newton found that force and mass are connected by two independent relations, $F = kma$, where F is the force which gives a mass m an acceleration a and k is a constant, and $F=k'\dfrac{m_1m_2}{r^2}$, where F is the force of attraction between masses m_1 and m_2 at distance r apart and k' is a constant. It was decided, quite arbitrarily, in the first equation to make k unity and to regard it as dimensionless, thereby defining the dimensions of force as MLT^{-2} (as above). Using these dimensions of force, the second equation can be made dimensionally correct only by regarding k' as having the dimensions $M^{-1}L^3T^{-2}$. The dimensions of the right-hand side of the equation are then

$$M^{-1}L^3T^{-2}\frac{M^2}{L^2}=MLT^{-2},$$

which are the same as the dimensions of force on the left-hand side of the equation. The constant k' is called the Gravitational Constant; it is usually denoted by G and changes in value according to the units of mass, length and time employed. Thus a dimensional constant, as its name implies, only remains constant for a particular system of units and must be adjusted if the system of units is changed.

Dimensions, then, enable us to see at a glance not only how a quantity has been defined in terms of the fundamental quantities mass, length and time, but how its numerical value is affected if the units of any of these fundamental quantities are changed; e.g. if a change is made from cm. gm. sec. to kilogramme metre second units the unit of force (dimensions MLT^{-2}) becomes equal to $10^3\times10^2=10^5$ dynes.

It is worth emphasising that the dimensions of physical quantities are arbitrary and conventional. It would be perfectly legitimate to consider force, length and time as the fundamental quantities (in which case the dimensions of mass would be $FL^{-1}T^2$) and to build up a complete system in place of the usual one based on mass, length and time.

Dimensions of electric charge

If the dimensions of any electrical quantity, say charge, are worked out in the electrostatic and electromagnetic systems it is found that they are different.

On the electrostatic system,

$$\text{Force}=\frac{q_1q_2}{r^2},$$

$$[q^2]=[\text{Force}]\times[\text{distance}]^2$$

$$=MLT^{-2}L^2.$$

$$\therefore\quad [q]=M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}.$$

On the electromagnetic system

$$\text{Force}=\frac{m_1 m_2}{r^2}.$$

$$\therefore\ [m]=M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}.$$

Magnetic field strength = Force per unit pole.

Magnetic field strength due to a current $=\dfrac{li}{r^2}$ or $\dfrac{i\,ds\sin\theta}{r^2}$ (see pp. 85, 95).

$$\begin{aligned}\therefore\ [i]&=\frac{[\text{Force}]}{[\text{Pole strength}]}\times\frac{[r]^2}{[l]}\\ &=\frac{MLT^{-2}}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}\,\frac{L^2}{L}\\ &=M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1},\\ [q]&=[i]\times[\text{time}]\\ &=M^{\frac{1}{2}}L^{\frac{1}{2}}.\end{aligned}$$

The quotient of the dimensions of charge on the two systems

$$\frac{[\text{Electrostatic charge}]}{[\text{Electromagnetic charge}]}=\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{M^{\frac{1}{2}}L^{\frac{1}{2}}}=LT^{-1}$$

is seen to have the dimensions of a velocity, which gives added significance to the value 3×10^{10} as the ratio of the units.

It is rather to be expected that the dimensions of charge on the two systems should differ since the systems start from quite different premises, just as an entirely different system of dimensions in mechanics would result if force were defined by means of the equation $F=\dfrac{m_1 m_2}{r^2}$ for the gravitational attraction of two masses, instead of by means of $F=ma$.

Dimensions

	Electromagnetic system	Electrostatic system	$\dfrac{\text{e.m.u.}}{\text{e.s.u.}}$ $(c=3\times10^{10})$
Current	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	c
Electric potential	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$1/c$
Resistance	LT^{-1}	$L^{-1}T$	$1/c^2$
Electric charge	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	c
Electric field	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}$	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$	$1/c$
Capacitance	$L^{-1}T^2$	L	c^2
Magnetic pole	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	$1/c$
Magnetic field	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}$	c
Magnetic induction	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{-\frac{3}{2}}$	$1/c$
Inductance	L	$L^{-1}T^2$	$1/c^2$

Dimensional requirements for a comprehensive system

In this book, therefore, two systems of units with entirely different dimensions have been used to deal with the phenomena of electrostatics on the one hand and of magnetism and the electric current on the other. If electromagnetic units of charge were used to measure static charges Coulomb's law would become $F=c^2\dfrac{q_1q_2}{r^2}$, where $c=3\times10^{10}$, since each electromagnetic unit of charge is equivalent to c electrostatic units. On substituting the dimensions of the electromagnetic unit of charge in this equation it is seen that c must be a dimensional constant with the dimensions of a velocity, LT^{-1}.

$$MLT^{-2}=[c^2]\frac{(M^{\frac{1}{2}}L^{\frac{1}{2}})^2}{L^2}.$$

$$\therefore\ [c^2]=L^2T^{-2},$$

$$[c]=LT^{-1}.$$

Thus if one system of units were to be used for the whole range of electrical and magnetic phenomena it would be necessary to introduce a dimensional constant. Let us therefore generalise the three fundamental equations for the forces *in vacuo* between two electric charges, between two magnetic poles, and between an electric current and a magnetic pole by inserting dimensional constants α, β, γ (one or two of which may be dimensionless) as follows:

$$F=\frac{q_1q_2}{\alpha r^2},\quad F=\frac{m_1m_2}{\beta r^2},\quad F=\frac{i\,ds\sin\theta}{\gamma r^2}\,m.$$

Since we are considering forces *in vacuo*, the dielectric constant k and magnetic permeability μ of a medium are not included in the equations; when it is necessary to insert them they will not affect dimensions since they are, by definition, pure numbers and dimensionless.

It can be shown as follows that $\dfrac{\gamma}{\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}}$ must have the dimensions of a velocity; in fact $\dfrac{\gamma}{\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}}=c$. From the first equation,

$$[q]=[F^{\frac{1}{2}}][\alpha^{\frac{1}{2}}]\,L,$$

But $i=q/t$, $\quad\therefore\ [i]=[F^{\frac{1}{2}}][\alpha^{\frac{1}{2}}]\,LT^{-1}.$

From the second equation,

$$[m]=[F^{\frac{1}{2}}][\beta^{\frac{1}{2}}]\,L.$$

Substituting the dimensions of i and m in the third equation (ds having dimensions L and $\sin\theta$ zero dimensions),

$$[F]=\frac{[F^{\frac{1}{2}}]\,[\alpha^{\frac{1}{2}}]\,LT^{-1}.L.[F^{\frac{1}{2}}]\,[\beta^{\frac{1}{2}}]\,L}{[\gamma]\,L^2}.$$

The right-hand side reduces to

$$[F]\,\frac{[\alpha^{\frac{1}{2}}]\,[\beta^{\frac{1}{2}}]}{[\gamma]}\,LT^{-1}.$$

Since this must have the same dimensions as the left-hand side, i.e. $[F]$, it follows that

$$\frac{[\gamma]}{[\alpha^{\frac{1}{2}}]\,[\beta^{\frac{1}{2}}]}=LT^{-1}.$$

In the electromagnetic system β and γ are equal to unity and dimensionless, and we have seen that $\alpha=\frac{1}{c^2}$, with the dimensions of (velocity)$^{-2}$, which confirms the equation $\frac{\gamma}{\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}}=c$. In the electrostatic system α and γ are equal to unity and dimensionless and $\beta=\frac{1}{c^2}$. There is a third c.g.s. system of electrical units due to Gauss, which may be mentioned for the sake of completeness, in which $\alpha=1$, $\beta=1$ and $\gamma=c$. Thus in the Gaussian system the units of charge and current are the same as in the electrostatic system, while the unit of magnetic pole strength is the same as in the electromagnetic system; hence a dimensional constant, $\gamma=c$, is necessary in the third fundamental equation.

System	Magnitude			Dimensions		
	α	β	γ	α	β	γ
Electrostatic	1	c^{-2}	1	0	$L^{-2}T^{2}$	0
Electromagnetic	c^{-2}	1	1	$L^{-2}T^{2}$	0	0
Gaussian	1	1	c	0	0	LT^{-1}

These three systems are merely the simplest possibilities; innumerable systems could be devised by giving suitable magnitudes and dimensions to α, β, γ subject to the relation $\frac{\gamma}{\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}}=c$.

Alternative treatment of dimensions of electrical quantities

There is another method of treating electrical dimensions which, although not followed in this book, is used in some others, and hence the student should be familiar with it.

The fundamental equations for a vacuum are written ·

$$F=\frac{q_1 q_2}{k_0 r^2},\quad F=\frac{m_1 m_2}{\mu_0 r^2},\quad F=\frac{ids\sin\theta}{Ar^2}m.$$

The quantity represented by k_0 is called the dielectric constant or permittivity of a vacuum and is regarded as having dimensions; μ_0 is called the permeability of a vacuum and is also regarded as having dimensions. The quantity A is taken as unity and regarded as dimensionless. Thus $\frac{1}{k_0^{\frac{1}{2}}\mu_0^{\frac{1}{2}}}=c$ and the product of the dimensions of k_0 and μ_0 is the same as that of the reciprocal of (velocity)². The individual dimensions of k_0 and μ_0 are not fixed separately, but their dimensions, written $[k]$ and $[\mu]$, are included in the dimensions of each electrical quantity. This is alternative to introducing a dimensional constant and dates from the time when the dimensions of a quantity were thought to be unique, rather than arbitrary, and to throw light on the physical nature of the quantity.

It results in different dimensions on the electromagnetic system for magnetic field strength, H, and magnetic induction, B, because $H=\frac{m}{\mu_0 r^2}$ and $B=\frac{m}{r^2}$; hence the equation connecting these quantities, in order to be dimensionally correct, must be written

$$B=\mu_0 H+4\pi J.$$

Other equations must similarly be rewritten; for example the force on a wire carrying a current *in vacuo* becomes $\mu_0 Hil$. The value of μ_0 on the electromagnetic system is unity, just as the value of k_0 on the electrostatic system is unity, but these quantities must be included in the appropriate equations because of their dimensions.

Metre-kilogramme-second (*m.k.s.*) *system*

Any arbitrary values of k_0, μ_0 and A can be selected and, so long as they satisfy the relation $\frac{A}{k_0^{\frac{1}{2}}\mu_0^{\frac{1}{2}}}=c$, a single system of units which can be used for both electrostatics and electromagnetics will result.

In 1901 Giorgi pointed out that if the metre, kilogramme and second are taken as fundamental units instead of the centimetre, gramme and second, if μ_0 is taken as 10^{-7} and A as 1, then the practical units, ampere, volt, ohm, etc., can be used in the system. Since the velocity of light, c, is 3×10^8 metres per sec.,

$$k_0=\frac{1}{9\times 10^{16}\times 10^{-7}}=\frac{1}{9\times 10^9}.$$

The advantage of the m.k.s. system is that it replaces three systems, the electrostatic, the electromagnetic and the practical. It was officially accepted in 1935 by the International Electrotechnical Commission and henceforward, presumably, will be used by electrical engineers although there is considerable opposition among physicists to its introduction.

The unit of force on the m.k.s. system is the newton which is that force which will give to a mass of 1 kg. an acceleration of 1 metre per sec. per sec. Since 1 kg. $=10^3$ g. and 1 metre $=10^2$ cm., 1 newton $=10^5$ dynes. The unit of work is the joule, the work done when a force of 1 newton acts through 1 metre. This is clearly correct since 1 newton $=$ 10^5 dynes, 1 metre $=10^2$ cm. and 1 joule $=10^7$ ergs.

To verify that, by taking $k_0=\dfrac{1}{9\times 10^9}$ and using m.k.s. units, the practical electrical units fit the formulae, consider the force between two charges, each of 1 coulomb at a distance of 1 metre apart. Using first the c.g.s. electrostatic system,

$$\begin{aligned}
F&=\frac{q_1q_2}{r^2} \\
&=\frac{3\times 10^9\times 3\times 10^9}{10^4}\text{ dynes} && \begin{array}{l}\text{1 coulomb}=3\times 10^9\text{ e.s.u.}\\ \text{1 metre}=10^2\text{ cm.}\end{array} \\
&=\frac{9\times 10^{18}}{10^4\times 10^5}\text{ newtons} && \text{1 newton}=10^5\text{ dynes} \\
&=9\times 10^9\text{ newtons.}
\end{aligned}$$

Using the m.k.s. system, $\quad F=9\times 10^9\,\dfrac{q_1q_2}{r^2}$.

Here $q_1=q_2=1$ m.k.s. unit, and $r=1$ m.k.s. unit.

$$\therefore\ F=9\times 10^9\text{ newtons.}$$

To make possible a development of the subject starting from current and potential difference rather than from the magnetic pole, and also to clarify dimensions, it has been suggested that a fourth absolute unit, additional to those of mass, length and time should be chosen. Giorgi himself favoured the ohm since an international standard of resistance could be made easily and kept with the standard kilogramme and the standard metre. The ampere could then be defined by Joule's law from the power dissipated in watts in the standard ohm. Power equals i^2R and, since the dimensions of power are $ML^{-2}T^{-3}$,

$$\begin{aligned}
[i]^2&=\frac{ML^{-2}T^{-3}}{R}, \\
[i]&=M^{\frac{1}{2}}L^{-1}T^{-\frac{3}{2}}R^{-\frac{1}{2}}.
\end{aligned}$$

The volt is then defined from Ohm's law, $V=iR$, so that

$$[V]=M^{\frac{1}{2}}L^{-1}T^{-\frac{3}{2}}R^{\frac{1}{2}}.$$

A four-dimensional system obviates the necessity for using the unspecified dimensions $[k]$ and $[\mu]$.

Magnetic units would be derived via electromagnetic induction; for example, pole strength would be obtained by considering the quantity of electricity induced when a pole is plunged into a coil. This avoids basing the whole of electrical theory on the somewhat artificial concept of an isolated magnetic pole. But, as Sir Richard Glazebrook has pointed out, both an ammeter and a voltmeter contain permanent magnetic poles.

The International Electrotechnical Commission, however, did not adopt the proposal of a fourth fundamental unit but decided to take $\mu_0=10^{-7}$ as fixing the m.k.s. system.

A further proposal that μ_0 should be taken as $4\pi\times10^{-7}$, giving the so-called rationalised system, finds considerable support. In this system 4π is transferred from some expressions which are commonly used by the practising engineer—for example, the field inside a solenoid becomes ni, the ampere turns, instead of $4\pi ni$—to expressions less commonly used such as the force between two poles, $F=\dfrac{m_1m_2}{4\pi\mu_0r^2}$.

Method of dimensions

The following example illustrates an application of the theory of dimensions.

Example

Given that the electrical power, W, expended in a circuit depends on the E.M.F., E, and the resistance, R, in the circuit, find by the method of dimensions the relation between these quantities.

Let $\quad W=kE^xR^y$, where k is a constant.

Substituting dimensions on the electromagnetic system

$$ML^2T^{-3}=(M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2})^x(LT^{-1})^y.$$

Equating the dimensions of M, L and T on each side,

$$1=\tfrac{1}{2}x. \qquad \therefore\ x=2.$$

$$2=\tfrac{3}{2}x+y. \qquad \therefore\ y=-1.$$

$$-3=-2x-y.$$

The same answer is obtained using dimensions on the electrostatic system.

Thus

$$W=k\frac{E^2}{R}.$$

The method of dimensions does not enable the value of a constant such as k to be determined.

QUESTIONS

1. Describe the tangent galvanometer, and describe briefly how you would use it to calibrate an ammeter, a voltmeter and a standard resistance in absolute units. (The mechanical equivalent of heat $=4{\cdot}2\times10^7$ ergs/calorie.) (O. S.)

2. Define in their correct order the units in the electromagnetic system up to and including the unit of capacity. You may assume that the ordinary mechanical units up to and including the unit of work are known. (C. S.)

3. Describe a method for the determining of current in absolute electromagnetic units. (C. S.)

4. Give an account of the electrostatic and electromagnetic systems of units. How would you determine experimentally the ratio of the units of potential in the two systems? (O. S.)

5. Write down in logical sequence the definitions, and work out the dimensions, of the units of quantities up to and including current, P.D. and resistance (*a*) on the electromagnetic, (*b*) on the electrostatic system.

6. Assuming that $T\propto I^xM^yH^z$, where T is the periodic time of a magnet, of moment of inertia I and magnetic moment M, in a magnetic field of strength H, find by the method of dimensions the value of x, y, z.

7. The energy of a charged condenser is proportional to the capacitance, C, and the potential difference between the plates V, each raised to some power, i.e. Energy $\propto C^xV^y$. Find x and y by the method of dimensions.

8. Inductance is the E.M.F. induced in a circuit divided by the rate of change of current with time. Find its dimensions.

The periodic time T, of electrical oscillations in a circuit containing inductance L, and capacitance C, is given by $T\propto L^xC^y$. Using the method of dimensions find x and y.

9. By means of the method of dimensions determine a, b, c and g in the following expression for the eddy current loss of energy W per centimetre length of round wire:

$$W\propto f^aB^bd^c\rho^g,$$

where $f=$frequency, $B=$flux density, $d=$diameter of wire, $\rho=$specific resistance of the material.

10. Assuming that

$$F=9\times10^9\,\frac{q_1q_2}{r^2}\quad\text{and}\quad F=10^7\,\frac{m_1m_2}{r^2},$$

where all the quantities are in m.k.s. units, define the units of the following on the m.k.s. system and work out their numerical relation to the corresponding units on the c.g.s. system: charge, electric field strength, electric potential, magnetic pole strength, magnetic field strength, magnetic induction (B). Show that there are two ways of defining the unit of electric current and that they give the same unit.

CHAPTER 10

ELECTROLYSIS

MOST liquids which conduct electricity are split up chemically by the current. The phenomenon is known as electrolysis and the liquids are called electrolytes. Typical electrolytes are solutions of acids, bases and salts; examples of liquid conductors which are not electrolytes are molten metals, mercury being the most common example.

The electrode by which the current enters the electrolyte is called the anode and that by which it leaves the cathode. A simple example of electrolysis is the passage of an electric current through a solution of hydrochloric acid; hydrogen is liberated at the cathode and chlorine at the anode:

$$2HCl \rightarrow \underset{\text{cathode}}{H_2} + \underset{\text{anode}}{Cl_2}.$$

It is found that hydrogen in acids and the metals in salts and bases are always liberated at the cathode; oxygen and the non-metals are liberated at the anode.

Secondary reactions

When an electric current is passed through an aqueous solution of copper sulphate, using copper electrodes, it is found that copper is deposited on the cathode and the anode is eaten away. It can be assumed that the copper sulphate dissociates as follows:

$$CuSO_4 \rightarrow \underset{\text{cathode}}{Cu} + \underset{\text{anode}}{SO_4}.$$

The SO_4 then attacks the anode in what is called a secondary reaction:

$$Cu + SO_4 = CuSO_4.$$

Using platinum instead of copper electrodes, copper is deposited on the cathode and oxygen is liberated at the anode; the SO_4 does not attack the platinum anode and the secondary reaction is

$$2SO_4 + 2H_2O = 2H_2SO_4 + O_2.$$

The idea of secondary reactions has now been replaced by the theory of discharge potentials which will be described in the next chapter. But, for simple electrolytic phenomena, secondary reactions provide a satisfactory working hypothesis and do not put such a tax on the memory as discharge potentials.

Faraday's laws of electrolysis

The quantitative investigation of electrolysis was performed by Faraday who summarised his results in two laws:

(1) *The mass of a substance liberated in electrolysis is proportional to the quantity of electricity passed, i.e. to the product of current and time.*

(2) *When the same quantity of electricity is passed through different electrolytes the masses of substances liberated are in the ratio of their equivalent weights.*

The quantity of electricity which liberates 1 gram-equivalent of all substances, e.g. 1·008 g. of hydrogen, 8 g. of oxygen, 31·8 g. of divalent copper, and 108 g. of silver, is 96,500 coulombs and is called the *Faraday.*

The electrochemical equivalent

The electrochemical equivalent of a substance is the mass liberated in electrolysis by 1 coulomb.

This is a quantity which can be determined accurately by experiment and from which the Faraday may be calculated:

$$\text{Faraday}=\frac{\text{Equivalent weight of substance}}{\text{Electrochemical equivalent of substance}}.$$

Accurate determination of the Faraday

The Faraday has been determined most accurately by means of the silver and iodine voltameters. A voltameter is an instrument for measuring the electrochemical equivalent of a substance or, alternatively, if the electrochemical equivalent is known, of measuring current.

A standard type of silver voltameter is shown in Fig. 168. A measured current is passed through a 15–20% solution of silver nitrate for a known time and the platinum cathode is weighed before and after the current is passed. The porous cup prevents particles from falling from the silver anode on to the cathode. The electrolysis may be represented by the equation

$$AgNO_3 \rightarrow \underset{\text{cathode}}{Ag}+\underset{\text{anode}}{NO_3}.$$

The anode is attacked by the NO_3

$$Ag+NO_3=AgNO_3,$$

so that the concentration of the silver nitrate remains unchanged.

An account of a very accurate determination of the electrochemical equivalent of silver, in which the current was measured by a Rayleigh current balance, was described on p. 170.

In the iodine voltameter a solution of potassium iodide is electrolysed:

$$\mathrm{KI} \rightarrow \underset{\text{cathode}}{\mathrm{K}} + \underset{\text{anode}}{\mathrm{I}}.$$

The iodine liberated at the anode can be measured to an accuracy of 5 or 6 parts per 1,000,000 by titrating with arsenious acid.

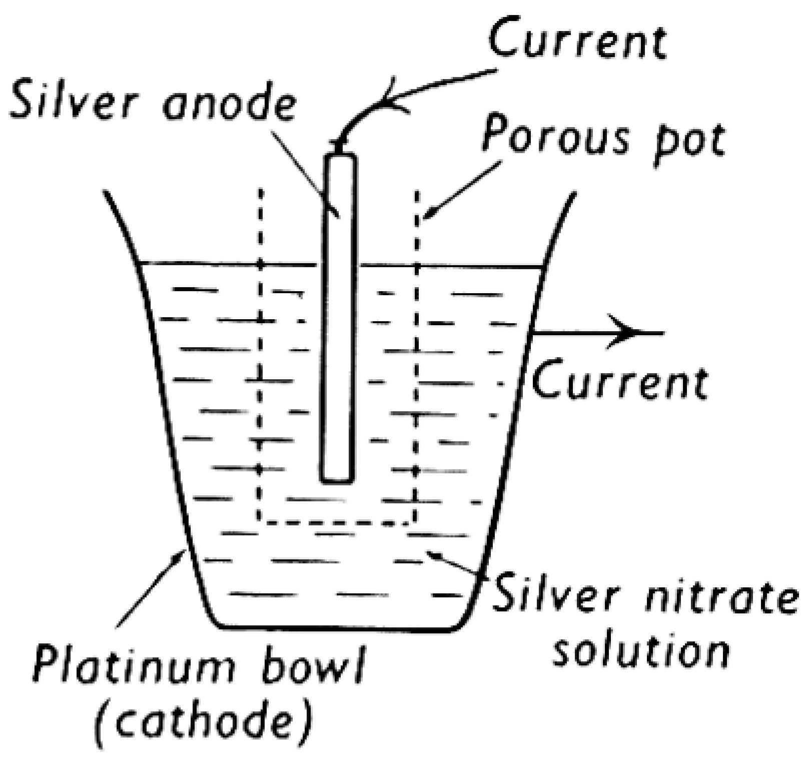


Fig. 168

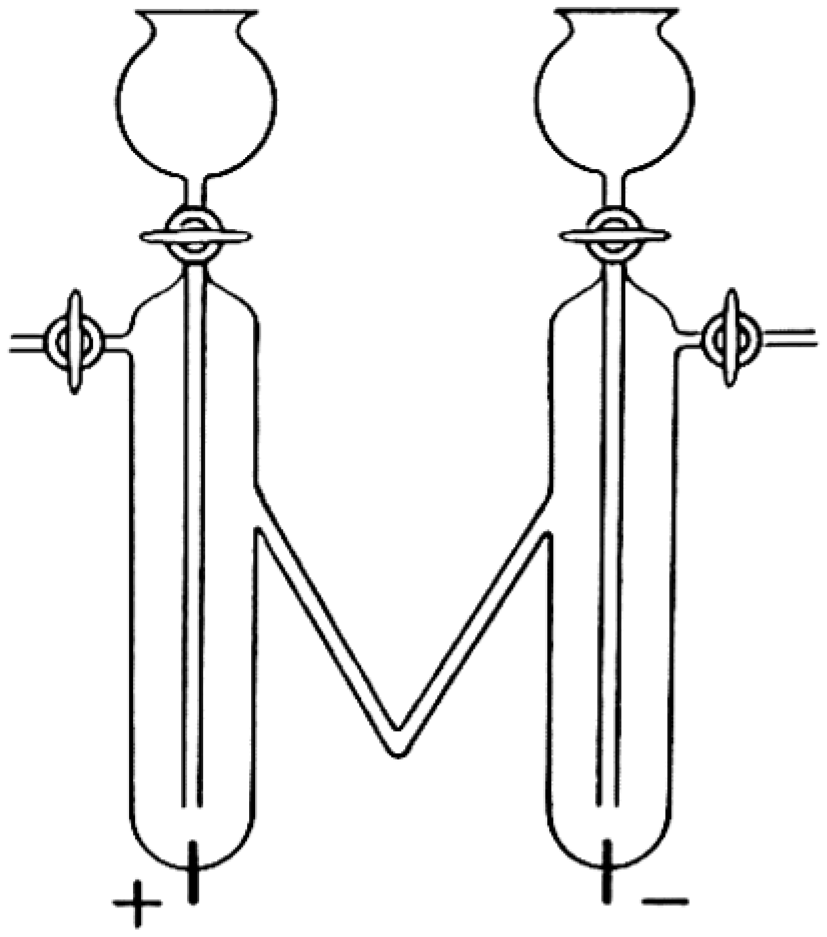


Fig. 169

If the potassium iodide solution round the cathode contains dissolved iodine, the potassium liberated in the electrolysis combines with the iodine, and the decrease in the iodine concentration may also be measured by titration. This gives a check on the other determination.

The voltameter is shown in Fig. 169. Two upright limbs connected by a V tube have electrodes of platinum-iridium wire sealed through their lower ends (pure platinum is attacked by iodine). The electrolyte is introduced through the vertical capillary tubes and removed after electrolysis by gentle suction from each limb through the capillary and side tubes into flasks for titration.

In 1914, Vinal and Bates put a silver and an iodine voltameter in series. They found that the values of the Faraday given by the two voltameters differed by more than 4·5 times the probable experimental error. This has given rise to considerable uncertainty and controversy.

R. T. Birge, who has devoted many years to a critical examination of the determinations of the physical constants, recommended in 1929 that the Faraday should be based solely on the silver voltameter. But it has been found that there are inclusions in the silver deposit; prolonged investigations have shown that the filter paper used during the preliminary purification of the silver nitrate solution may affect the weight of the deposit by as much as 1 part in 10,000. It has also been suggested that there may be a preferential deposit of the two isotopes* of silver. Birge, therefore, in 1941 recommended a weighted average of the values obtained from the silver and iodine voltameters and gave as the most probable value

$$F = 96{,}487 \pm 10 \text{ absolute coulombs.}^\dagger$$

THEORIES OF ELECTROLYSIS

A theory of electrolysis must account for the way in which an electrolyte carries a current and how the electrolyte is decomposed.

It was very early suggested that the molecule of an electrolyte must consist of positively and negatively charged parts and that the positive part is attracted to the cathode and the negative part to the anode.

When it was discovered that Ohm's law holds for electrolytes and that, if polarisation is avoided, the most minute E.M.F. will cause some current to flow, it was realised that part of the E.M.F. could not be employed in decomposing the electrolyte. Clausius, in 1857, suggested that some of the electrolyte might be split up in solution into positively and negatively charged particles before an E.M.F. was applied. Faraday gave the name *ions* to these charged particles.

The theory of Arrhenius or ionic theory

In 1887 Arrhenius put forward his theory of electrolytic dissociation. He assumed that when an electrolyte enters solution part of it is dissociated into ions. Thus when sodium chloride, NaCl, is dissolved in water some of the molecules dissociate into positively charged sodium ions, Na^+, and negatively charged chlorine ions, Cl^-. Ions are continually recombining and molecules dissociating, there being a dynamic equilibrium represented by the equation

$$NaCl \rightleftharpoons Na^+ + Cl^-.$$

The theory met with violent opposition at first. Sodium and chlorine are extremely active chemically, and their separate existence in a solution was incredible to many chemists; it was not realised that ions have very different chemical properties from the uncharged atoms. We can now understand clearly how the sodium and chlorine ions gain their charges and why their chemical properties differ from those of the corresponding uncharged atoms. A sodium atom has a nucleus, with a positive charge

---

[*] See p. 317.

[†] This is based on the chemical scale of atomic weights, taking the ordinary mixture of oxygen isotopes O = 16·0000. There is also a slightly different scale known as the physical scale, taking one isotope, ^{16}O, as of atomic weight 16·0000.

of 11, and 11 electrons round the nucleus. It will readily part with one electron, leaving it with a resultant positive charge and with the very stable arrangement of 10 electrons possessed by its predecessor in the periodic table, the inert gas neon. Similarly, a chlorine atom readily gains an electron, obtaining a negative charge and increasing its 17 electrons to the stable configuration of 18 electrons, possessed by the inert gas argon.

When an E.M.F. is applied between electrodes dipping into an electrolyte the positive ions, or cations, are attracted to the cathode while the negative ions, or anions, are attracted to the anode. The two streams of oppositely charged ions, travelling in opposite directions, carry the current through the electrolyte. Anions give up their surplus electrons to the anode, and cations receive electrons from the cathode, thus maintaining the flow of electrons in the external circuit. Having given up their charges the ions are liberated as uncharged atoms and molecules.

The charge on the ion

If e is the charge on the hydrogen ion and N is the number of ions in 1 g. of hydrogen,

$$Ne = 96{,}500,$$

since 1 g. of hydrogen is liberated by 96,500 coulombs. N is known as Avogadro's number and may be determined from the kinetic theory of gases; its value is $6{\cdot}06 \times 10^{23}$.

$$\therefore\ e = \frac{96{,}500}{6{\cdot}06 \times 10^{23}}$$

$$= 1{\cdot}59 \times 10^{-19} \text{ coulomb.}$$

This agrees with the value for the charge of the electron determined by other methods which will be described in Chapter 17.

In the case of a divalent substance such as copper, a gramme-atomic weight contains N atoms, but a gramme-equivalent weight contains only $\frac{1}{2}N$ atoms. Since 96,500 coulombs discharges $\frac{1}{2}N$ ions of a divalent substance, it must be assumed that each ion carries a charge $2e$. Thus

$$CuSO_4 \rightarrow Cu^{++} + SO_4^{--}.$$

The ions of any substance have a surplus or deficiency of electrons equal to their valency.

In this way Faraday's second law can be accounted for on the ionic theory.

Evidence for dissociation

There is strong evidence of the dissociation of electrolytes in solution quite apart from electrolysis. The particles of a solute may be considered as distributed in a solution rather like the molecules of a gas. They set up a pressure, called the osmotic pressure, which can be measured in some cases with the help of a membrane, which is permeable only to the solvent molecules. For very dilute solutions the osmotic pressure is proportional to the concentration and to the absolute temperature. Van't Hoff showed theoretically that the gas laws can be applied to the solute in very dilute solutions. He found that the gas equation $pv=RT$ holds for non-electrolytes such as cane sugar. But the osmotic pressure of electrolytes is much greater than would be expected from a consideration of the number of molecules; in the case of sodium chloride it is nearly twice as great. This can be explained by assuming that each molecule of sodium chloride has dissociated into two ions, and hence there are twice as many particles contributing to the osmotic pressure. Similarly, the depression of the freezing-point and elevation of the boiling-point of solutions of electrolytes is abnormally large owing to dissociation.

Again, solutions of electrolytes exhibit the characteristics of their individual ions. All acids have a sour taste which is ascribed to the hydrogen ion, all cupric salts are blue due to the cupric ion, all dichromates are yellow due to the dichromate ion, and so on.

Degree of dissociation

The electrical conductivity of a solution containing a given mass of an electrolyte increases with increasing dilution and eventually approaches a maximum value. Arrhenius explained this by saying that on dilution the electrolyte becomes more dissociated; at infinite dilution it is completely dissociated. The ratio of the equivalent conductivity* of a solution of an electrolyte to its equivalent conductivity at infinite dilution thus gives the fraction of the electrolyte dissociated. This is known as the degree of dissociation and is represented by α. Equivalent conductivity is usually represented by the Greek letter, capital lambda, Λ.

Thus

$$\alpha=\frac{\Lambda}{\Lambda_\infty}.$$

---

* The exact definition of this term is given on p. 192.

Ostwald's dilution law

If, then, as Arrhenius assumed, the degree of dissociation depends on the concentration (or dilution), there must be, at every concentration, an equilibrium between the dissociation of molecules into ions and the recombination of ions to form molecules, represented by an equation such as

$$AB \rightleftharpoons A^+ + B^-.$$

Ostwald proceeded to apply the law of mass action to this process in the same way that it is applied to chemical actions. The law states that the velocity of a chemical action is proportional to the product of the 'active masses' of the reacting substances.

The active masses of the ions and molecules of an electrolyte may be represented by their concentrations. Thus

Velocity of dissociation $= k_1$ (concentration of undissociated molecules),

Velocity of recombination $= k_2$ (concentration of anion) (concentration of cation),

k_1 and k_2 being constants.

For equilibrium the velocity of dissociation must equal the velocity of recombination.

Hence

$k_1 \times$ concentration of undissociated molecules $= k_2$ (concentration of anion) (concentration of cation).

$$\therefore \frac{\text{(concentration of anion) (concentration of cation)}}{\text{concentration of undissociated molecules}} = \frac{k_1}{k_2}.$$

Suppose that 1 g.-equivalent of the electrolyte is dissolved in a volume V of solution and that a fraction α is dissociated. The concentration of each of the ions is α/V and that of the molecules is $(1-\alpha)/V$.

Substituting in the above equation and writing k for k_1/k_2,

$$\frac{(\alpha/V)^2}{(1-\alpha)/V} = \frac{\alpha^2}{(1-\alpha)V} = k.$$

By finding α from the ratio of equivalent conductivities, the value of k, called the dissociation constant, can be determined.

The test of the theory is, of course, the constancy of k. For some electrolytes k is a true constant, but for others it is not constant at all. As a result, electrolytes have been divided into two main classes: weak electrolytes, such as organic acids and bases, for which the theory holds, and strong electrolytes, such as inorganic acids, bases and salts, for which the theory fails. Since the theory works well for weak electrolytes it was long felt that it should be possible to adjust it to fit strong electrolytes, which were regarded as anomalous. This has not happened however; a new theory has been developed in which strong electrolytes are regarded as normal and weak electrolytes as anomalous.

The theory of Debye and Hückel

It is now known, as a result of the investigation of crystal structure with X-rays, that the crystals of many compounds are completely ionised in the solid state. Fig. 310 on page 324 represents a crystal of rock salt, NaCl; an electron has been transferred from each of the sodium atoms to a chlorine atom, and the cubic lattice is composed solely of sodium and chlorine ions, held in position by their electrostatic attraction.

The dielectric constant of water is very high, being about 80. This explains why rock-salt dissolves readily in water; the sodium and chlorine ions become easily detached from their positions in the crystal lattice, since the electrostatic force between them in water is reduced to $\frac{1}{80}$th of its ordinary value. In organic solvents of low dielectric constant the rock-salt is insoluble.

Debye and Hückel put forward in 1923 the theory that substances such as rock-salt, being completely ionised in the solid state, are also completely ionised in solution at all concentrations. How, then, can the marked change in conductivity with dilution be explained?

Arrhenius had assumed that an increase in conductivity is caused solely by an increase in the number of ions present in the solution; he believed that the mobilities of the ions do not vary as a solution is diluted. Debye and Hückel, on the other hand, started by assuming that the number of ions of strong electrolytes is constant at all concentrations and accounted for the change in conductivity on dilution by a change in the mobility of the ions.

An ion in solution is surrounded by other ions. Round any positive ion there must be a slightly negative 'atmosphere', since it attracts negative ions and repels positive ones. When an E.M.F. is applied the positive ion tends to go in one direction and the negative atmosphere in the opposite direction, thus causing a drag on the positive ion. The magnitude of the drag depends on the dilution, since the latter determines the closeness of the ions.

Ions may attract solvent molecules, and the resulting increase in size of the ions will reduce their mobility.

In concentrated solutions oppositely charged ions will be sufficiently close to attract each other and form ion pairs or undissociated molecules; on the Debye–Hückel theory, however, such a condition is anomalous, whereas on the theory of Arrhenius it is normal.

The Debye–Hückel theory was developed mathematically, but it can be used to justify the empirical law (i.e. a law based solely on experimental observations), discovered by Kohlrausch,

$$\Lambda_\infty - \Lambda_c = k\sqrt{c},$$

where Λ_∞ and Λ_c are the equivalent conductivities of strong electrolytes at infinite dilution and at concentration c respectively, k being a constant.

Owing to the complicated and mathematical nature of the Debye-Hückel theory, the simpler Arrhenius theory has survived side by side with it. The latter is still used when dealing with weak electrolytes, and the former is used for strong electrolytes.

Conductivity of electrolytes

Much of the evidence on which the foregoing theories of electrolysis is based consists of the change in conductivity of electrolytes with dilution. A brief account of the experimental determination of the conductivity of electrolytes will now be given.

Conductivity or conductance is the reciprocal of resistance, i.e. conductivity = current/voltage. Specific conductivity is the reciprocal of specific resistance, i.e. the value of current/voltage between two parallel electrodes of area 1 sq.cm. and 1 cm. apart. We need, however, another term to deal with the change in conductivity of a fixed mass of solute when the solution is diluted. The *equivalent conductivity* is the value of current/voltage between two parallel electrodes, 1 cm. apart, of sufficient area to contain between their surfaces the volume of solution which contains 1 g.-equivalent of the solute. If 1 g.-equivalent is dissolved in V c.c. the area of the plates must be V sq.cm. Thus the equivalent conductivity is V times the specific conductivity.

The fundamental experimental determination is that of specific resistance (the reciprocal of the specific conductivity) by means of a Wheatstone Bridge. In the earliest attempts to find the resistance of a solution the results were so erratic that it was thought that Ohm's law could not apply. The disturbing factor was found to be the back E.M.F. due to polarisation (see p. 209), and once this was eliminated the results were consistent.

Between 1869 and 1880 Kohlrausch carried out a classic series of experiments on the conductivity of electrolytes. He eliminated polarisation by using alternating current, with a frequency of about 1000 cycles per second, provided by an induction coil. The products of electrolysis responsible for polarisation are produced in one half-cycle of the A.C. and removed in the next. A valve oscillator is now used in preference to an induction coil, since the A.C. produced by the latter is asymmetrical. Kohlrausch used platinum electrodes coated with finely divided platinum; these also reduce polarisation because of their large area and because they act as a catalyst in the union of hydrogen and oxygen liberated by the current.

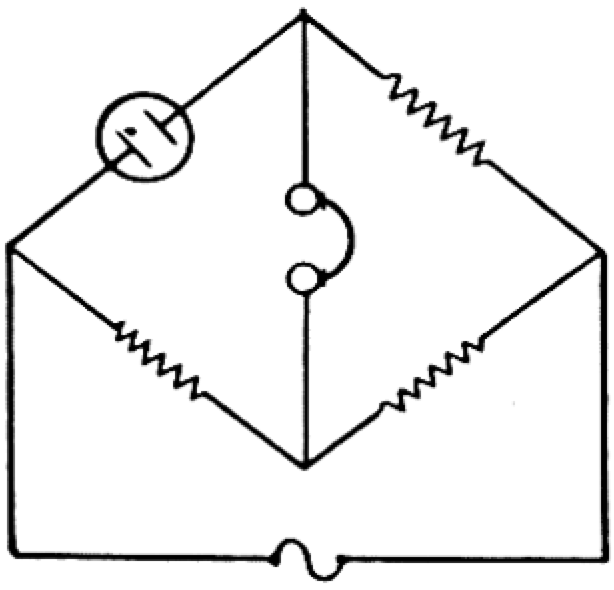


Fig. 170

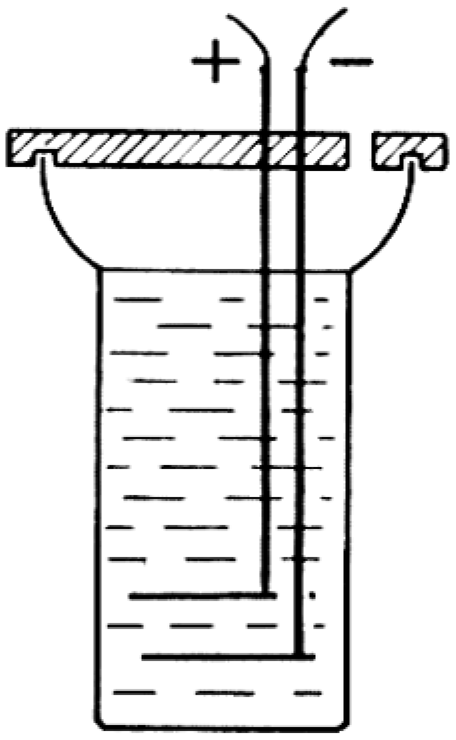


Fig. 171

Fig. 170 represents the Wheatstone Bridge arrangement used; the usual battery is replaced by a source of A.C. and the galvanometer by a telephone. When the bridge is balanced the sound in the telephone is a minimum.

Fig. 171 shows a special conductance cell to contain the electrolyte. Circular, horizontal platinum electrodes are fixed firmly in position and attached to platinum wires sealed through glass. The hole in the cover enables dilution to be effected by means of a pipette.

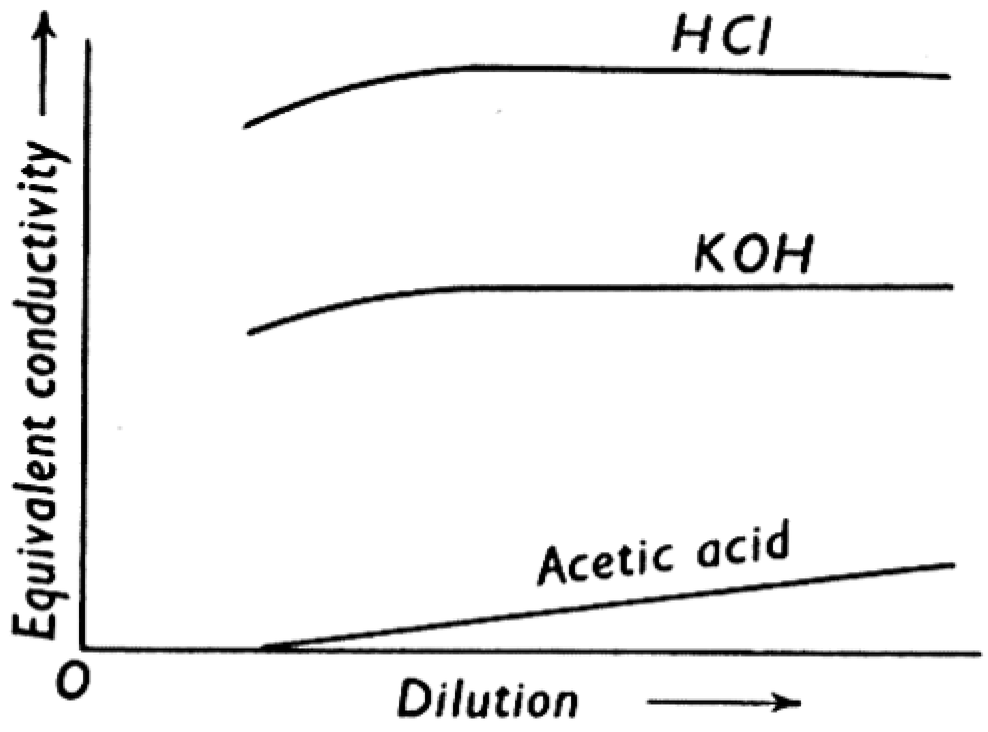


Fig. 172

Fig. 172 shows how the equivalent conductivity of strong electrolytes tends towards a maximum with increasing dilution. The weak electrolyte acetic acid, however, shows no sign of reaching its maximum. Its equivalent conductivity at infinite dilution is calculated from the sum of the equivalent conductivities of the hydrogen and acetate ions; these are obtained from measurements with a strong acid and sodium acetate respectively.

The calculation is based on the fact, established by Kohlrausch, that the equivalent conductivity at infinite dilution is the sum of two independent parts, one due to the anion and the other to the cation. The evidence can be seen clearly from the following equivalent conductivities at infinite dilution, at 18° C., measured in mhos or reciprocal ohms:

				Difference
Potassium chloride	130·0	Sodium chloride	108·9	21·1
Potassium nitrate	126·3	Sodium nitrate	105·2	21·1

The differences for corresponding potassium and sodium salts are the same, 21·1, and must be the difference between the contributions of potassium and sodium. Similarly, the differences between the two potassium and the two sodium salts are both 3·7, representing the difference between the contributions of the chloride and the nitrate ions.

The mobilities of the ions

Changes in the concentration of the electrolyte near the two electrodes, after electrolysis, may be detected and measured. Hittorf in 1853 showed that these changes could be explained by the different velocities of the ions. The faster moving ion causes a greater diminution in concentration round the electrode from which it moves.

Fig. 173 represents diagrammatically how the changes in concentration occur. In the top line, before electrolysis begins, there are equal numbers of positive and of negative ions in all three compartments. Suppose that the positive ion or cation has a velocity 2 and the negative ion or anion a velocity 3. The second line represents the situation after electrolysis has occurred. The positive ions have moved two spaces to the right and the negative ions three spaces to the left. There are five positive ions and five negative ions which are unpaired and which are discharged at the electrodes. The decreases in concentration of the electrolyte in the anode and cathode compartments, however, represented by the reductions in the number of paired ions, are in the ratio 2:3. Let the velocities of the cation and anion be represented by u and v respectively (instead of 2 and 3):

$$\frac{\text{Decrease of concentration in anode compartment}}{\text{Decrease of concentration in cathode compartment}}=\frac{u}{v}.$$

This ratio may be measured with the apparatus in Fig. 174. Suppose the vessel contains a solution of silver nitrate and platinum electrodes. After passing a current for some time the solutions in the anode and cathode compartments are titrated and their decrease in silver content found. Hence the ratio of the velocities of the nitrate and silver ions may be determined.

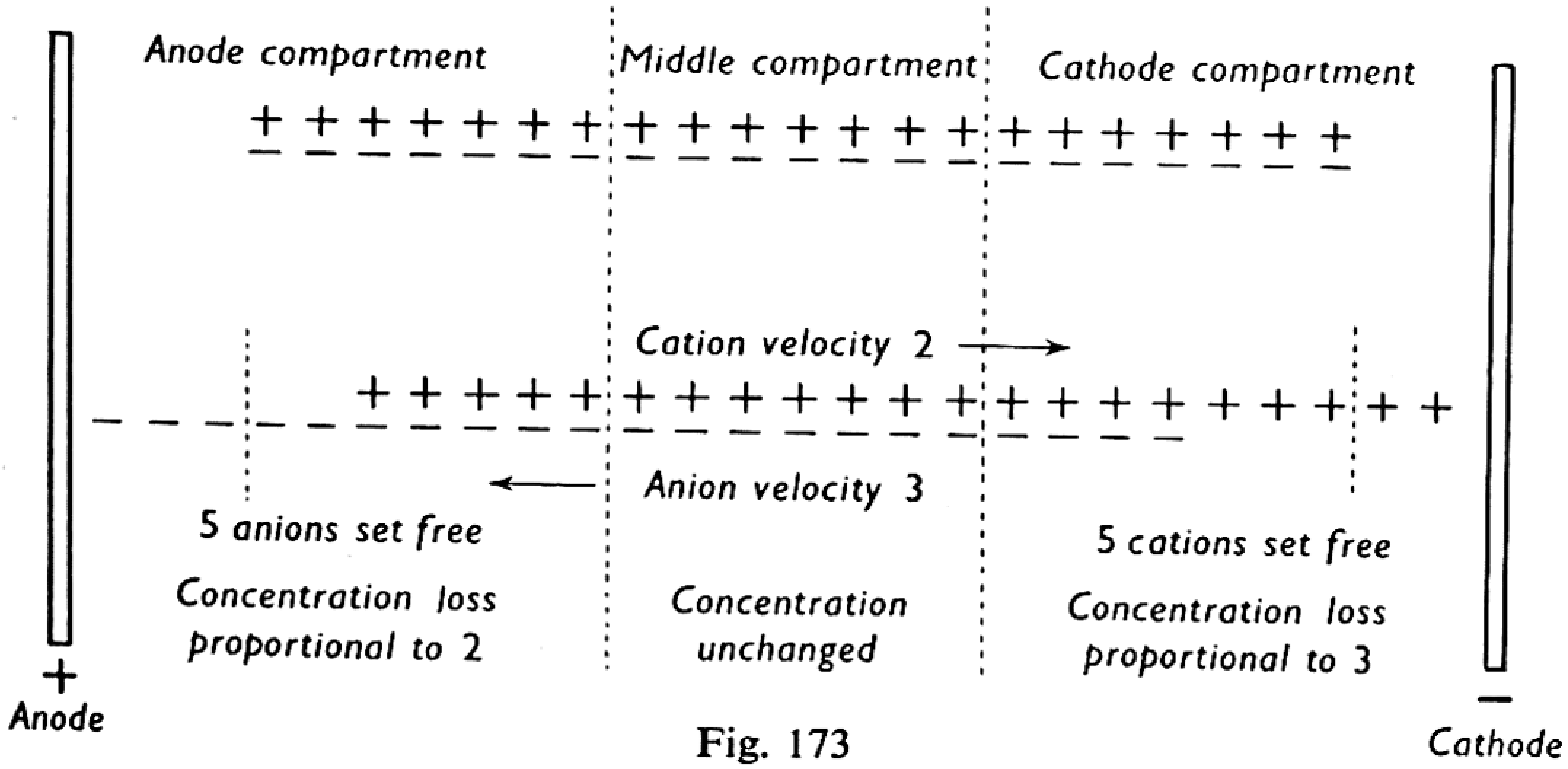


Fig. 173

It is possible to find the sum of the velocities of the ions from the equivalent conductivity at infinite dilution, Λ_∞, since

$$\Lambda_\infty = F(u+v), \quad \text{where } F = \text{the Faraday, 96,500 coulombs.}$$

In this expression u and v represent the velocities of the ions under a field of unit potential gradient, 1 volt per cm., and are called the *mobilities* of the ions.

To prove the expression it must be remembered that the equivalent conductivity is current/voltage between two plates, 1 cm. apart, of sufficient area to contain 1 g.-equivalent of solute between them. Suppose there are N molecules in 1 g.-equivalent. At infinite dilution all these molecules dissociate and hence there are N ions of each kind. Suppose the ions are monovalent with a charge e. When the P.D. between the plates is 1 volt, the charge carried per second, i.e. the current, is $Ne(u+v)$. This is, by definition, Λ_∞. But

$$Ne = F.$$

$$\therefore \Lambda_\infty = F(u+v).$$

Once the values of u/v and $u+v$ have been determined, u and v can easily be found. The table below gives the mobilities of various ions at 18° C. in cm. per sec. for a potential gradient of 1 volt per cm.

Ion	Mobility	Ion	Mobility
Hydrogen	0·00330	Hydroxyl (OH)	0·00180
Sodium	0·000452	Chlorine	0·000678
Potassium	0·00067	Iodine	0·000688
Copper	0·000245	NO_3	0·00064
Silver	0·00056	SO_4	0·000355

It is not always possible to measure the decrease in concentration in the anode compartment. If, however, the decrease in concentration at the cathode can be found and also the mass deposited on the cathode, it is still possible to calculate u/v.

In Fig. 173, $2+3=5$ ions are discharged at each electrode. Thus the mass liberated and also the quantity of electricity passed are proportional to $u+v$.

$$\therefore \frac{\text{Quantity of electricity carried by anions}}{\text{Total quantity of electricity passed}}=\frac{v}{u+v}.$$

Hittorf called this the transport number of the anion, and it is given the symbol n_a. Similarly for the cation, $n_c=u/(u+v)$.

In the experiment with silver nitrate in Fig. 174, n_a can be found from

$$n_a=\frac{\text{Loss of mass of silver in cathode compartment}}{\text{Mass of silver deposited on the cathode}}.$$

Since $\dfrac{u}{u+v}=n_a$, therefore $\dfrac{u}{v}=\dfrac{n_a}{1-n_a}$.

Direct determination of the mobilities of the ions

It is possible to make a direct determination of the mobility of some ions by timing the motion of a colour boundary. Fig. 175 represents a method, due to Whetham, for finding the mobility of the Cr_2O_7 ions which cause an orange coloration. The lower part of the U tube is filled with the orange-coloured solution of potassium dichromate, $K_2Cr_2O_7$, and the upper parts of each limb with colourless potassium carbonate solution. When a P.D. is maintained between the electrodes the Cr_2O_7 ion moves towards the anode; hence the colour boundary at P moves up and that at Q moves down. To prevent diffusion of the liquids the potassium dichromate solution is mixed with a little agar-agar jelly, which has no effect on the mobility of the ions.

Another example of the use of the method is the determination of the mobility of the hydrogen ion. The lower part of the U tube is filled with a dilute alkali, impregnated with a few drops of phenolphthalein to give it a red colour. In the upper half of each limb there is an acid, the hydrogen ions of which decolorise the phenolphthalein. When the current flows, the hydrogen ions move to the cathode and hence the boundary at *P* moves down.

The results obtained are in good agreement with those given by Hittorf's method.

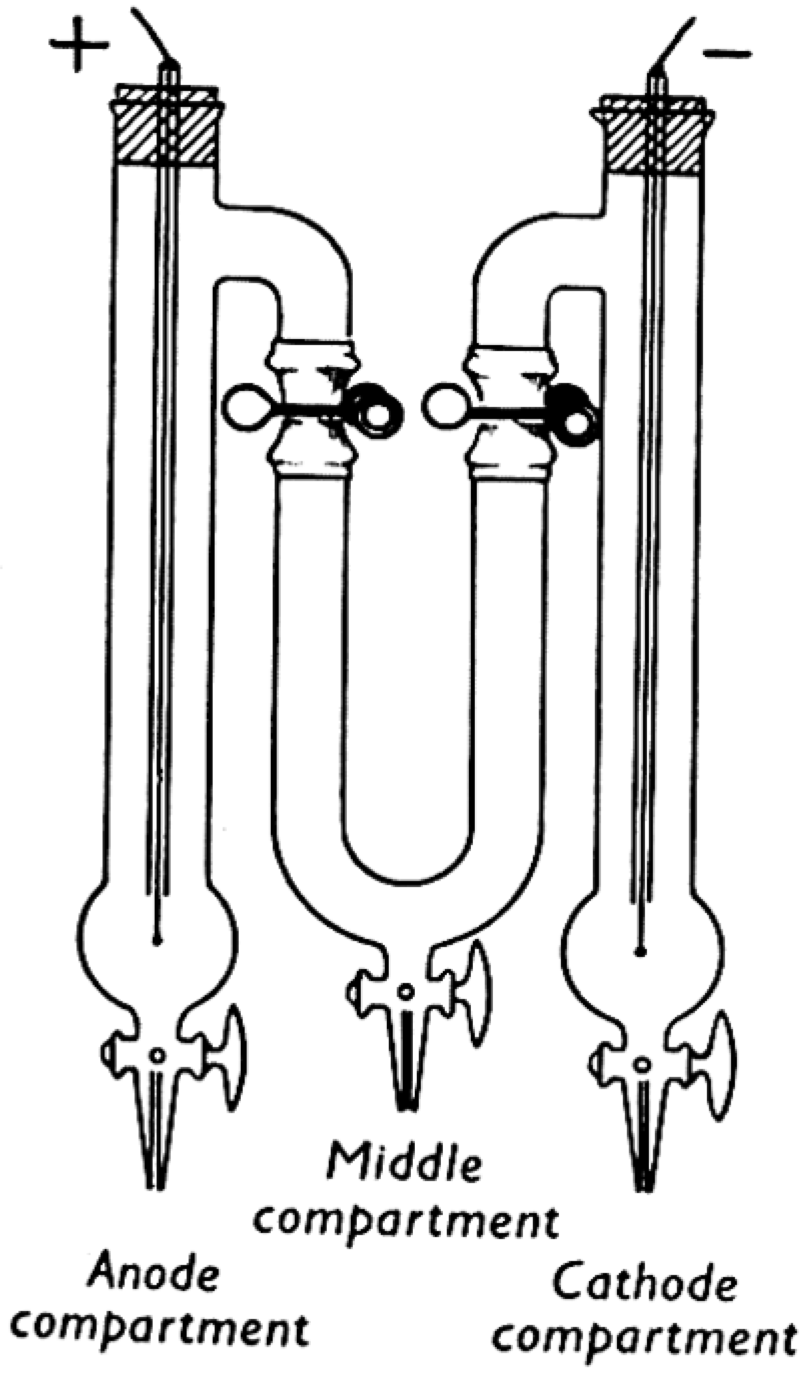


Fig. 174

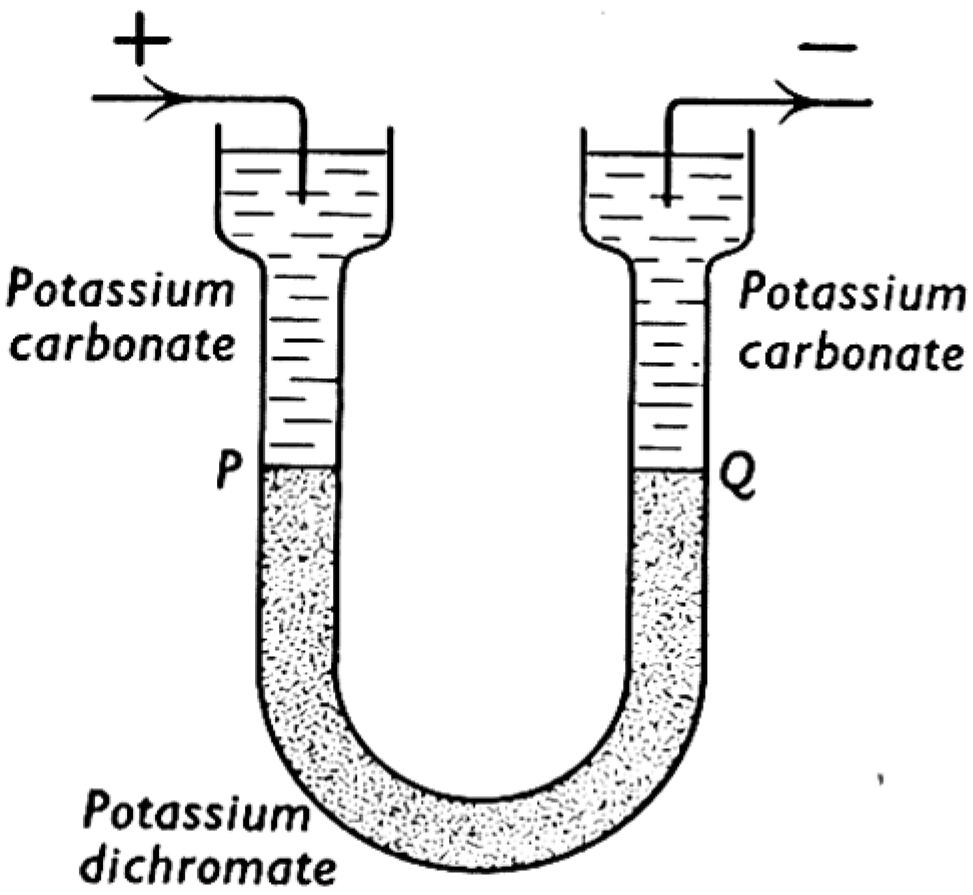


Fig. 175

QUESTIONS

1. State Faraday's laws of electrolysis.

A brass plate of total area 100 sq.cm. is to be given a copper plating of thickness 0·01 mm. How long will the process take if the current is 1 ampere? (Density of copper = 8·93 g. per c.c.; electrochemical equivalent of hydrogen = 0·0000104 g. per coulomb; chemical equivalent of copper = 31·6.) (N.)

2. Calculate the volume of mixed gases liberated in 10 minutes by a current of 1 ampere passing through a water voltameter. The temperature of the gases is 20° C. and their pressure is 765 mm. of mercury. The electrochemical equivalent of hydrogen is 0·0000104 g. per coulomb, the density of hydrogen at N.T.P. is 0·09 g. per litre and of oxygen 1·43 g. per litre. (B.)

3. One form of electricity quantity meter depends upon the electrolytic liberation of mercury from a solution of a divalent mercury salt. A meter of this type is provided with a shunt whose resistance is $\frac{1}{99}$th of that of the meter. What quantity of electricity has been used when 1·39 c.c. of mercury is deposited? Assume that the electrochemical equivalent of hydrogen is 0·00001045 g. per coulomb, that the atomic weight of mercury is 200·6 and that the density of mercury is 13·57 g. per c.c. (N.)

4. Give a succinct account of the elementary theory of electrolysis and show that it is consistent with Faraday's laws of electrolysis.

What current will liberate 100 c.c. of hydrogen at 20° C. and 78 cm. of mercury pressure in 5 minutes? (E.C.E. of hydrogen = 0·00001044 g. per coulomb; density of hydrogen = 0·00009 g. per c.c. at N.T.P.) (L.)

5. A current when passed through a tangent galvanometer and a copper voltameter in series produces a deflection of 45° in the galvanometer and liberates 0·30 g. of copper in 20 minutes. The galvanometer coil consists of four turns of wire, each of 10 cm. radius. What value does this experiment give for the horizontal component of the earth's magnetic field? (The electrochemical equivalent of copper is 0·000329 g. per coulomb.) (O. & C.)

6. A sheet of copper is suspended by copper threads midway between two copper electrodes in a bath of copper sulphate solution and these two electrodes are connected to the poles of a Daniell cell. Describe and explain what will happen to all three sheets of copper in the course of a day. (D.)

7. The master discs of gramophone records are usually covered with a thin coating of copper deposited electrolytically. It is found that when the discs are placed directly in the plating bath the coating is much thicker at the edges than at the centre. Suggest a method of preventing this defect. (B.)

8. In calibrating an ammeter by means of a copper voltameter it was found that the gain plate increased by 5·9 g. per hour when the average reading under test was 4·5 amperes. What was the percentage error of the ammeter? (Chemical equivalent of copper = 31·65; 1 Faraday = 96,540 coulombs.) (O. S.)

9. Given that the Faraday is $9{\cdot}64 \times 10^4$ coulombs per gram atom, and the atomic charge $1{\cdot}59 \times 10^{-19}$ coulombs, calculate the number of atoms in a gram atom, and the mass in grams of the hydrogen atom. (B.)

10. Give an account of an experimental method for determining the electrical conductivity of an electrolyte. What do you understand by *equivalent conductivity*? How does it vary with the concentration of the electrolyte? (N.)

11. Write an account of electrolytic and metallic conduction pointing out their resemblances and differences.

Discuss the dissociation theory of electrolytic conduction, paying particular attention to the experimental facts which support it. (O. & C.)

12. What are Faraday's laws of electrolysis? Describe how you would verify them.

What evidence do these laws provide for the belief that there is a fundamental unit of electric charge, and how could the results of the experiments you describe be used to estimate its magnitude? (C. S.)

CHAPTER 11

CELLS

Primary cells

When plates of zinc and copper are placed in dilute sulphuric acid there is a P.D. between them of about 1 volt, the copper being positive, and a current will flow if they are connected with a wire. The arrangement is called a voltaic cell, since it was discovered by Volta.

While the current is flowing zinc dissolves and hydrogen collects at the copper plate. The chemical action in the cell may be represented by

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

Owing to polarisation set up by the hydrogen on the copper plate, the current falls to a low value. The hydrogen may be removed by an oxidising agent known as a depolariser.

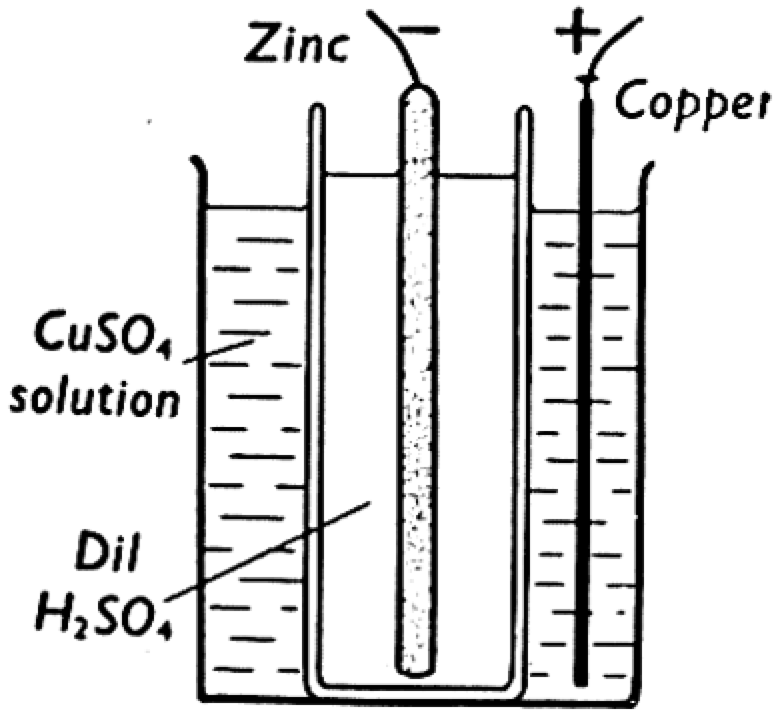


Fig. 176

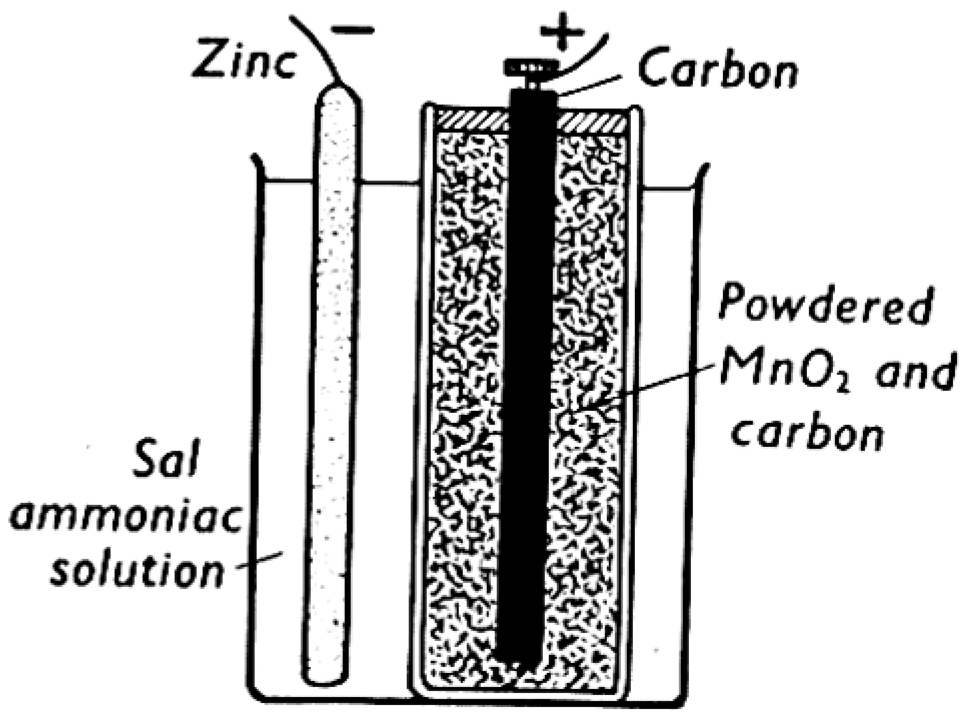


Fig. 177

The Daniell cell (Fig. 176) consists of the above simple cell with the addition of a concentrated solution of copper sulphate in contact with the copper plate. The copper sulphate acts as a depolarising agent by reacting with the hydrogen to form a deposit of copper on the copper plate and sulphuric acid:

$$CuSO_4 + H_2 = H_2SO_4 + Cu.$$

Since zinc reacts readily with copper sulphate a porous pot is used to separate the zinc and sulphuric acid from the copper sulphate. The depolariser works very efficiently and the cell will give a steady current. Its E.M.F. is about 1·1 volts.

The Leclanché cell (see Fig. 177) consists of a positive plate of carbon, a negative plate of zinc, an electrolyte consisting of a solution of ammonium chloride in water, and a depolariser consisting of manganese dioxide mixed with carbon to render it more conducting. Since the depolariser is solid it must be held round the positive plate by means of a porous pot. In the dry form of the cell the solution is replaced by a moist paste of ammonium chloride mixed with glycerine and the porous pot is replaced by a muslin bag. The chemical reactions occurring in the cell may be represented by the equation

$$\mathrm{Zn+2NH_4Cl+2MnO_2=Zn(NH_3)_2Cl_2+H_2O+Mn_2O_3}.$$

The depolariser is not very efficient, especially in the wet form of the cell, and the Leclanché cell will not give such a steady current as a Daniell cell. Its E.M.F. is about 1·5 volts.

The Lalande cell also has plates of copper and zinc but its electrolyte is an alkali, for example, NaOH. The depolariser is copper oxide, and in one form of the cell the positive plate consists of copper gauze filled with copper oxide. The E.M.F. is 1·0–1·1 volts and the internal resistance is very low, 0·03–0·05 ohm. The Lalande is perhaps the most efficient primary cell; it has been used extensively for railway signalling.

Calculation of the E.M.F. of a Daniell cell

Lord Kelvin assumed that the electrical energy delivered by a cell is equivalent to the chemical energy released during its working.

If 1 g.-equivalent (i.e. 32·7 g.) of powdered zinc is shaken up with a solution of copper sulphate the zinc displaces the copper with the release of 25,000 calories of heat:

$$1\text{ g.-equivalent }\mathrm{Zn+CuSO_4=ZnSO_4+Cu}+25{,}000\text{ calories.}$$

This can be verified by a simple calorimetric experiment. The above chemical action is, in effect, what occurs when a Daniell cell is working; the zinc dissolves to form zinc sulphate and copper is deposited.

It is found experimentally that just as 96,500 coulombs of electricity deposit 1 g.-equivalent of zinc in electrolysis, so in a cell, during the generation of 96,500 coulombs of electricity, 1 g.-equivalent of zinc is consumed.

If E volts = E.M.F. of Daniell cell,

Electrical energy when 1 g.-equivalent of Zn is consumed = $96{,}500E$ joules.

Chemical energy when 1 g.-equivalent of Zn is consumed = 25,000 calories = $25{,}000 \times 4{\cdot}2$ joules.

LUIGI GALVANI

ALESSANDRO VOLTA

LUIGI GALVANI (1737–1798), *professor of anatomy at Bologna, during the course of experiments on the nerves of the frog, discovered that a freshly dissected frog's leg twitched when placed near to an electrical machine which was generating sparks. Subsequently he found that the leg twitched when touched by two dissimilar metals such as copper and zinc. He believed that the source of the electricity was in the frog and was thus led into controversy with Volta who held, and later proved, that this was not 'animal electricity', but of the same nature as that generated by rubbing. Nevertheless, Galvani's single-minded investigation of a phenomenon of wholly unforeseen importance amply earned the fame of commemoration in the name 'galvanometer'. During eleven years of arduous experiment, mainly with frogs, he observed the effects* (*as is now known*) *of electromagnetic induction and of electromagnetic waves.*

ALESSANDRO VOLTA (1745–1827), *professor of physics at Pavia, invented an electroscope consisting of two straws—the forerunner of the gold leaf electroscope—and also the electrophorus. He investigated the effect discovered by Galvani and showed first that a frog's leg twitched when a discharge from a condenser was passed through the nerve. He then demonstrated that different pairs of metals, when touching the frog's leg, gave effects of different magnitudes and he arranged the metals in an electromotive series. Finally, in a brilliant experiment, considering the smallness of the* P.D. *involved, he deflected an electroscope, by combining it with a condenser, from the contact of a single pair of metals, thus dispensing with the frog. His invention of the 'voltaic' cell opened a new era in the history of electricity. He travelled widely, was a friend of Voltaire, Lavoisier and Priestley, and was a favourite of Napoleon.*

Equating, $$E=\frac{4{\cdot}2\times 25{,}000}{96{,}500}=1{\cdot}09 \text{ volts.}$$

This agrees very well with the experimental determination of the E.M.F., but the agreement is fortuitous. Kelvin pointed out that heat may be lost to, or absorbed from, the surroundings while a cell is working. In fact:

Energy of chemical action = Electrical energy produced + heat given out in working of cell.

The last term is nearly zero in the case of a Daniell cell, but this is not so for all cells.

The origin of the E.M.F.

The origin of the E.M.F. of a cell was a matter of controversy during the whole of the nineteenth century and has still not been completely settled.

Volta, the discoverer of the cell, always maintained that the seat of the E.M.F. lay in the contact of the two different metals. He showed by experiment that when two dissimilar metals are put in contact and then separated by means of insulating handles, they possess positive and negative charges of electricity respectively. We now know that, when plates of copper and zinc are put in contact, electrons pass from the zinc to the copper, making the copper negatively charged and the zinc positively charged, and that the P.D. between them, known as the *contact potential difference*, is about 0·6 volt. This appears to account for about half of the E.M.F. of a Daniell cell. The phenomenon is the same as the production of electrostatic charges by rubbing; the rubbing merely promotes closer contact.

The opponents of the contact theory, however, maintained that the seat of the E.M.F. lay in the junctions between the metals and the solutions in the cell. They claimed that there was no source of energy at the junction of the metals, and that all the energy came from the chemical reactions with the solutions. In the light of modern knowledge, however, as will be seen later, a release of energy at the junction of the metals can be accounted for.

Contact potential difference

The contact P.D. between two metals may be measured by the apparatus shown in Fig. 178. A is a quadrant electrometer (an instrument for measuring slight changes of potential), B and C are plates of the two different metals forming a condenser, D is a potentiometer, and

E an earth connection. The condenser *BC* is surrounded by an earthed metal shield, not shown in the diagram, to protect it from stray electrostatic fields.

Let us suppose that plate *B* is copper, plate *C* zinc, that the connecting wires are copper and that the sliding contact is at the earthed end of the potentiometer wire. Where the copper wire is connected to the zinc plate *C* there will be a flow of electrons from the zinc to the copper. The electrons do not spread and there is what is known as an *electrical double layer* at the junction. This consists of a layer of electrons in the copper held in position by a corresponding layer of positive charge on the zinc. The two layers are very close together, like the plates of a charged condenser, the separation being of the order of an atomic diameter. An electrical double layer is the seat of a sudden P.D. The potential throughout the copper must be the same and the potential throughout the zinc must be the same but there is a P.D. between the two metals.

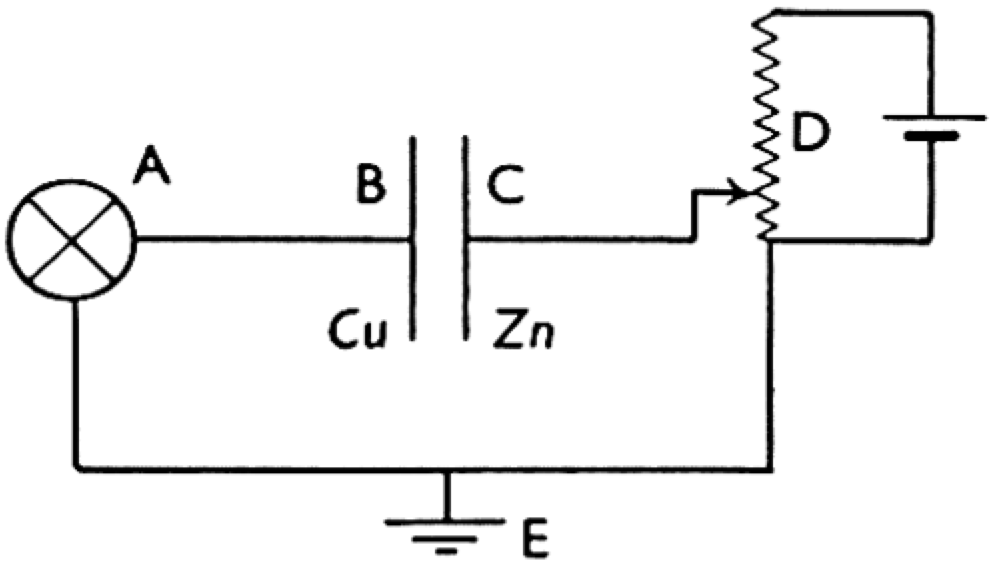


Fig. 178

A few electrons must spread throughout the copper and be drawn from the whole of the zinc to make the alterations in the potentials of the plates. Clearly an electric field exists between the condenser plates *B* and *C*.

If one of the condenser plates *B* or *C* is moved relative to the other, the electric field between them will change and there will be a deflection of the electrometer. If, for example, *C* is moved away from *B*, the influence of its positive charge on *B* will be reduced and the potential of *B* will change from earth potential to a negative potential.

Suppose, however, that the sliding contact along *D* is moved so that the potential of *C* is made the same as that of *B*. There will be no field between *B* and *C* so that relative movement will not cause a deflection of the electrometer. The reading of the potentiometer will then be equal to the contact P.D. between copper and zinc. The value is about 0·6 volt.

Electronic work function

Why do electrons tend to pass from zinc to copper until the counter-effect of the electrical double layer prevents further migration?

Knowledge bearing on this problem has been greatly increased by the investigation of the work required to remove electrons from a metal by heat (thermionic effect) and by light (photoelectric effect), which will be discussed more fully in a later chapter.

The conductivity of a metal is due to the free, valence electrons which can move about readily inside the metal. If these electrons are free why do they not spill out of the metal? An electron just outside the surface of a metal tends to repel the other free electrons inside the metal and to give rise to an induced positive charge. To escape it must overcome the attraction of this induced positive charge. If the electron is in front of a large, plane, conducting surface, the force between it and the surface is the same as if there were an equal, induced charge of opposite sign at an equal distance behind the plane, known as the electrical image. Thus the force between the electron and a surface is sometimes called the image force. The work done per unit charge against the image force in removing electrons from the surface is called the *electronic work function*. Its value differs for different substances; it is denoted by ϕ and is usually measured in volts. Thus the work required to remove an electron, charge e, from a surface, is ϕe *electron-volts*. An electron-volt is the work done on an electron as it passes through a P.D. of 1 volt. Since the charge on an electron is $4{\cdot}80 \times 10^{-10}$ e.s.u. and 1 volt $= \frac{1}{300}$ e.s.u.,

$$1 \text{ electron-volt} = 4{\cdot}80 \times 10^{-10} \times \tfrac{1}{300} = 1{\cdot}60 \times 10^{-12} \text{ ergs}.$$

In Chapter 15 it will be seen how electronic work functions can be obtained experimentally. To quote the experimental results, the electronic work function of copper is 4·16 volts and of zinc 3·57 volts. The difference between them is 0·59 volt, which is equal to the contact P.D. between copper and zinc.

When two pieces of metal are put into contact, electrons escape from the metal with the smaller electronic work function to the metal with the larger, and energy is released. Work is done by electrons on escaping from zinc, but more work is done on them on entering copper.

When a Daniell cell is working, zinc atoms are continually giving up electrons and passing into solution as ions while copper ions are continually being deposited from solution and receiving electrons. There is, therefore, a source of energy in the copper-zinc combination of a Daniell cell due to the difference in the electronic work functions of copper and zinc.

When powdered zinc is allowed to displace copper from a solution of copper sulphate some of the energy released as heat is due to the difference in the electronic work functions of copper and zinc.

Electrode potentials

The theory which puts the seat of the E.M.F. of a cell at the junctions between the metal plates and the solutions must now be considered. At the end of the nineteenth century this was thought to be the whole story. It is certainly part of the story, but in view of what has been said above, only part.

When a plate of pure zinc is placed in a solution of sulphuric acid or of zinc sulphate it tends to pass into solution. Neutral zinc atoms from the plate become positively charged zinc ions in solution leaving the plate with a resultant negative charge. Only a minute quantity of zinc passes into solution because an electrical double layer is set up at the junction between the plate and the solution as in Fig. 179 (*a*), which prevents further dissolution. The plate is all at the same potential and the solution is all at the same potential but, owing to the electrical double layer, the potential of the zinc plate is negative with respect to the solution. The value of the P.D. is about 0·76 volt.

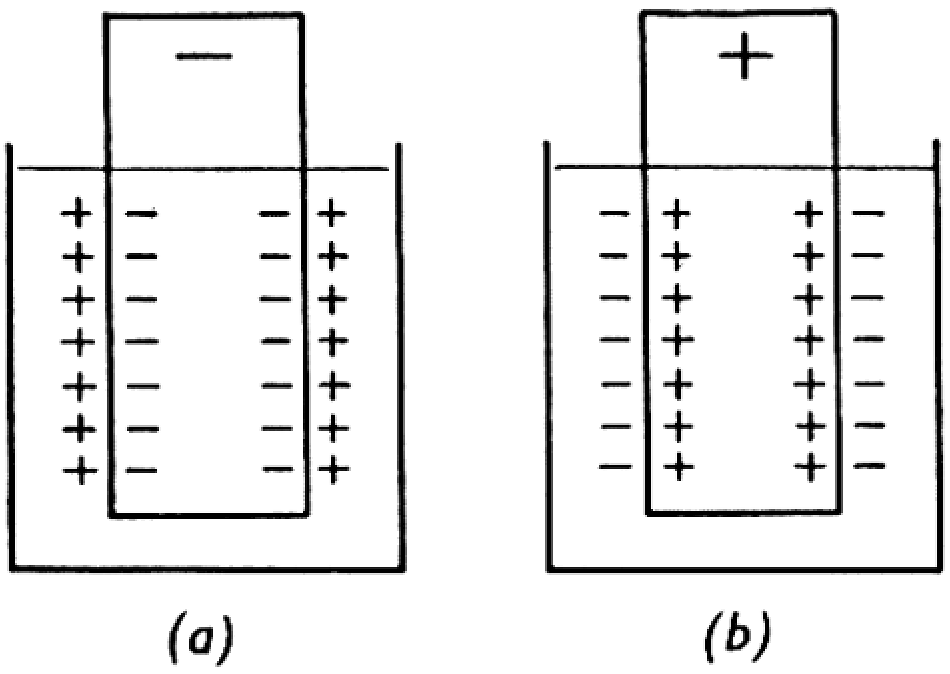


Fig. 179

On the other hand, when a plate of copper is placed in a solution of copper sulphate, copper ions tend to be deposited on the copper plate and an electrical double layer is set up as in Fig. 179 (*b*). The copper acquires a positive potential with respect to the solution of about 0·34 volt.

In 1888 Nernst put forward a mathematical theory of the above phenomena. He called the tendency of ions to pass into solution the solution pressure and the tendency of ions to be deposited was measured by the osmotic pressure. In the cases of zinc, manganese, cadmium and the alkali metals, the solution pressure is greater than the osmotic pressure and the metal becomes negatively charged when placed in a solution of its ions; in the cases of copper, silver, mercury and gold, the reverse is true. The equilibrium between any metal and its ions is a dynamic one; ions are continually passing into solution and being deposited.

The P.D. between the metal and the solution, often described briefly as the *electrode potential*, depends on the concentration of the ions in solution because the osmotic pressure or tendency to deposit depends on the concentration. Indeed, a cell can be made with its two plates of the same metal dipping into solutions of different concentrations. Such a cell is called a concentration cell.

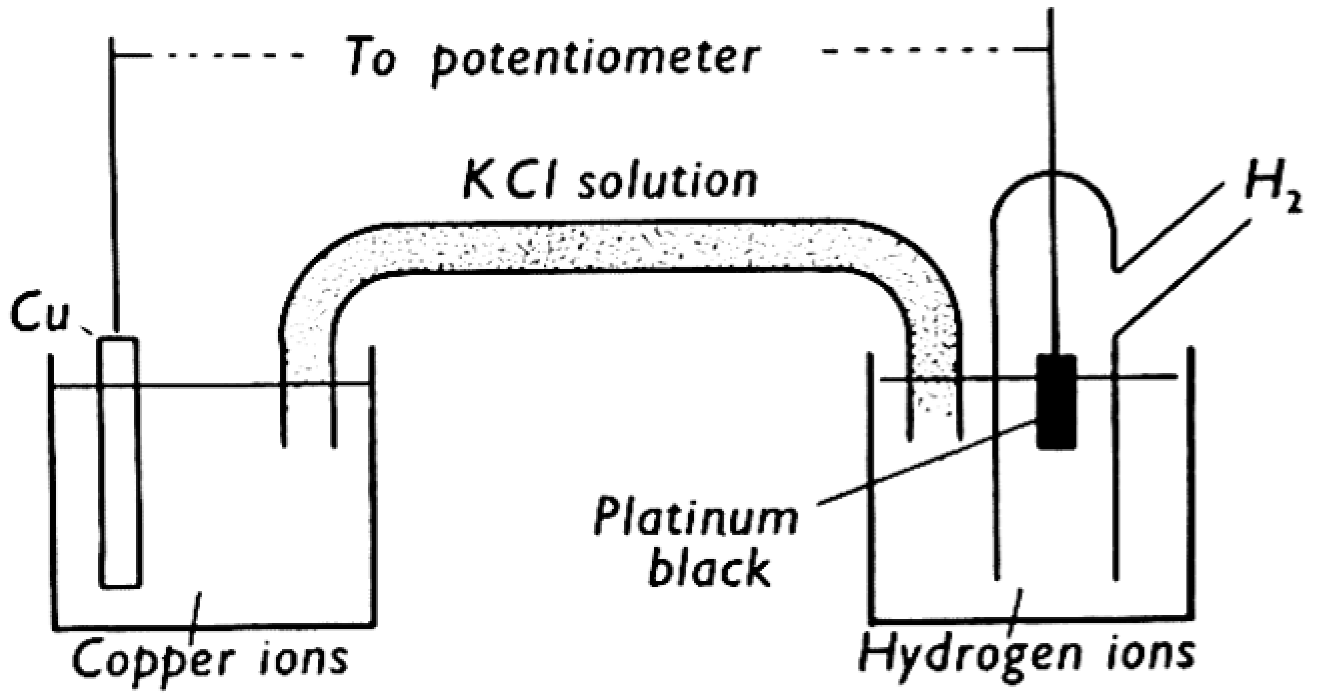


Fig. 180

Measurement of electrode potentials

Every cell may be regarded as two half-cells which act independently of each other. Thus the Daniell cell comprises a half-cell of copper in a solution of copper sulphate and a half-cell of zinc in a solution of sulphuric acid or zinc sulphate. The E.M.F. of a cell is the algebraic difference of its two electrode potentials. In the case of the Daniell cell the E.M.F. is $0{\cdot}34-(-0{\cdot}76)=1{\cdot}10$ volts.

It has been found impossible to make a satisfactory direct measurement of an electrode potential. The electrode and its solution must be combined with another electrode and solution whose P.D. is known and the E.M.F. of the resulting cell measured by the usual potentiometer arrangement. It is clear that some standard electrode must be chosen and given an arbitrary value. Then differences in electrode potentials will give the E.M.F.'s of cells although the actual value of each electrode potential is an arbitrary one. The standard internationally agreed upon is that suggested by Nernst; the P.D. between a hydrogen electrode with gas at 1 atmospheric pressure and a solution containing hydrogen ions at a concentration of 1 g.-ion per litre is considered as zero at all temperatures.

The hydrogen electrode consists of a platinum wire coated with platinum black, which readily adsorbs hydrogen, in contact with hydrogen gas at 1 atmosphere pressure and immersed in a normal hydrogen-ion solution.

Fig. 180 represents the arrangement for finding the electrode potential of copper. The copper and hydrogen electrodes are in separate vessels connected by a U tube containing a concentrated solution of potassium chloride. The object of the potassium chloride solution is to reduce the difference of potential at the liquid/liquid junction to a negligibly small value.

The table below gives the standard electrode potentials of various elements in equilibrium with normal solutions (i.e. containing 1 g.-ion per litre) of their ions.

Standard electrode potentials

Element	Ion	Volts	Element	Ion	Volts	Element	Ion	Volts
Li	Li^+	−2·96	Fe	Fe^{++}	−0·44	O_2	OH^-	0·40
K	K^+	−2·92	Cd	Cd^{++}	−0·40	I_2	I^-	0·54
Ca	Ca^{++}	−2·76	Co	Co^{++}	−0·29	Ag	Ag^+	0·80
Na	Na^+	−2·72	Ni	Ni^{++}	−0·23	Hg	Hg^{++}	0·80
Mg	Mg^{++}	−1·55	Sn	Sn^{++}	−0·14	Br_2	Br^-	1·07
Al	Al^{+++}	−1·33	Pb	Pb^{++}	−0·12	Cl_2	Cl^-	1·36
Mn	Mn^{++}	−1·1	Fe	Fe^{+++}	−0·04	Au	Au^{+++}	1·36
Zn	Zn^{++}	−0·76	H_2	H^+	0·00	F_2	F^-	1·90
Cr	Cr^{++}	−0·56	Cu	Cu^{++}	0·34			

Such a list of elements forms an electromotive series. It has long been known that any member of this list will displace from solution another member below it in the list.

When designing a cell it is desirable to select two metals as far apart in the list as possible. The algebraic difference of the two electrode potentials gives the E.M.F. of the cell.

The action of a cell

The action of a Daniell cell can now be explained on the basis of electrode potentials. When the cell is on open circuit the zinc plate has a negative potential with respect to the liquid of about 0·76 volt, and the copper plate has a positive potential with respect to the liquid of about 0·34 volt, the exact values depending on the concentrations of the solutions. The copper and zinc plates have therefore a P.D. of about

1·1 volts and, when connected by a wire, electrons will flow through the wire from the zinc to the copper. The differences in potential between both plates and the liquid are thus reduced below their original equilibrium values. Hence more zinc ions pass from the zinc plate into the solution and more copper ions are deposited on the copper plate, thereby tending to restore their negative and positive potentials, respectively. By this means also electrons are replenished in the zinc plate and neutralised on the copper plate, so enabling the flow of electrons to continue in the wire.

There is an electrical double layer and a slight P.D. between the two liquids, the zinc sulphate and copper sulphate solutions, but this is so small that it can be neglected.

How does the contact potential between the two metals, copper and zinc, fit into this theory? We have seen that electrons tend to pass from zinc to copper until the copper has a negative contact potential with respect to the zinc of about 0·6 volt. Since the copper plate in the Daniell cell is maintained at a positive potential of 1·1 volts with respect to the zinc because of the electrode potentials, it is clear that electrons will pour continuously from the zinc to the copper. In the Daniell cell, therefore, the contact effect between the metals is not essential to its working, although it must make a contribution to the supply of energy. However, a cell might be devised in which most of the E.M.F. came from the metal/metal contact and in which the electrolyte merely served to prevent the flow of electrons from stopping in the way already described.

Discharge potential

Further use can be made of the table of standard electrode potentials to explain which ions will be discharged in electrolysis. The modern theory, which has replaced the old theory of secondary reactions discussed on p. 184, is that the potential at which an ion is discharged, known as the *discharge potential*, is in general the same as the electrode potential at the particular concentration of the electrolyte.

Thus suppose two copper plates are dipping into a normal solution of copper sulphate; each plate will be at a potential of +0·34 volt with respect to the solution. If the potential of one plate is reduced slightly below +0·34 volt copper will be deposited from the solution upon it, and if the potential of the other plate is increased to slightly above +0·34 volt the copper will tend to go into solution.

The theory explains, far more satisfactorily than the old secondary reaction theory, deposition from a mixed electrolyte. Suppose a solution of silver nitrate is carrying a current. When solutions of copper, lead and sodium nitrates are added successively, the current increases on each occasion showing that each electrolyte contributes to carrying the current. But only silver is deposited at the cathode. This can be explained in terms of the figures in the table on p. 207. The potential of the cathode is reduced sufficiently by the applied E.M.F. for silver ions to be discharged but not sufficiently for copper, lead and sodium ions to be discharged.

In every aqueous solution there are present small quantities of hydrogen and hydroxyl ions from the ionisation of the water molecules

$$H_2O \rightleftharpoons H^+ + OH^-.$$

If some of these ions are removed, more water molecules dissociate.

Electrolysis of copper sulphate solution with copper electrodes

At the cathode, copper ions are discharged, but if the applied E.M.F. is high, hydrogen may be liberated as well since the potential of the cathode may become sufficiently low for hydrogen ions to be discharged. At the anode there are three possibilities—copper ions may enter the solution, SO_4 ions may be discharged or hydroxyl ions may be discharged. The discharge potential of SO_4 ions is not known, but it is likely that it is extremely high. The discharge potential of the hydroxyl ions is considerably higher than that given in the table on p. 207, since the concentration is low. Hence the raising of the potential of the anode by the applied E.M.F. is usually sufficient only to cause the copper ions to enter the solution.

The electrolysis may be summarised as follows, e representing the charge of the electron, and hence also the charge on a monovalent ion:

$$\text{At the cathode} \quad Cu^{++} + 2e \rightarrow Cu,$$
$$\text{At the anode} \quad Cu - 2e \rightarrow Cu^{++}.$$

Polarisation

A current will begin to flow between two copper electrodes dipping into a solution of copper sulphate when only a minute E.M.F. is applied. But, except momentarily, no appreciable current will flow between two electrodes of sheet platinum dipping into dilute sulphuric acid until the applied P.D. between the electrodes is about 1·7 volts.

The two platinum electrodes become coated with hydrogen and oxygen and behave as gas electrodes. The voltaic cell thus produced sets up a back E.M.F. known as a *polarisation E.M.F.*

The effect may be investigated by the experimental arrangement of Fig. 181. The applied E.M.F. is varied by sliding K along the potentiometer wire AB; C represents the platinum electrodes dipping into dilute sulphuric acid and G is a galvanometer or milliammeter to measure the current. The voltmeter V is excluded from the circuit by the two-way switch S for this part of the experiment. The graph of current and applied E.M.F. is shown in Fig. 182. D is the discharge potential.

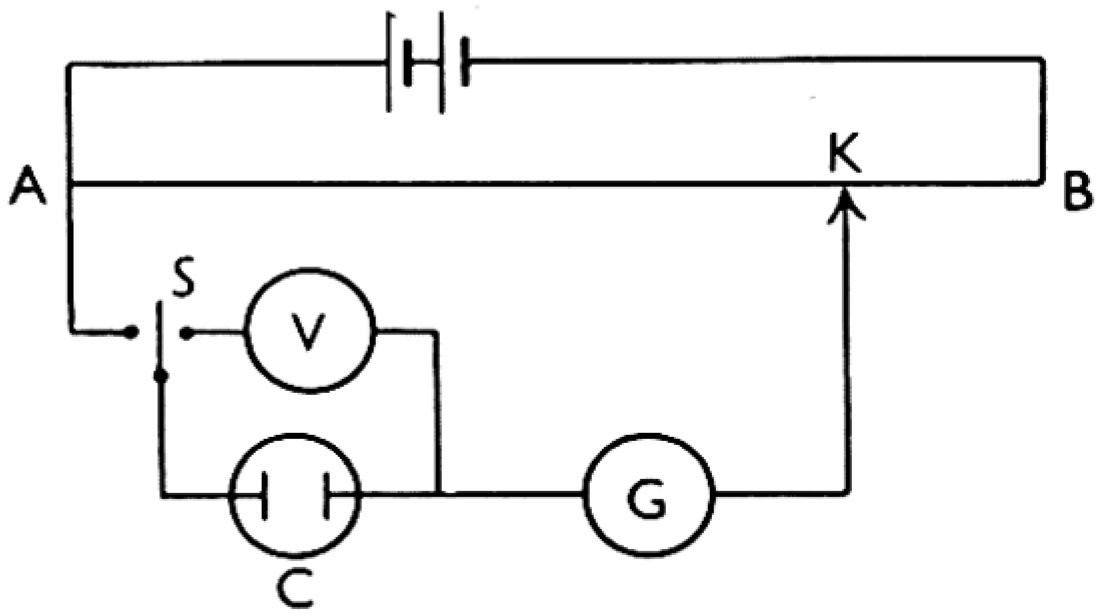


Fig. 181

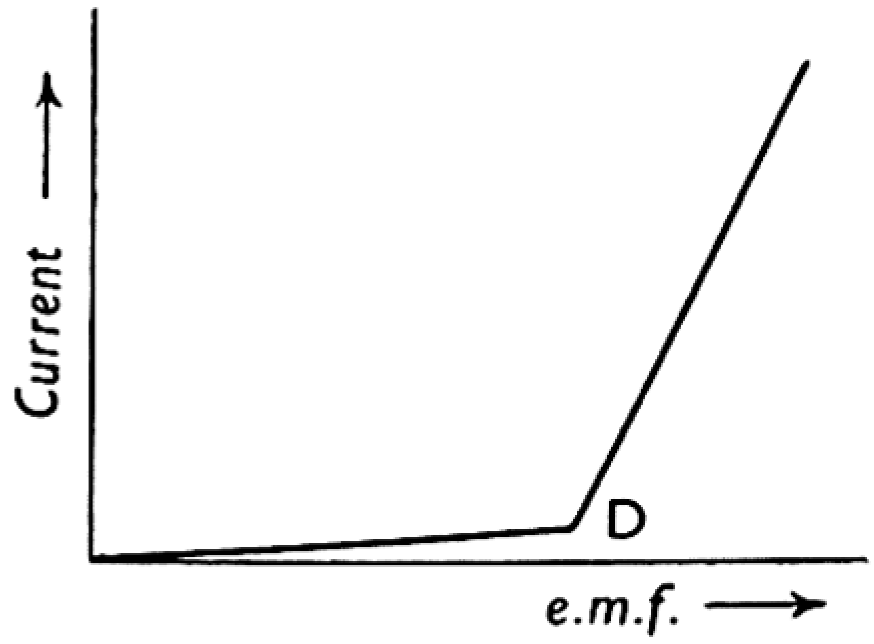


Fig. 182

The small residual current in the first part of the graph is made possible by the diffusion of the gases from the electrodes. But an appreciable current cannot flow until the applied E.M.F. V is greater than the polarisation E.M.F., v. The effective E.M.F. driving the current, i, through the resistance of the electrolyte R, is then $V-v$.

Ohm's law becomes

$$V-v=iR.$$

This is clearly the equation of the steeper part of the graph, which is a straight line cutting the E.M.F. axis at a value v.

If the arm of the switch S in Fig. 181 is moved from left to right the voltmeter V will register the back E.M.F. set up in C; the voltmeter reading falls rapidly.

Now the E.M.F. of a cell with platinum electrodes containing adsorbed hydrogen and oxygen, represented as Pt (H_2 1 atmosphere)/Normal acid/(O_2 1 atmosphere) Pt, is about 1·2 volts. Electrolysis of sulphuric acid with platinum electrodes would be expected to take place at this P.D. in view of the statement on p. 208 that the discharge potential is generally equal to the electrode potential. This statement, however, does not apply when polarisation occurs. The difference

$$1{\cdot}7-1{\cdot}2=0{\cdot}5 \text{ volt}$$

is called the *over-voltage*.

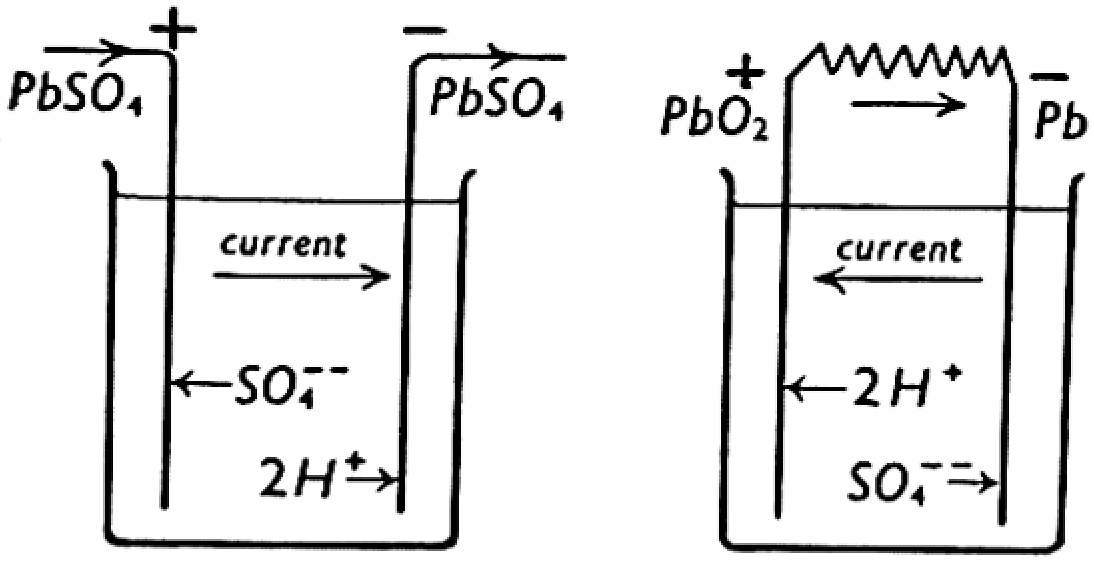
The over-voltage varies with electrodes of different metals. Thus with a cathode of mercury instead of platinum the applied voltage has to exceed 2·3 volts. Several theories of over-voltage have been put forward.

Concentration polarisation

When a current is passed between copper electrodes dipping into a solution of copper sulphate there is at first no back E.M.F., but after a time a small and slowly increasing back E.M.F. can be detected. This is due to the decrease in concentration of the copper sulphate round the cathode, and the increase in concentration round the anode; it is known as concentration polarisation. Similarly, the E.M.F. of a Daniell cell will fall gradually as it delivers a current owing to the increase in concentration of the zinc sulphate solution round the zinc plate which lowers the P.D. between the zinc plate and the solution, and the decrease in concentration of the copper sulphate solution, which lowers the P.D. between the copper plate and the solution. In order to obtain a steady E.M.F. a saturated solution of zinc sulphate can be used instead of sulphuric acid (thereby reducing somewhat the E.M.F. of the cell), and the copper sulphate solution can be maintained at saturation point by keeping solid crystals of copper sulphate in the solution.

Accumulators

The lead accumulator consists of a positive plate of lead peroxide PbO_2 and a negative plate of lead in dilute sulphuric acid (see Fig. 183). It is known as a secondary cell, since it can be recharged by passing a current through it in the opposite direction to that of discharge.



Charging Discharging

Fig. 183

On discharge the two plates are converted to lead sulphate, $PbSO_4$. The chemical reactions for the two electrodes may be written as follows:

$$Pb+PbO_2+2H_2SO_4 \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftharpoons}} 2PbSO_4+2H_2O.$$

During discharge the electrolyte loses sulphuric acid and hence its specific gravity falls. Some accumulators are fitted with hydrometers to indicate the specific gravity of the acid and hence the state of charge.

The specific gravities when the cell is charged and discharged are about 1·25 and 1·1.

When fully charged the E.M.F. of an accumulator is about 2·05 volts. This falls slightly during use, but one of the great advantages of the cell is that its E.M.F. remains very steady during most of the discharge. The E.M.F. should not be allowed to fall below 1·80 volts. The capacity of an accumulator is measured in ampere-hours; thus a cell with a capacity of 10 ampere-hours will deliver a current of 1 ampere for 10 hours, or ½ ampere for 20 hours. The use of several plates placed close together causes the cell to have a low internal resistance of the order of 0·01 ohm.

The nickel-iron accumulator is lighter in weight and more robust than the lead accumulator, and has the advantage of not deteriorating, like the lead accumulator, when stored in a half-charged condition. Its E.M.F., 1·2 volts, is smaller than that of the lead accumulator, and does not remain so steady during discharge. The positive plate consists of nickel hydrate, the negative plate of cadmium and iron oxides, and the electrolyte a solution of potassium hydroxide. The chemical reactions may be summarised as follows:

$$2Ni(OH)_3 + Fe \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftharpoons}} 2Ni(OH)_2 + Fe(OH)_2.$$

The electrolyte, KOH, does not enter into chemical combination with the plates, and hence its specific gravity remains constant during charge and discharge.

QUESTIONS

1. Calculate the maximum quantity of electricity which can be obtained from a Daniell cell with a zinc plate weighing 100 g. (Atomic weight of zinc = 65·4, valency = 2, Faraday = 96,500 coulombs.) (B.)

2. The negative pole of a Daniell cell sending a steady current through a circuit decreases in weight by 0·080 g. in 34 minutes. Calculate the increase in weight of the positive pole in this time, and the value of the current in amperes. (At.wt. of copper = 63·6; at.wt. of zinc = 65·4; E.C.E. of hydrogen = 0·0000104 g. per coulomb.) (L.)

3. A certain simple cell consists of a positive member X, a negative member Y, and an exciting liquid Z. There is no chemical action between X and Z, but when 1 g. of Y dissolves in the exciting liquid h calories are liberated; the electrochemical equivalent of Y is y g. per coulomb. Find the E.M.F. of the cell. (1 calorie = 4·2 joules.) (N.)

4. Calculate the value of the smallest E.M.F. which can produce continuous electrolysis in a water voltameter, given that the electrochemical equivalent of hydrogen is $1{\cdot}04 \times 10^{-5}$ g. per coulomb; that the heat of combustion of 1 g. of hydrogen in oxygen is 35,000 calories, and that 1 calorie equals 4·2 joules. (C.)

5. Explain the phenomenon of polarisation in a simple Volta cell and describe some experiments which support your statement.

Two accumulators, each of E.M.F. 2 volts and of negligible internal resistance, are connected in series with a voltameter containing dilute sulphuric acid. The current which flows is 1·012 amperes. A third 2-volt accumulator is then added in series and the current rises to 1·812 amperes. Calculate the resistance of the dilute acid between the electrodes. (D.)

6. A water voltameter and a copper voltameter of resistances 2 ohms and 1 ohm respectively are connected in parallel, and a battery of E.M.F. 6 volts and internal resistance 0·5 ohm is used to send current through them. Find the current in each of the voltameters, assuming that the E.M.F. of polarisation of the water voltameter is 1·8 volts. (N.)

7. Two platinum plates are immersed in acidulated water and connected to the terminals of a voltameter. They are also connected through a key and an ammeter to a battery of E.M.F. about 4 volts. The key is held down for a few minutes and then raised. Describe and account for the indications of the voltmeter and of the ammeter from the time when the key is first closed until a few minutes after it had been opened. (D.)

8. Describe briefly the lead secondary battery and explain the chemical reactions which take place in it during charging and discharging.

Such a battery is charged at different voltages (E) and the current (I) which flows through it is measured. The following observations are made:

E (volts)	2·3	2·6	3·0
I (amperes)	0·4	1·6	3·2

How do you reconcile these facts with Ohm's law? If this battery is charged at 3·5 volts, what is the maximum efficiency of the process (i.e. ratio of energy of chemical transformation to total energy supplied)?

9. Describe the changes which take place in the lead storage cell during charging and discharging.

Why is it bad for the cell to charge or discharge at too high a rate or for too long a time? Consider each case separately. Contrast the value of a hydrometer and a voltmeter in determining the extent to which a cell is discharged.

The electrochemical equivalent of lead is 0·001073 g. per coulomb; lead combining to form $PbSO_4$ gives up 380 calories per g.; the corresponding weight of the appropriate lead oxide combining to form $PbSO_4$ gives up 80 calories. Calculate an approximate value for the E.M.F. of a storage cell from these facts, and say why you do not expect it to agree with the observed value. (C.)

CHAPTER 12

ELECTROMAGNETIC INDUCTION

A CHANGE in the magnetic flux in the neighbourhood of a conductor causes an induced E.M.F. to be set up in the conductor. The change may be caused by relative motion between the magnetic flux and the conductor or by a variation in the strength of the magnetic flux.

The following two laws, giving the direction and magnitude of the induced E.M.F., have already been discussed (pp. 90, 118).

Lenz's law. The direction of the induced E.M.F. is such that it tends to oppose the change to which it is due.

Faraday's law. The magnitude of the induced E.M.F. in e.m.u. is equal to the number of lines of magnetic flux cut per second.

When dealing with a coil (or a circuit), as opposed to a straight conductor, it is more convenient to consider the rate of change of the flux passing through or 'threading' the coil than the rate of cutting. A change in the magnetic flux threading the coil causes an E.M.F. to be induced in each turn of the coil. If magnetic flux, denoted by ϕ, threads through a coil of N turns, there is said to be a *magnetic flux linkage* of $N\phi$. The quantity $N\phi$ is also called the flux turns.

Faraday's law applied to a coil or a circuit may be written as follows:

Induced E.M.F. = Rate of change of linkages of magnetic flux, i.e.

$$E = -\frac{d(N\varphi)}{dt} \text{ e.m.u.}$$

$$= -\frac{d(N\varphi)}{dt} \times 10^{-8} \text{ volt} \quad (1 \text{ volt} = 10^{8} \text{ e.m.u.}).$$

The negative sign indicates that the induced E.M.F. tends to set up a current which would produce a flux opposing that which causes it.

Quantity of electricity induced

The induced current is proportional to the rate of change of magnetic flux; the quantity of electricity induced, however, is proportional to the change in magnetic flux. If the rate of change is high, the current is large and the time for which it flows is short; if the rate of change of the same quantity of flux is low, the current is smaller and the time for which it flows is longer.

Suppose a quantity of electricity, Q, is induced by a change in magnetic flux linkage of $N\phi$ in a circuit of resistance R ohms, and that the induced current at time t is i.

Then
$$Q=\int i\,dt.$$

But
$$i=E/R, \quad \text{where } E=\text{induced E.M.F. at time } t$$
$$=-\frac{1}{R}\frac{d(N\phi)}{dt}\times 10^{-8}\text{ ampere.}$$
$$\therefore\ Q=\int -\frac{1}{R}\frac{d(N\phi)}{dt}\,10^{-8}\,dt$$
$$=-\frac{10^{-8}}{R}\int d(N\phi)$$
$$=-\frac{\mathbf{N\varphi}}{\mathbf{R}}\,\mathbf{10^{-8}\text{ coulomb,}}$$

i.e.
$$\text{Quantity induced}=\frac{\text{Total change of flux turns}}{\text{Resistance}}\times 10^{-8}\text{ coulomb.}$$

Measurement of the pole strength of a magnet

The pole strength of a magnet may be determined by plunging one of the poles into a coil and measuring, by means of a ballistic galvanometer, the quantity of electricity induced.

A pole of strength m c.g.s. units has $4\pi m$ lines of induction or flux. If it is plunged into a coil of N turns and resistance R ohms (including the whole resistance of the galvanometer circuit), then the quantity of electricity induced, Q coulombs, is given by

$$Q=\frac{4\pi mN}{R}\times 10^{-8}. \tag{1}$$

Since a ballistic galvanometer is highly damped when connected to a coil of comparatively low resistance, it is convenient to calibrate the galvanometer with an earth inductor (see p. 68) and to keep the circuit in Fig. 184 unchanged both for calibration and for plunging the magnet into the coil.

The coil of the earth inductor is made vertical and then turned sharply through 90° between a position in the magnetic meridian and a position at right angles. The change in the magnetic flux linkage is

$N'AH$, where N' and A are the number of turns and area of cross-section of the earth inductor coil respectively and H is the horizontal component of the earth's magnetic field. Hence the quantity of electricity induced, Q', is given by

$$Q' = \frac{N'AH}{R} \times 10^{-8} \text{ coulomb.} \tag{2}$$

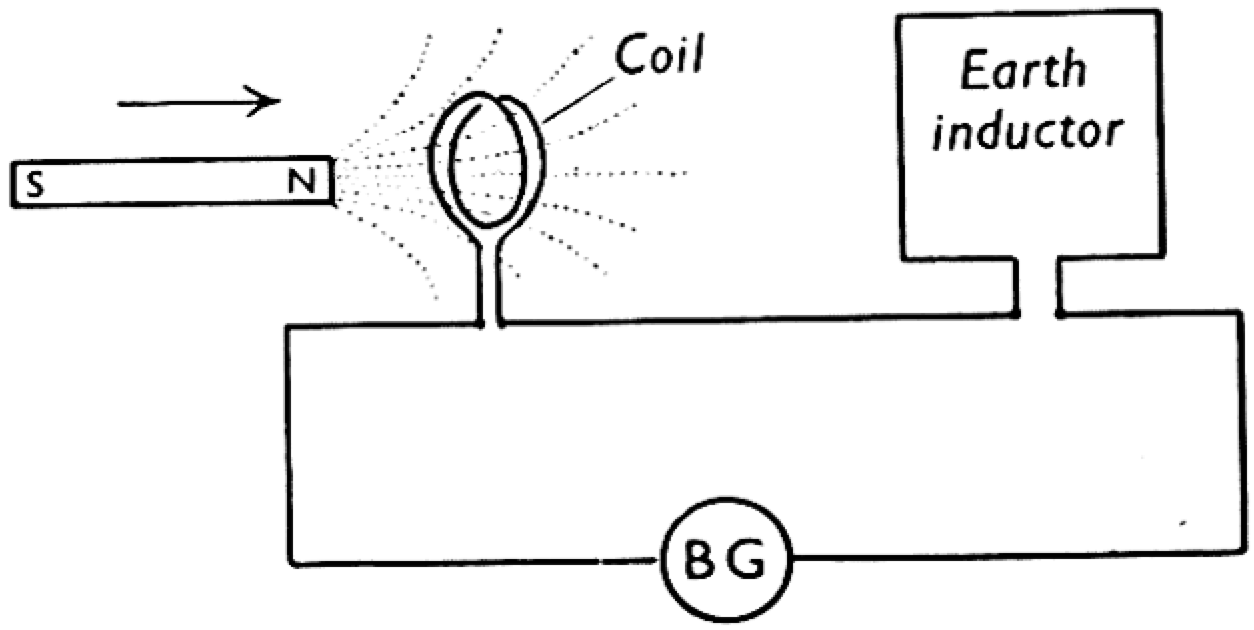


Fig. 184

Dividing equation (1) by (2)

$$\frac{Q}{Q'} = \frac{4\pi mN}{N'AH}.$$

The ratio Q/Q' is equal to the ratio of the throws of the ballistic galvanometer. Hence, knowing N, N', A and H it is possible to calculate m.

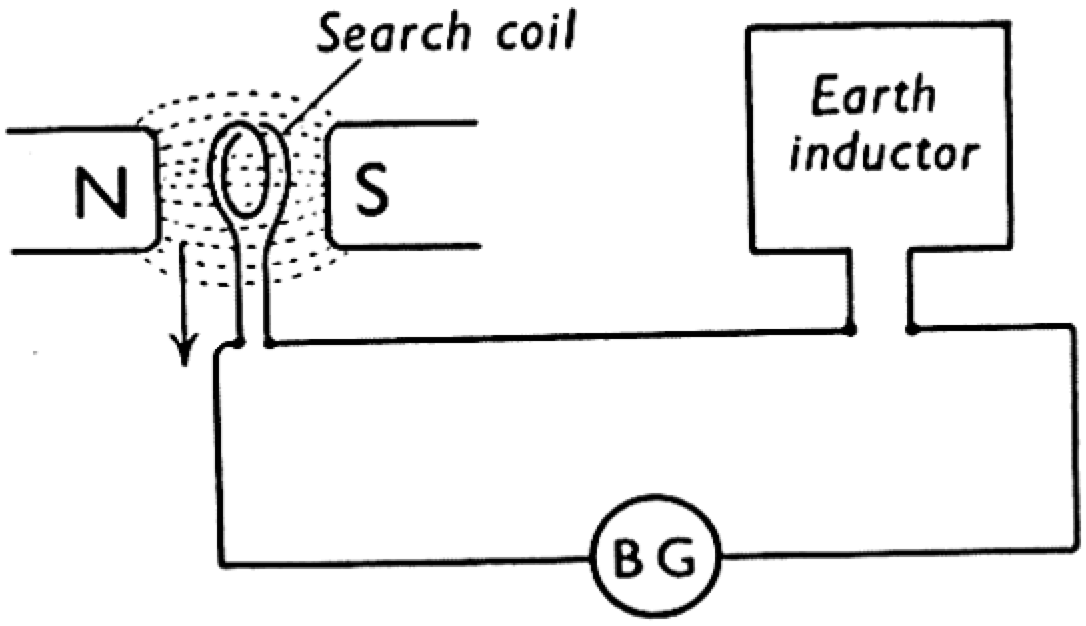


Fig. 185

Measurement of the magnetic field of an electromagnet with a search coil

A convenient method of measuring a magnetic field such as that between the poles of an electromagnet is to use a small coil, called a search coil, connected to a ballistic galvanometer (Fig. 185). The search coil is placed at right angles to the field so that the maximum flux passes through it and then quickly removed from the field. The quantity of electricity, Q, induced in the search coil is given by

$$Q = \frac{NAH}{R} \times 10^{-8},$$

where N = number of turns of search coil, A = area of cross-section of search coil, H = strength of magnetic field, R = total resistance of circuit.

The ballistic galvanometer may be calibrated with an earth inductor as already described.

Mutual inductance

A current changing in one circuit will induce an E.M.F. in a nearby circuit. Some of the magnetic flux due to the current in the first circuit, called the *primary*, must interlink the second circuit, called the *secondary* (Fig. 186). The phenomenon is called *mutual induction*.

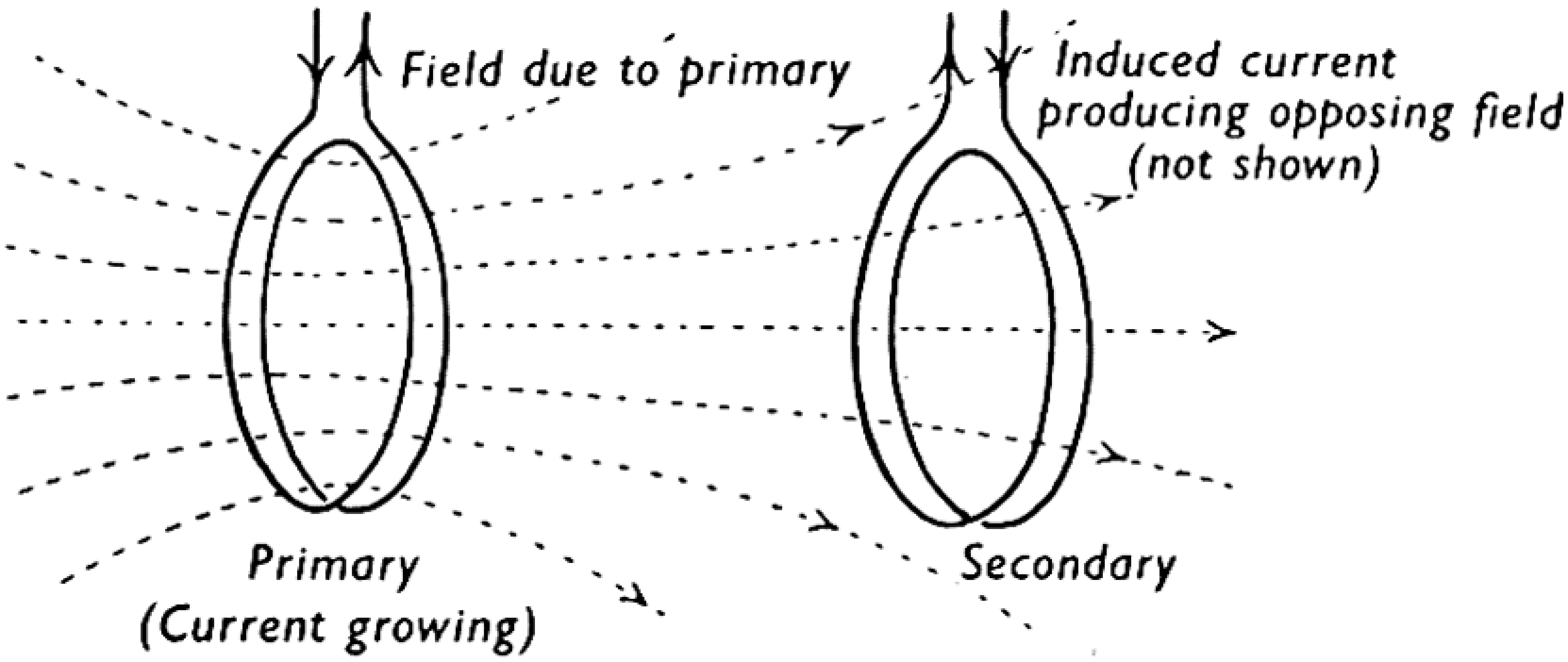

Field due to primary
Induced current producing opposing field (not shown)
Primary (Current growing)
Secondary

Fig. 186

The direction of the induced current in the secondary is such that it tends to oppose the change in the magnetic field of the primary. Thus when the current in the primary is growing, as in Fig. 186, the induced current in the secondary sets up an opposing magnetic field; when the current in the primary is dying away the induced current in the secondary sets up a supporting magnetic field in the same direction as that of the primary. The induced current in the secondary only lasts while that in the primary is changing.

Not all the magnetic flux from the primary in Fig. 186 links the secondary. The mutual induction could be increased to a maximum by moving the two coils together so that their faces were touching.

If an induced E.M.F. of 1 *volt is induced in the secondary by a current in the primary changing at the rate of* 1 *ampere per second, the circuits are said to possess a mutual inductance of* 1 *henry.*

The mutual inductance, M, in henries may be defined by the equation

$$M=\frac{-e}{di/dt},$$

where e = E.M.F. induced in secondary in volts, and di/dt = rate of change of current in primary in amperes per second. When primary and secondary are interchanged M is the same.

If the current in the primary changes by 1 ampere in 1 second then, in the above equation for M, $di/dt=1$ and $M=-e$. But e is equal, in volts, to the change in flux linkage through the secondary in 1 second times 10^{-8}. Thus

> *M (in henries) = Magnetic flux linkage through the secondary, due to a primary current of* 1 *ampere, times* 10^{-8}.

This may be regarded as an alternative definition of M and is useful in calculating mutual inductances.

Mutual inductance of two concentric solenoids with an air core

Suppose a short secondary coil is wound round the outside and in the middle of a long primary coil (Fig. 187). It is possible to calculate the mutual inductance from the dimensions of the coils.

Let i = current through the primary in amperes, n = number of turns *per unit length* of the primary, A = area of cross-section of the primary, N = *total* number of turns of the secondary.

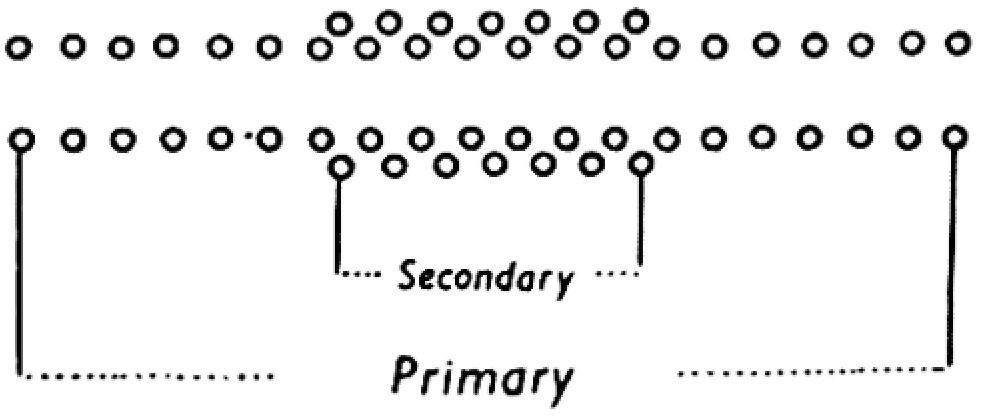


Fig. 187

$$\text{Field inside the primary} = \frac{4\pi ni}{10} \text{ oersteds} \qquad \text{(see p. 94)}.$$

$$\text{Flux through the primary} = \frac{4\pi niA}{10} \text{ maxwells} \qquad (\mu = 1).$$

$$\therefore \text{ Magnetic flux linkage through secondary} = \frac{4\pi niAN}{10}.$$

But M = magnetic flux linkage × 10^{-8}, when $i=1$ ampere.

$$\therefore\ M = \frac{4\pi nAN}{10} \times 10^{-8}$$

$$= 4\pi nNA \times 10^{-9} \text{ henry}.$$

It should be noted that the primary coil has been assumed to be so long that its external field, which is opposite in direction to the internal field, is extremely small in the region of the secondary. In practice, the primaries of mutual inductances are not very long and the formula becomes more complicated.

Self-inductance

When a current in a coil (or indeed in any circuit) is changing, the magnetic flux links with the turns of the coil and gives rise to an induced E.M.F. The phenomenon is known as *self-induction*. When the current is increasing the induced E.M.F. opposes it, and when the current is decreasing the induced E.M.F. aids it.

If an E.M.F. *of* 1 *volt is induced when the current changes at the rate of* 1 *ampere per second, the coil or circuit is said to have a self-inductance of* 1 *henry.*

The self-inductance, L, in henries, may be defined by the equation

$$L=\frac{-e}{di/dt},$$

where e = induced E.M.F. in volts and di/dt = rate of change of current in amperes per second. By reasoning similar to that used in the case of mutual inductance it can be shown that

L (in henries) $= 10^{-8}$ (magnetic flux linkage due to a current of 1 ampere).

Inductance of an air-cored solenoid

The self-inductance, often abbreviated to inductance, of an air-cored solenoid can be calculated from its dimensions.

Consider a long solenoid, such that the field inside it can be assumed to be uniform. Using the same nomenclature as in the case of mutual inductance,

$$\text{Field inside the solenoid}=\frac{4\pi ni}{10}\ \text{oersteds},$$

$$\text{Flux threading each turn of the solenoid}=\frac{4\pi niA}{10}\ \text{maxwells},$$

$$\text{Magnetic flux linkage in the solenoid}=\frac{4\pi niA}{10}\times nl,$$

where l is the length of the solenoid, and hence nl is the total number of turns.

But $L=10^{-8}$ (magnetic flux linkage), when $i=1$ ampere.

$$\therefore\ L=4\pi n^2 lA\times 10^{-9}\ \text{henry}.$$

With an iron core L is μ times the value with an air core. Since μ may be of the order of 3000 the self-inductance may be so large as to reduce an alternating current to a very low value. Such an iron-cored coil is called a *choke*.

Demonstration of the action of a choke

The great importance of self-inductance in the case of alternating current may be demonstrated by the apparatus in Fig. 188.

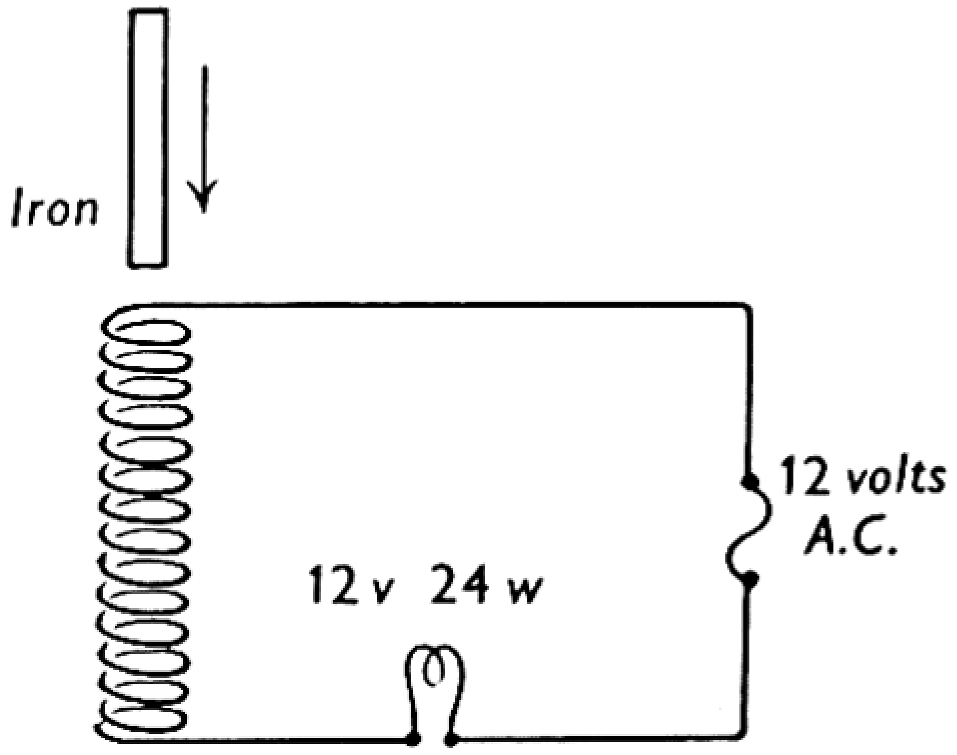


Fig. 188

The solenoid has sufficient self-inductance, even with an air core, to prevent the lamp from reaching its full brightness. But when an iron rod is inserted into the solenoid the lamp dims and eventually glows very feebly owing to the enormously increased self-inductance and consequent back E.M.F. in the solenoid.

The advantage of using a 12-volt, 24-watt lamp is that it requires the comparatively large current of 2 amperes to bring it to full brightness.

Use of standard inductances

The National Physical Laboratories, in addition to the concrete standards of resistance and E.M.F. already described in Chapter 9, construct standards of capacitance and inductance; the entire electromagnetic system is maintained by means of these four standards.

There are, in fact, two procedures for establishing all the units of the electromagnetic system, the one already described in which current and resistance are determined in terms of mechanical units, and one based on inductance and time.

By maintaining more than the necessary minimum number of standards, and using the two procedures, it is possible to make checks and to test the stability of the different standards.

An inductance can be measured in terms of resistance and time, or in terms of resistance and capacitance. Thus if the value of an inductance is known from its dimensions, it can be used to standardise resistance and capacitance. The methods employ a Wheatstone Bridge; for details of these methods, reference must be made to more advanced books.

The concrete standard of self-inductance consists of a coil of copper wire wound on a non-magnetic former; its absolute value can be calculated from its dimensions by a method similar to that already indicated. At the N.P.L. a concrete standard of mutual inductance consisting of two concentric coils is maintained as the chief standard of inductance.

Effect of inductance on growth and decay of a current

The current in a circuit containing inductance takes time to grow and to decay. In a highly inductive circuit, such as the field windings of a dynamo or motor, the current may take many seconds to grow.

The slow growth of the current in an inductive circuit can be demonstrated with the apparatus in Fig. 189. Two 2·5-volt pea lamps, B_1 and B_2, are connected to a 4-volt battery through a resistance and a high inductance respectively. When the switch S is depressed B_1 lights first and B_2 afterwards. The inductance L can conveniently take the form of the two coils of an electromagnet; it should, of course, be as high in value as possible.

The lamp B_2 can be replaced by a milliammeter, and the slow movement of its pointer, when the current is switched on, observed.

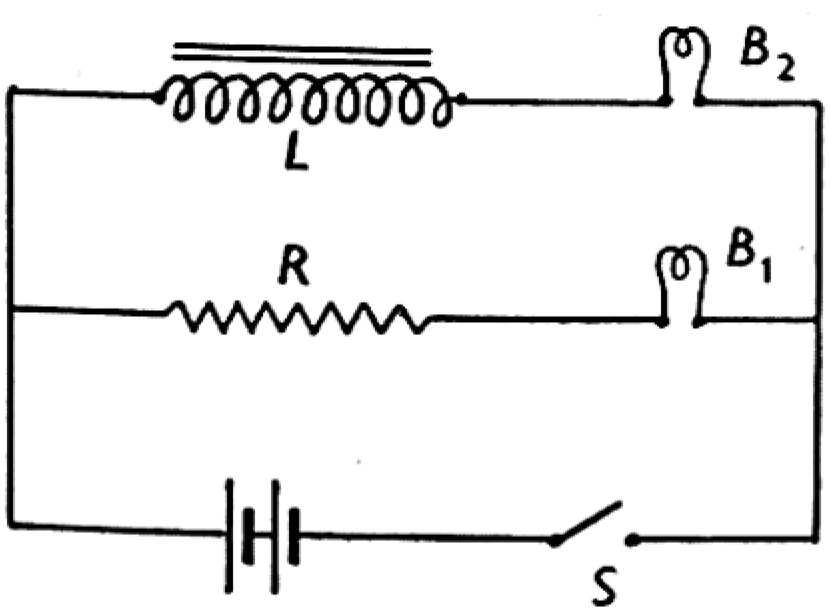


Fig. 189

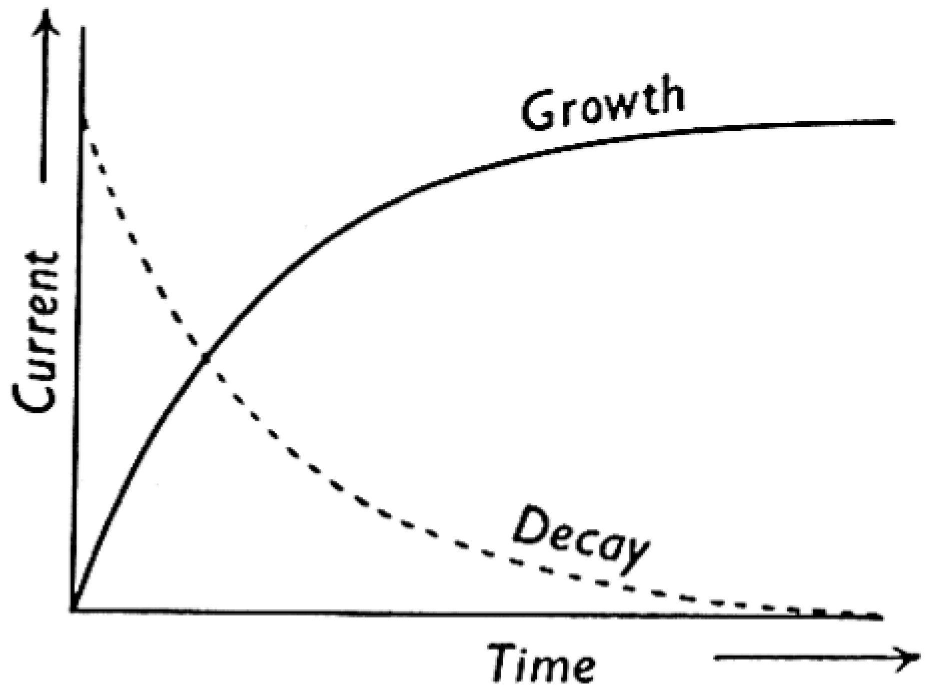


Fig. 190

The time taken by a current to grow can be calculated as follows. Suppose a coil of resistance R and inductance L is connected to a steady E.M.F., E. After times t and $t+\delta t$ let the currents be i and $i+\delta i$ respectively. The back E.M.F. due to self-induction is $L(di/dt)$. Hence, applying Ohm's law,

$$E-L\frac{di}{dt}=iR.$$

The solution of this equation enables the current at any time to be calculated. Rearranging terms and integrating,

$$\int\frac{di}{E/R-i}=\int\frac{R}{L}dt$$

$$\therefore\quad -\log_e\left(\frac{E}{R}-i\right)+K=\frac{R}{L}t,$$

where K is a constant of integration. If $i=0$ when $t=0$,

$$K=\log_e E/R,$$

$$\therefore \log_e \frac{E/R-i}{E/R}=-\frac{R}{L}t,$$

$$\frac{E/R-i}{E/R}=e^{-Rt/L},$$

$$i=\frac{E}{R}(1-e^{-Rt/L}).$$

The graph of this equation is the continuous curve in Fig. 190.

The maximum value of the current, when the current has ceased to grow and hence is unaffected by the self-inductance, is E/R. The ratio L/R is called the *time constant* of the circuit and is equal to the time in seconds for the current to grow from zero to 0·63 of its final value; when $t=L/R$, $1-e^{-Rt/L}=1-1/e=1-0{\cdot}37=0{\cdot}63$. Thus a coil of inductance 1 henry and of resistance 1 ohm requires 1 second to grow to 0·63 of its final value.

A current decays at the same rate that it grows, provided that the resistance of the circuit remains unchanged when the source of E.M.F. is removed. Applying Ohm's law when $E=0$,

$$0-L\frac{di}{dt}=iR.$$

Solving as before and assuming that when $t=0$, $i=E/R$,

$$i=\frac{E}{R}e^{-Rt/L}.$$

The dotted graph in Fig. 190 represents this equation. The time constant L/R equals the time taken for the current to decay to 0·37 of its maximum value.

When a circuit is broken by a switch the resistance of the air gap makes R very large and hence the time constant L/R is small. The decay is much more rapid than is shown in Fig. 190. It is dangerous to try to stop a current in a highly inductive circuit by means of a switch, since the induced E.M.F. may be of the order of thousands of volts, the pitted burns on many laboratory switches being evidence of the effect on a small scale.

The work done in establishing a current

When a current is growing in a coil of resistance R and inductance L

$$E-L\frac{di}{dt}=iR \quad \text{(see above)},$$

i.e.
$$E=iR+L\frac{di}{dt}.$$

Multiplying by $i\delta t$

$$\underset{\substack{\text{Energy supplied}\\ \text{in time } \delta t}}{Ei\delta t} = \underset{\substack{\text{Energy wasted}\\ \text{as heat}}}{i^2R\delta t} + \underset{\substack{\text{Work done}\\ \text{against back}\\ \text{E.M.F.}}}{Li\delta i.}$$

The energy liberated as heat is dissipated to the surroundings, but the work done against the back E.M.F. is recovered when the current is switched off and the induced E.M.F. aids the external source of E.M.F.

Suppose the current rises from zero to I. The work done in establishing the current against the back E.M.F.

$$=\int_0^I Li\,di=\tfrac{1}{2}LI^2.$$

This energy may be regarded as stored in the magnetic field or as the energy of motion of the electrons in the wire and of their electric field. The analogy with kinetic energy, $\frac{1}{2}mv^2$, is obvious, and self-inductance is often compared with inertia.

Discharge of a condenser through a non-inductive resistance

The discharge of a condenser through a non-inductive resistance obeys a law very similar to that for the decay of a current in an inductive circuit.

Suppose a condenser of capacitance C is charged to a potential V and is then connected to a resistance R as in Fig. 191. Let the charges on the positive plate at times t and $t+\delta t$ be q and $q-\delta q$ respectively,

$$\text{Current flowing in } R=-\frac{dq}{dt},$$

$$\text{P.D. across condenser}=\text{P.D. across } R,$$

i.e.
$$\frac{q}{C}=-R\frac{dq}{dt}.$$

When $t=0$, $q=CV$,

$$\therefore \int_{CV}^{q} \frac{dq}{q} = -\int_{0}^{t} \frac{dt}{CR},$$

$$\log_e \frac{q}{CV} = -\frac{t}{CR}$$

or
$$q = CVe^{-t/CR}.$$

The time constant is CR; thus a condenser of capacitance 1 microfarad, after connection to a resistance of 1 megohm, takes 1 second for its charge to fall to 0·37 of its original value.

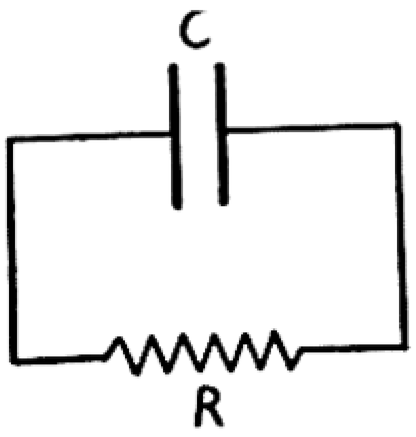


Fig. 191

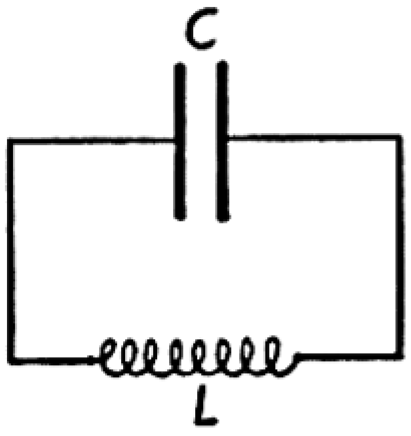


Fig. 192

Discharge of a condenser through an inductance

The discharge of a condenser through an inductance of low resistance is oscillatory. Suppose a charged condenser, of capacitance C, is connected to an inductance L of negligible resistance (Fig. 192). Let the charge on the condenser be q, and current i, at time t.

P.D. across the condenser = Back E.M.F. due to self-inductance,

$$\frac{q}{C} = L\frac{di}{dt}.$$

$$\text{But } i = -\frac{dq}{dt}.$$

$$\therefore \frac{q}{C} = -L\frac{d^2q}{dt^2},$$

$$\text{i.e. } \frac{d^2q}{dt^2} = -\frac{q}{LC}.$$

This is the usual equation for simple harmonic motion, and the periodic time, T, is given by
$$T = 2\pi\sqrt{(LC)}.$$

If the resistance of the circuit were, in fact, zero, the electrical oscillations would continue with undiminished amplitude. In practice the inductance has an appreciable resistance and hence the amplitudes of the oscillations decrease gradually. If the resistance of the circuit is too large no oscillations occur and the discharge of the condenser is unidirectional.

The circuit is of fundamental importance in radio and is discussed further on pp. 246, 298.

The induction coil

The induction coil, an instrument for producing a high voltage from a low one, is used frequently in the laboratory, and a similar instrument supplies the sparks for the explosion strokes in most motor-car engines. It is shown in Fig. 193. A primary coil, consisting of a few thick turns of wire, is wound on a soft-iron core, and round it is wound a secondary coil consisting of many turns of fine wire. The current in the primary is switched on and off rapidly by means of a hammer make and break similar to that of an electric bell. The iron armature is attracted by the magnetised iron core and, as a result, the circuit is broken at the point of contact with the adjustable screw; the attraction then ceases and the springy steel causes the armature to fly back. This is repeated several times each second. In the ignition system of a motor-car engine the circuit is made and broken by a rotating cam geared to the engine.

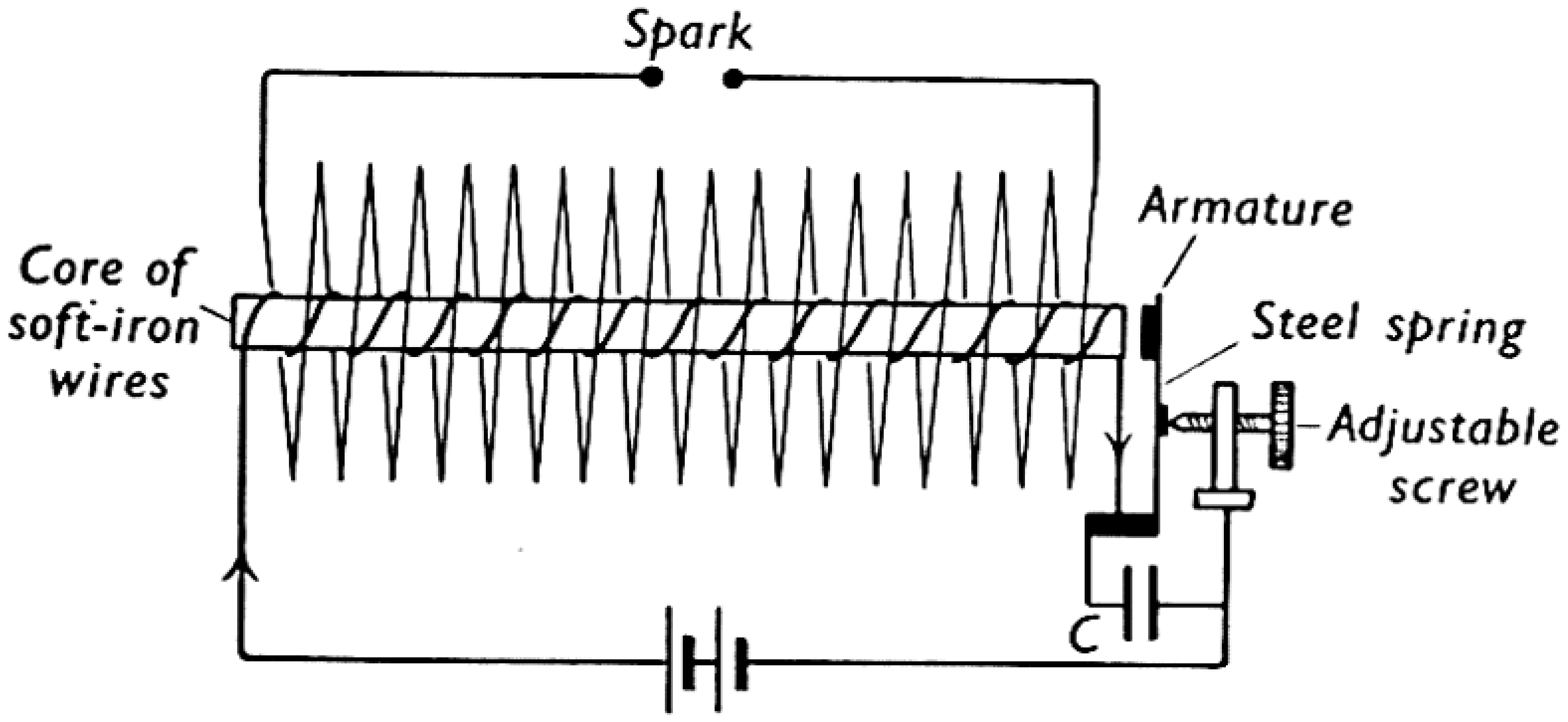
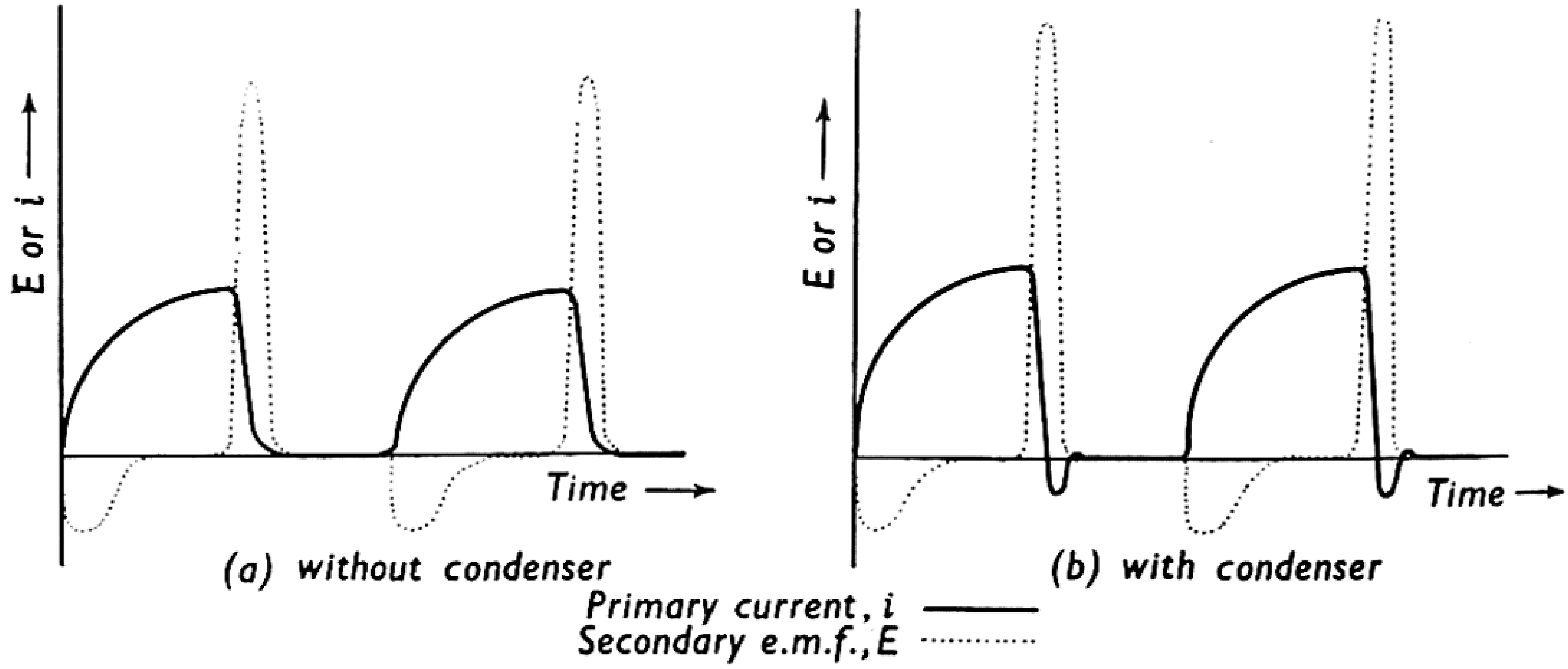


Fig. 193

The magnetic flux due to the primary links with the turns of the secondary and by its variation induces currents in the secondary which are in such a direction that they set up a magnetic flux to oppose the change. If there are 1000 times as many turns in the secondary as in the primary the induced E.M.F. in the secondary is approximately 1000 times the E.M.F. in the primary.

The magnitude of the secondary E.M.F. depends greatly on the rate at which the current changes in the primary. The self-inductance of the primary causes the current, at make, to grow comparatively slowly; at break the primary current dies away more rapidly. The E.M.F. due to self-induction causes arcing between the adjustable screw and the contact on the springy steel, thus prolonging the duration of the primary current. This arcing can be reduced, and the induction coil made to work more efficiently, by means of the condenser *C* which has a fairly large capacitance, usually about 1 microfarad. Instead of jumping across the gap in the form of a spark the primary current charges the condenser, and the latter then discharges through the primary coil, giving a small current in the opposite direction and causing an increase in the induced E.M.F. in the secondary. The effect of the condenser is illustrated in Fig. 194. The secondary E.M.F. is much greater at the break than at the make of the primary current, and hence the induction coil acts almost as a unidirectional source of P.D.



(a) without condenser (b) with condenser

Fig. 194

Currents are induced in the iron core as well as in the secondary coil; these unwanted currents, known as *eddy currents*, can be reduced by using a bundle of soft-iron wires, insulated from each other; the eddy currents tend to flow in circles round the axis of the core (as in the secondary coil), and the insulation between the iron wires blocks their flow.

The transformer

The transformer is an instrument for changing the voltage of an alternating current. In its simplest form it consists of an iron ring round which primary and secondary coils are wound (Fig. 195). The dotted lines represent the magnetic flux due to the primary current, and, since this links with the secondary, a current will be induced in the secondary whenever the flux changes. The magnetic flux due to an alternating current in the primary is continually changing, so that an alternating E.M.F. of the same frequency is induced in the secondary. If the number of turns in the secondary is greater than the number in the primary, the voltage is *stepped up*; if it is less, the voltage is *stepped down*. Assuming that the flux leakage is negligible,

$$\frac{\text{Voltage across secondary}}{\text{Voltage across primary}}=\frac{\text{Number of turns in secondary}}{\text{Number of turns in primary}}.$$

The efficiency of transformers may be as high as 99%. Hence it can be assumed, with reasonable accuracy, that

$$\text{Power in primary}=\text{Power in secondary}.$$

Thus if the voltage in the secondary is n times that in the primary, the current in the secondary is $1/n$ times that in the primary.

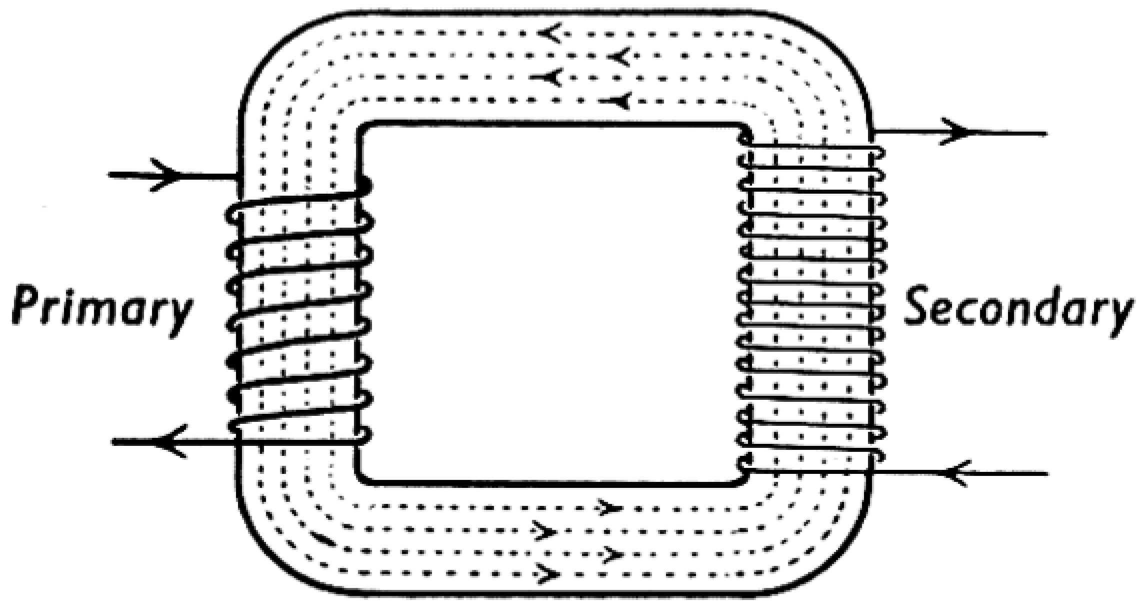


Fig. 195

The current in the secondary sets up a magnetic flux of its own in a direction opposite to that of the primary when the primary current is increasing, and in the same direction when the primary current is decreasing. Hence an increase in the current drawn from the secondary causes a reduction in the back E.M.F. due to the self-inductance of the primary with a consequent rise in the primary current.

Eddy currents in the iron core are reduced by constructing the latter of thin, soft-iron sheets, called laminations, insulated from each other, and of shape similar to the cross-section of the core shown in Fig. 195. The eddy currents tend to circulate in the same direction as the currents in the secondary, and hence are blocked by the insulation between the laminations.

The electric motor

The principle of the electric motor is illustrated in Fig. 196. A current is passed through a coil free to rotate between the two poles of a magnet. Applying Fleming's left-hand rule it will be seen that the coil in Fig. 196

tends to rotate in an anti-clockwise direction; the left-hand side of the coil is urged downwards and the right-hand side upwards. The rotation is made continuous by reversing the current each time the coil is vertical. This is done by means of a commutator, consisting of a metal ring split into two insulated halves connected to the ends of the coil, against which press carbon brushes. The current is led in and out through the brushes, and the latter change contact from one half-ring to the other half-ring, each time the coil is vertical.

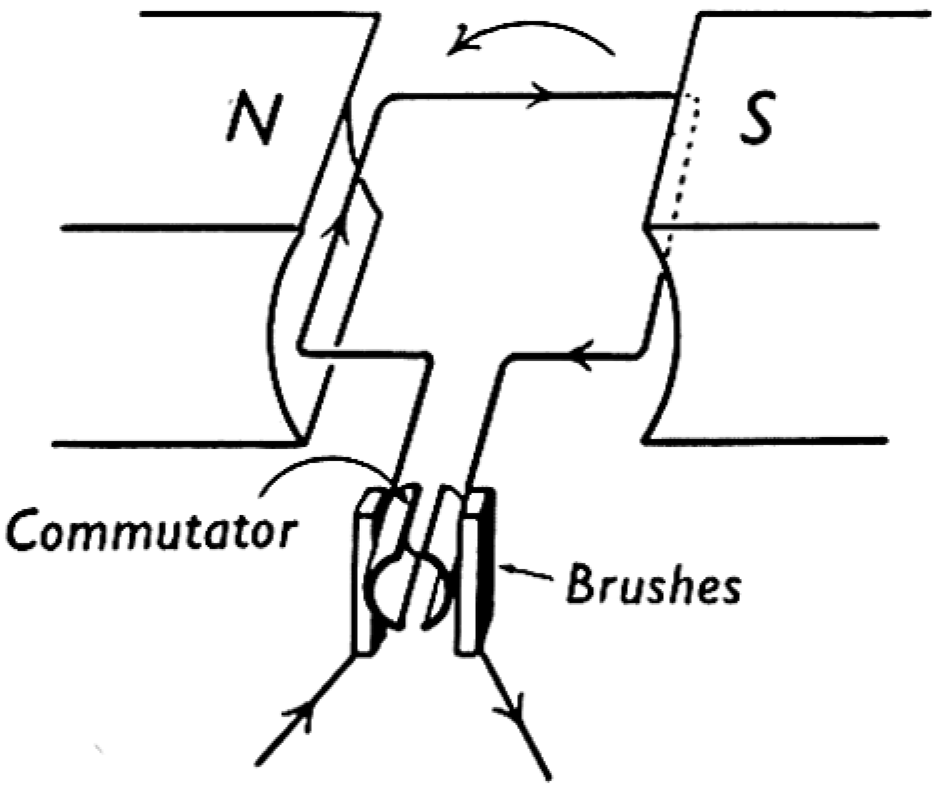


Fig. 196

A back E.M.F. is induced in the coil because the quantity of magnetic flux threading through it changes as it rotates. The motor, in this respect, behaves as a dynamo.

Let V = voltage applied across the coil,
i = current through the coil,
R = resistance of the coil,
E_b = back E.M.F. in coil.

Then $$i=\frac{V-E_b}{R}.$$

In a practical motor for which $V=100$ volts, the back E.M.F., E_b, may be of the order of 95 volts. Multiplying the above equation by i and rearranging the terms,

$$\underset{\text{Power supplied}}{Vi} = \underset{\substack{\text{Power dissipated}\\ \text{as heat}}}{i^2R} + \underset{\substack{\text{Power to rotate}\\ \text{coil}}}{E_b i}.$$

The work done by the source of supply in driving the current against the back E.M.F. is equal to the work done in rotating the coil.

If the rate of rotation of the coil is slowed down by making it do mechanical work (say against a brake), the back E.M.F. is reduced because the coil is cutting the lines of force more slowly, and hence the current through the coil increases. Thus an electric motor automatically regulates the amount of current and power required. A practical motor starting from rest must be connected in series with a resistance which is gradually reduced as the speed of the motor increases; otherwise owing to the absence of a back E.M.F., a very large, initial current will flow, and perhaps burn out the windings.

Practical motors

The torque or turning moment produced by the single coil in Fig. 196 is small and it fluctuates from a maximum when the coil is horizontal to zero when the coil is vertical. Hence to increase the torque in a practical motor, several coils are wound in slots in a soft-iron armature. Fig. 197 illustrates a drum-wound armature consisting of eight thick copper conductors interconnected as shown. The currents in the four conductors on the right-hand side are all passing into the paper, and those on the left-hand side out of the paper. Thus all tend to rotate in a clockwise direction. Four segments are required on the commutator.

The air gap between the soft-iron armature and the poles of the field magnet is kept as small as possible in order to make the magnetic flux as large as possible. Eddy currents are reduced by constructing the armature of laminations.

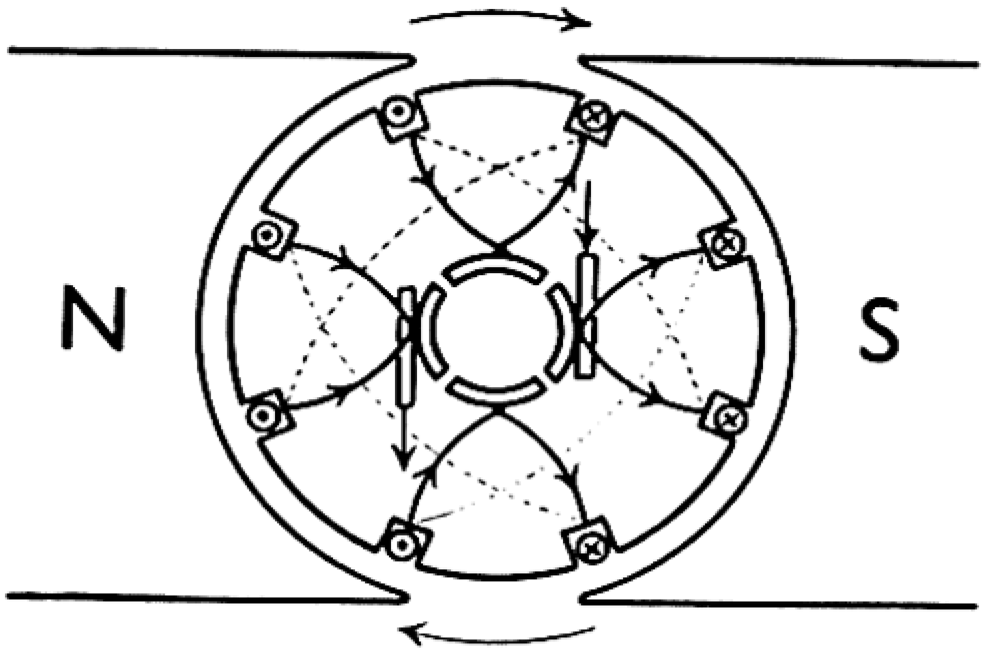
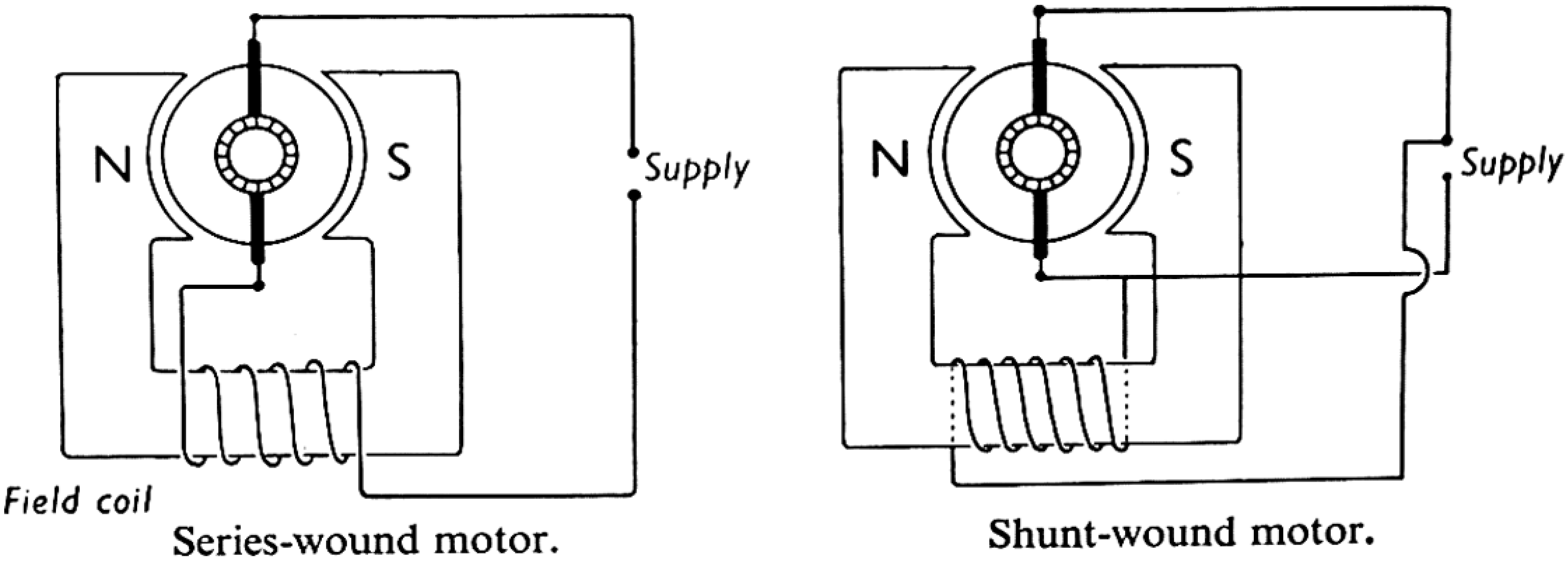


Fig. 197

The field magnet is usually an electromagnet and its coils may be connected either in series or in parallel with the armature coil (Fig. 198). *Series-wound* motors produce a large torque at starting, but the torque falls off with the load; parallel- or *shunt-wound* motors are not such good starters, but they are steadier with increasing load (see Fig. 199).

The torque of a motor is proportional to the product of the armature current and the magnetic flux, and hence to the product of the armature and field currents. The series motor is a good starter, because when there is a small back E.M.F., a large current flows in both the armature and field coils. In the case of the shunt-wound motor the current in the field coil is unaffected by the back E.M.F. in the armature coil, and only the armature current is large at starting.

Now consider the effect of increasing the load on the two motors. Each motor slows down and the back E.M.F. in the armature coil decreases, thus causing an increase in the armature current. In the case of the shunt-wound motor the current in the field coil is unaffected and the motor is able to provide the necessary increased torque at the decreased speed. But in the case of the series-wound motor the current in the field coils is also increased, causing a larger back E.M.F. in the armature coil, and hence reducing the current and causing a further drop in speed.



Series-wound motor. Shunt-wound motor.

Fig. 198

In order to obtain the advantages of both series- and shunt-wound motors, compound winding is employed; part of the field coil is in series and part in parallel with the armature coil.

The direction of rotation of a motor can be changed by reversing the current in either the armature or the field coil, but not in both. Thus a D.C. motor will run on A.C., but inefficiently, owing to the large self-induction of its coils.

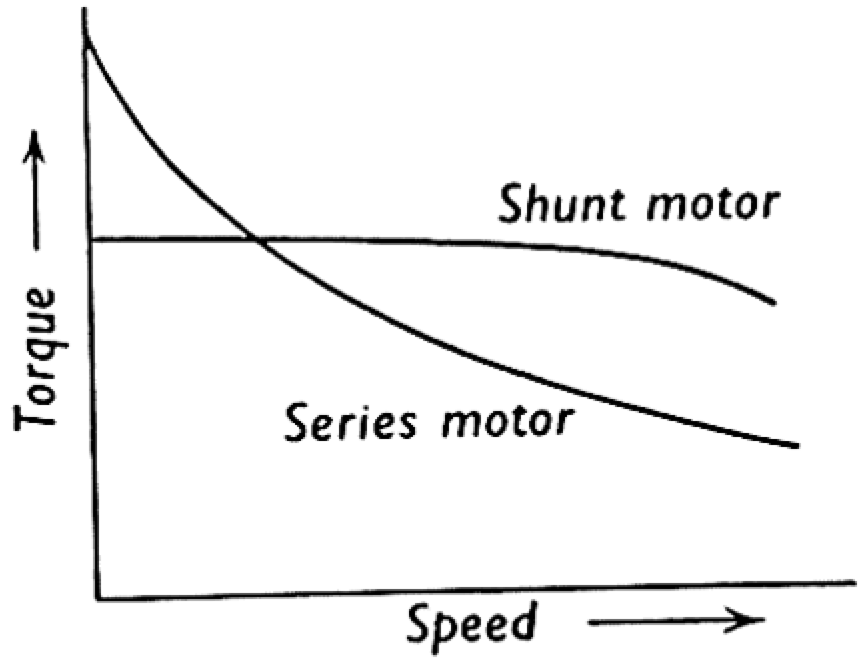


Fig. 199

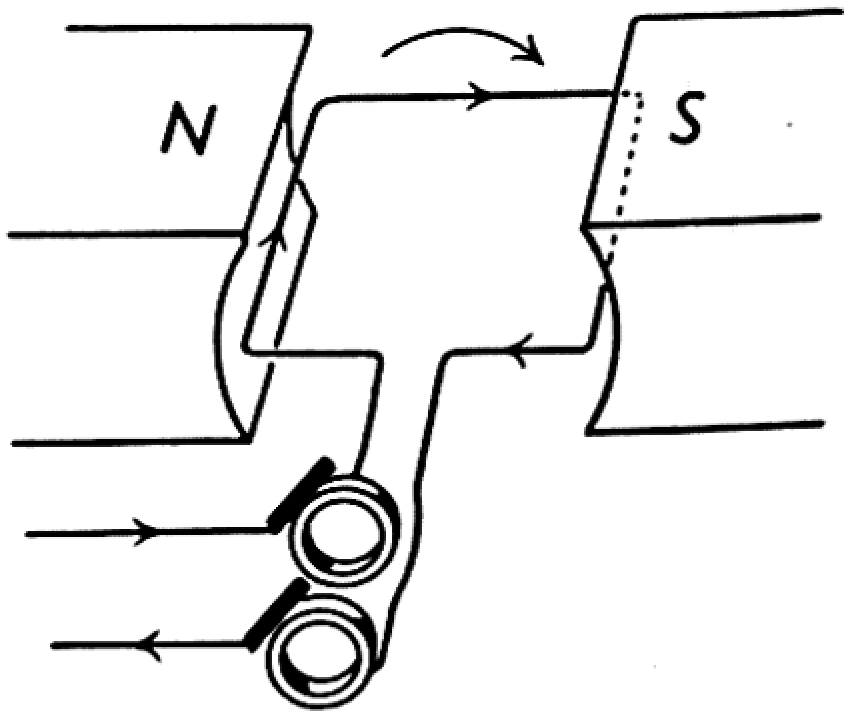


Fig. 200

The A.C. dynamo

If a coil is rotated by some external source of mechanical power between the two poles of a magnet there will be generated in it an induced current which can be led away by slip rings. The direction of the induced current in Fig. 200 can be verified by Fleming's right-hand rule. The current is alternating as shown in Fig. 201, being a maximum when the coil is horizontal and zero when it is vertical. Consider the left-hand side of the coil; it is cutting the lines of force most rapidly when the coil is horizontal, and it starts to cut the lines of force downwards instead of upwards as the coil passes the vertical position. The letters *V* and *H* in Fig. 201 represent the vertical and horizontal positions of the coil during one complete revolution or cycle. The number of cycles per second is called the *frequency* of the A.C.

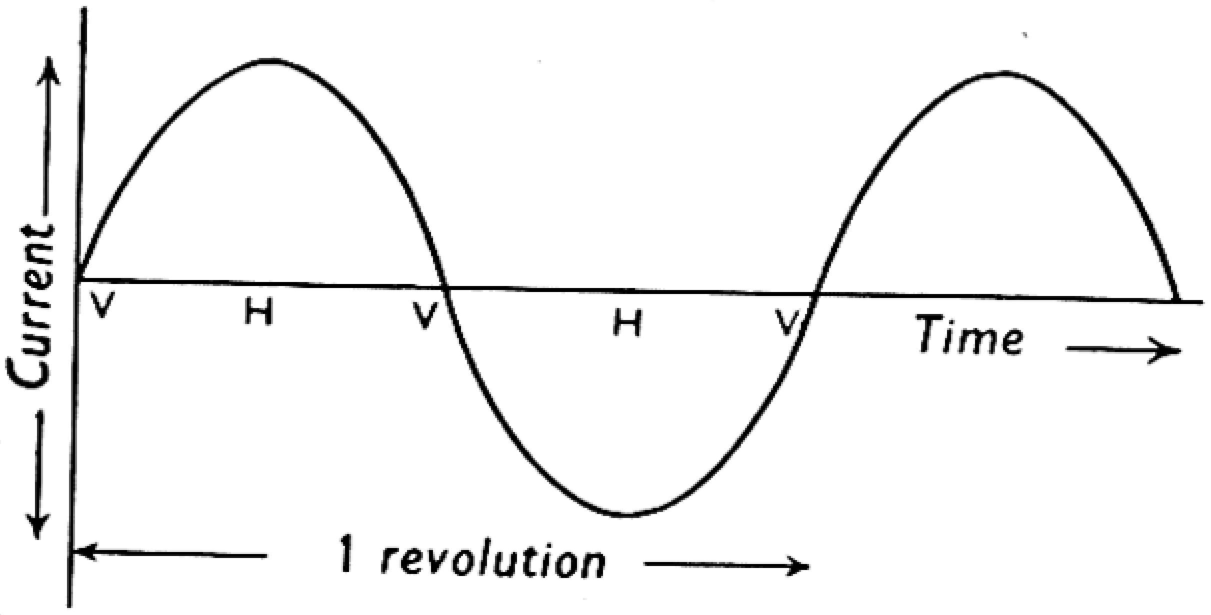


Fig. 201

The induced current in the coil will tend to make it behave like a motor and to rotate in the opposite direction. The work which is done against the reverse motor effect, as it is called, is equal to the energy of the induced current. If the coil is connected to a high external resistance the induced current will be small and the reverse motor effect will likewise be small. If, however, the coil delivers a large current the reverse motor effect will be large and the coil will be harder to turn than before.

The magnitude of the induced E.M.F. in the coil depends on the rate at which it is rotated and also on the density of the magnetic flux. Thus, in order to keep the voltage and the frequency constant a dynamo must be run at a constant speed. When it is called upon to deliver a larger current the torque provided, say, by a steam engine must be increased by a greater consumption of steam.

The current generated by a single coil is known as single-phase A.C. The current in the Grid system is three-phase A.C. which, in effect, consists of three separate A.C.'s differing in phase by 120°, which are generated in three separate coils, making angles of 120° with each other and rotating together on the same iron armature core. In large turbo-alternators the coils in which the current is generated are at rest (the stator) while the poles (the rotor) are rotated.

The D.C. *dynamo*

The current generated in the coil of a dynamo can be rectified by the use of a commutator as in Fig. 196 instead of slip rings. The current delivered to the external circuit will then be similar to Fig. 202. By the use of a drum-wound armature similar to Fig. 197, a current which fluctuates only slightly, as in Fig. 203, can be obtained.

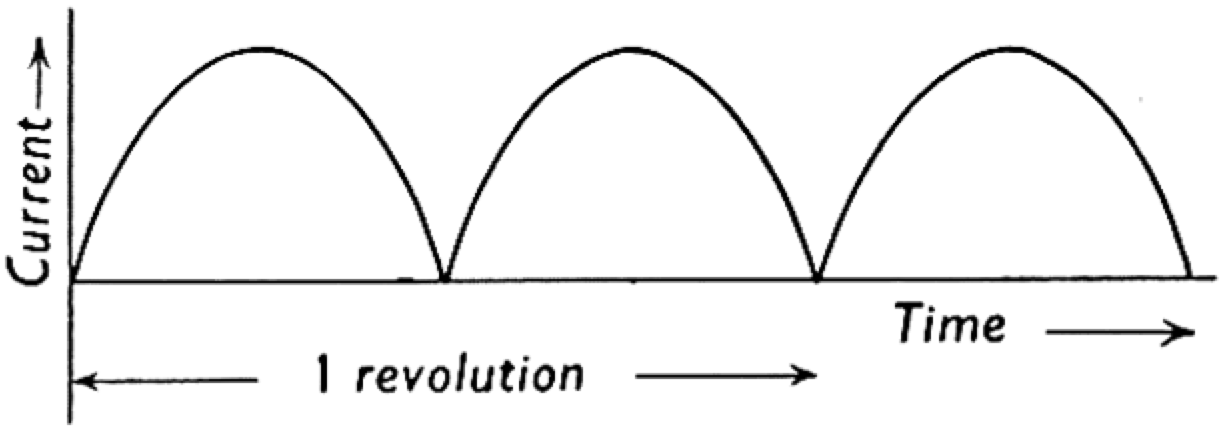


Fig. 202

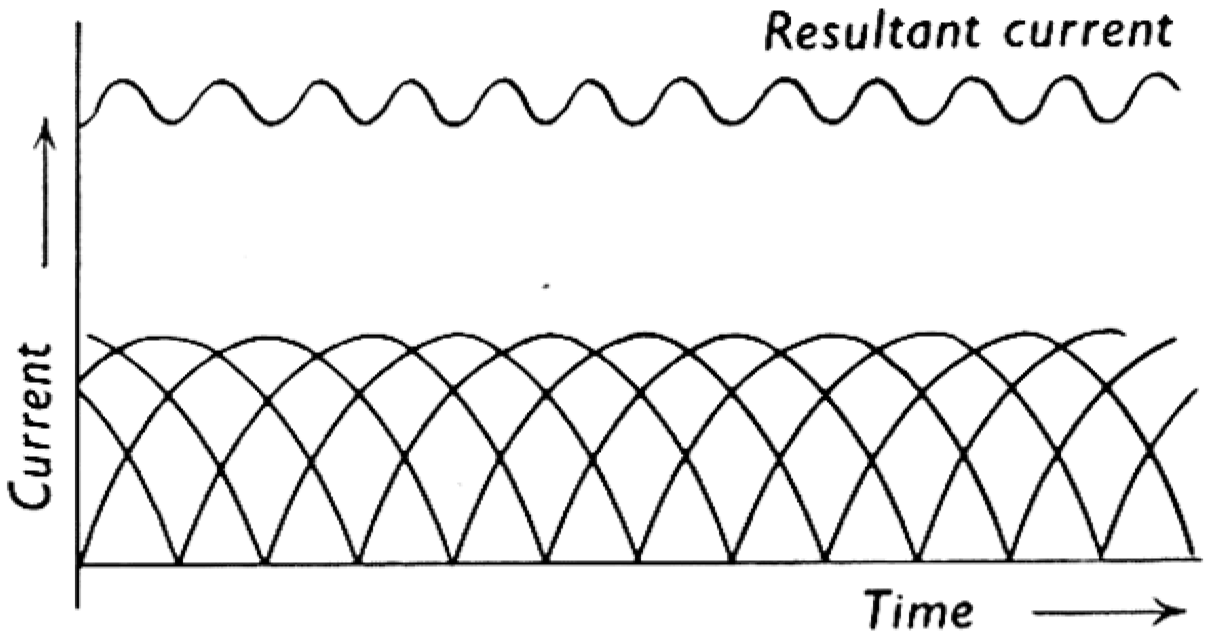


Fig. 203

The armature and field coils of a D.C. dynamo can be connected in series or in parallel as in the case of a motor. Indeed Fig. 198 represents D.C. dynamos as well as motors. The voltage generated by a series-wound dynamo increases with the current it supplies while the voltage of a shunt-wound dynamo decreases (see Fig. 204). It is possible, by incorporating the two windings in a compound-wound dynamo, to keep the voltage approximately constant and independent of the load.

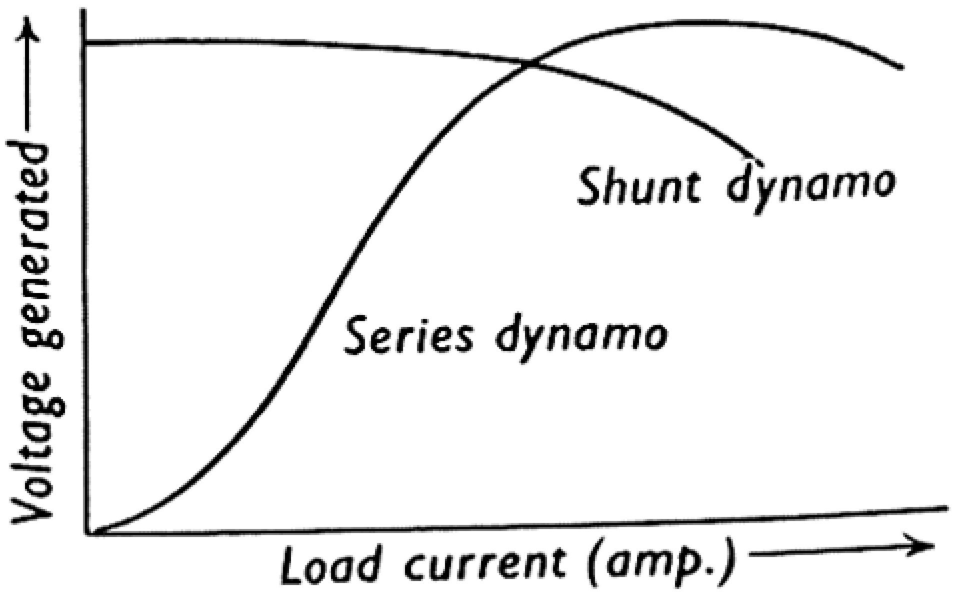


Fig. 204

The voltage generated by a dynamo, as has already been stated, is proportional to the rate at which the armature coil cuts the magnetic flux. Since the speed of rotation of the armature coil is kept constant the voltage depends only on the current in the field coils. In the case of a series-wound dynamo an increase in the load (or current supplied) means an increase in the magnetic flux since the whole load current must pass through the field coil. Hence the voltage increases. In the case of a shunt-wound dynamo the field coil is in parallel with the external circuit, the E.M.F. being generated, of course, in the armature coil. A decrease in the resistance of the external circuit (i.e. an increase in the load) will cause this circuit to take a larger proportion of the current. Hence the voltage generated by a shunt-wound dynamo will fall when the load is increased owing to a decrease in the current in the field coil.

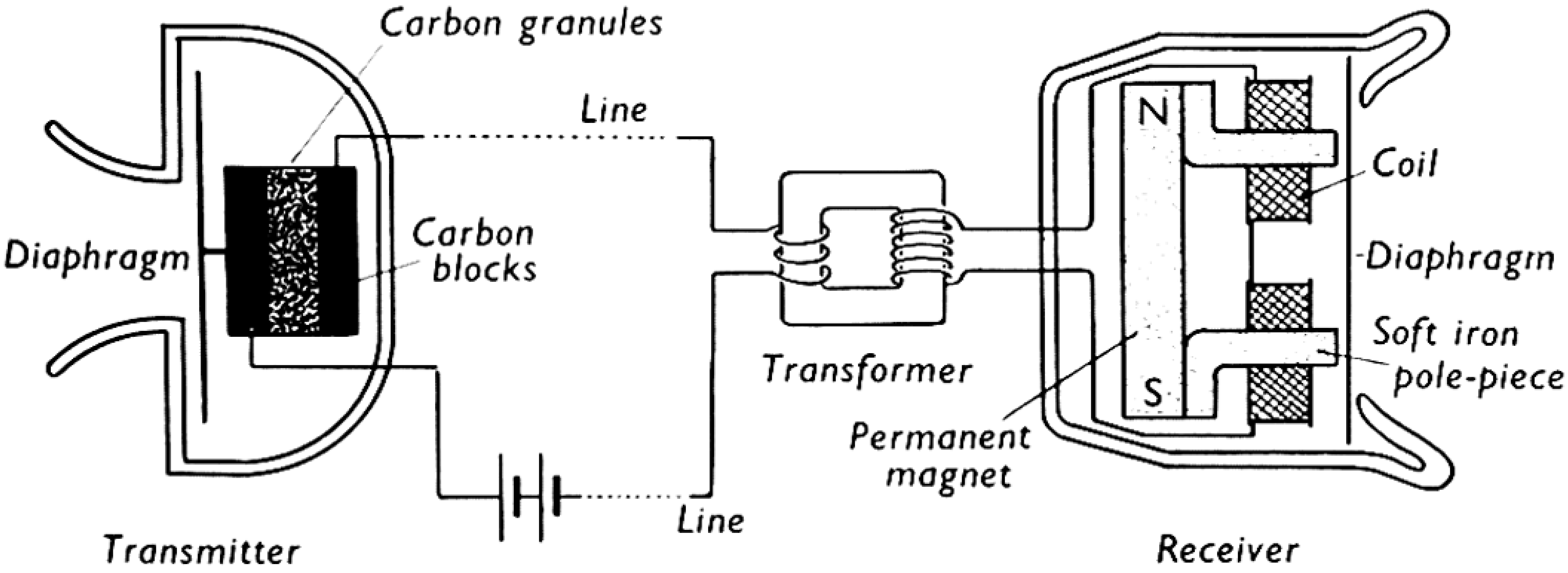


Fig. 205

The telephone transmitter and receiver

Fig. 205 represents a telephone transmitter connected to a receiver. The transmitter contains carbon granules between two carbon blocks, one of which is rigidly attached to a diaphragm. Sound waves falling on the diaphragm cause it to vibrate and hence to subject the system to changes of pressure. Compression of the packing of the granules results in a comparatively large reduction in their electrical resistance. Thus the current in the line fluctuates in sympathy with the pressure changes in the sound waves.

The receiver consists of a permanent magnet, having two soft-iron pole-pieces wound with coils, and a steel diaphragm. When the fluctuating current set up by the transmitter passes through the coils of the receiver the force of attraction on the diaphragm fluctuates and hence the diaphragm vibrates, giving out sound waves corresponding to those which fall upon the transmitter. The purpose of the permanent magnet in the receiver is to maintain a continuous attraction on the diaphragm and thus to enable the diaphragm to respond more vigorously to changing currents in the coils. Moreover, the absence of a permanent magnet would result in the response being an octave above the transmitted signal.

The resistance of the transmitter is low, of the order of an ohm, whereas that of the receiver is of the order of 100 ohms, since the coils of the latter are wound with many turns of fine wire. The transformer passes the fluctuations from the line to the receiver while, at the same time, preventing the high resistance of the receiver from reducing the current in the line.

QUESTIONS

1. Deduce the relations between the quantity of electricity flowing through a circuit and the flux change producing it.

A flat coil of 150 turns, each of area 300 sq.cm., and of total resistance 50 ohms, is connected to a circuit whose resistance is 40 ohms. Starting with its plane horizontal, the coil is rotated quickly through a half-turn about a diametral axis pointing along the magnetic meridian. If the quantity of electricity which then flows round the circuit is 4 microcoulombs, find the intensity of the vertical component of the earth's magnetic field. (1 volt $= 10^8$ e.m.u.) (N.)

2. A metal-framed casement window, of length 120 cm. and breadth 50 cm., is in a wall lying parallel to the magnetic meridian. Regarding the window frame as a conducting circuit of resistance 0·01 ohm, calculate the quantity of electricity which will pass round the frame as it swings open through 90°. ($H = 0{\cdot}18$ oersted.) (B.)

3. A coil of 30 turns of wire, each of area 10 sq.cm., is placed with its plane at right angles to a magnetic field of 1000 gauss. When the coil is suddenly withdrawn from the field a galvanometer in series with the coil indicates that 10^{-5} coulomb passes round the circuit. What is the combined resistance of the coil and the galvanometer? (C.)

4. A small search coil with 8 turns of mean area 1·5 sq.cm. is placed between the poles of an electromagnet with its plane perpendicular to the magnetic field. It is connected to a ballistic galvanometer, the total resistance of the circuit being 1000 ohms. When the coil is suddenly removed to a place where the magnetic field is negligibly small, the throw of the galvanometer is 23 divisions. When a condenser of capacity 1 microfarad charged to a potential difference of 1 volt is discharged through the galvanometer, the throw is 25 divisions. Calculate the strength of the magnetic field between the poles of the magnet. Damping may be neglected. (1 volt $= 10^8$ e.m.u.) (C. S.)

5. Two galvanometers, *A* and *B*, available for use with a search coil of resistance 5 ohms, have the following constants:

	Resistance (ohms)	Sensitivity (divisions per microcoulomb)
A	10	70
B	500	350

Which of these will be the more sensitive when used with the coil and to what extent? (N.)

6. Describe in detail how you would measure the magnetic moment of a small horseshoe magnet, showing how to calculate the result.

One pole of a magnet of pole strength 70 c.g.s. units is passed through a small coil of 100 turns of wire connected in a circuit of total resistance 5 ohms. What quantity of electricity in microcoulombs will flow as an induced current in the circuit? (1 volt $= 10^8$ c.g.s. units.) (O.)

7. A circular coil of wire consisting of 30 turns, each of radius 5 cm., is rotated about a vertical axis in its plane in the earth's field four times per second. The coil is connected to a circuit of total resistance 2 ohms. If the horizontal component of the earth's magnetic field is 0·18 oersted, calculate the average current in the coil irrespective of direction.

8. A long solenoid of radius 3 cm. has 15 turns per cm. length and a current of 4 amperes flows through it. A secondary coil of 1000 turns and resistance 60 ohms is wound on the middle part of the solenoid. Find how much electric charge will circulate in the secondary coil if the 4-ampere current is reversed. (L.)

9. A ballistic galvanometer is connected in series with a coil of 50 turns and an earth inductor, which has 200 turns of mean area 1000 cm.^{2} The coil is wound over a long solenoid of radius 10 cm. having 10 turns per cm. The maximum deflection obtainable by a 180° rotation of the earth inductor is 10 divisions. What current, when reversed in the solenoid, will give the same deflection? ($H = 0{\cdot}18$ oersted, dip$=67°$.) (C.)

10. A long solenoid, having 10 turns per cm. length, has an iron core of area 4 sq.cm. and permeability 1000, and carries a current of 2 amperes. Calculate the quantity of electricity induced in a secondary, of 100 turns and total resistance 2 ohms, wound round the outside of the solenoid, when the iron core is withdrawn.

11. State Faraday's Laws of Electromagnetic Induction.

O is the centre of a horizontal copper disc and A a point such that OA is vertical. The disc is free to rotate about OA. A magnetic needle is pivoted at A so that it is free to turn in a horizontal plane. Explain what is observed when the disc is rapidly rotated. How does the nature of the effect change when the material of the disc is iron? (C. S.)

12. The primary of a transformer is connected through a fuse to a source of supply, the secondary being on open circuit. If the source is D.C. the fuse is blown, but not if the source is A.C. Why is this? (C.)

13. Describe the construction of a transformer and of an induction coil, and explain the essential differences between them.

What is the function of the condenser in the induction coil? (O. S.)

14. Describe the construction and mode of action of an A.C. transformer.

A power station has an output of 5000 kilowatts at 400 volts. The power is transformed up to 120,000 volts for transmission along a line of total resistance 25 ohms, and is then transformed down to 200 volts. If each transformer has an efficiency of 96%, what power output and current will be available in the 200-volt circuit? (O. & C.)

15. The coefficient of mutual induction of an induction coil is 30 henries. A current of 1·5 amperes in the primary is extinguished in 0·002 second. Calculate the E.M.F. induced in the secondary coil.

16. What is meant by the coefficient of mutual induction of two coils?

A secondary of n turns is wound over the central section of a long solenoid of radius r having N turns per cm. Calculate the coefficient of mutual induction between the solenoid and the secondary.

Show that the coefficient of mutual induction of two coils having their axes in the same straight line necessarily decreases as the axial separation of the coils increases, and trace the connection between this result and the fact that if a current is passed through the two coils in series a force of attraction acts between them. (C. S.)

17. Explain what is meant by self-induction.

A circuit is made up of a battery, a switch, and the coils of an electromagnet arranged in series. The switch is open at first, it is then closed, allowed to remain closed for 1 minute, and then opened. Show by means of a sketch graph the relation between current and time, and discuss the graph. Point out the source from which the energy for the spark at break is drawn. (N.)

18. When a bundle of iron wires is placed in a coil of wire carrying A.C. a rise in temperature of the iron is observed. Explain the causes of this heating and show how the magnitude of the effect depends on (*a*) the magnetic properties of the iron, (*b*) the electrical conductivity of the iron, (*c*) the frequency of the A.C., (*d*) the diameter of the iron wires forming the bundle. (C. S.)

19. What factors determine the self-inductance of a coil?

Calculate the self-inductance of a coil of 100 turns if a current of 3 amperes gives rise to a magnetic flux of 6000 maxwells through the coil.

20. A coil having a self-inductance of 2 henries has a current of 0·5 ampere flowing through it. What was the energy required to establish the current?

21. Prove that the energy set up in the field surrounding a circuit of self-induction L is $\frac{1}{2}Lc^2$, where c is the current in the circuit.

A light spiral spring of copper is suspended from an insulated support, and at its lower end is hung a small weight. A current c is passed through the coil. Discuss and explain the energy changes which take place in the parts of the system. (O. S.)

22. Give a brief explanation of each of the following and state the important general principle illustrated by them:

(*a*) The oscillations of a moving-coil ballistic galvanometer can be stopped very quickly by short-circuiting the coil.

(*b*) When a plate of copper or aluminium is pushed into a strong magnetic field between the poles of a powerful electromagnet, considerable resistance to the motion is felt, but no such effect is observed with a sheet of glass.

(*c*) A thick copper ring can be made to jump off the pole of an electromagnet if a large A.C. is used. (C. S.)

23. A coil of self-inductance 10 henries, in series with a wire of negligible resistance which fuses at a current of 10 amperes, is connected to a 100-volt D.C. mains supply. How long after making the connection will the wire fuse?

Describe the changes of current and voltage before and after the fusing. (O. S.)

24. A condenser of capacity 1 microfarad is charged to 200 volts. If its leakage resistance is 10^8 ohms, in what time will the P.D. fall to 100 volts when the condenser is isolated? (C. S.)

25. An electrolytic condenser has a capacity of 200 microfarads. If the leakage resistance is 10^6 ohms, how long will it take to lose half its charge? (O. S.)

26. A condenser of capacity C is charged to a difference of potential V and its plates are then connected together through a non-inductive resistance, R. Find an expression for the quantity of electricity remaining in the condenser after a time t.

Indicate in a general way what difference it would have made to the results if an inductive resistance had been used and mention a practical application. (C. S.)

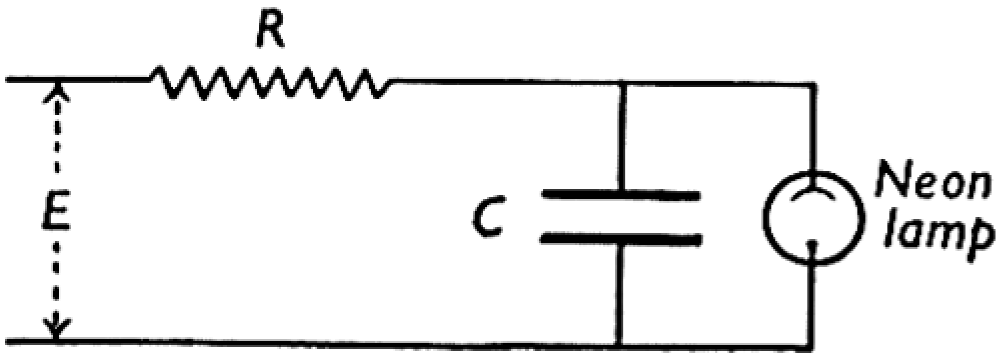


Fig. 206

27. A steady potential difference E is suddenly applied to a capacity C in series with a resistance R. Obtain an expression for the P.D. between the condenser plates at a time t after the instant of application.

In a neon lamp a discharge starts when the applied P.D. reaches a value V_1; the discharge is not extinguished until the P.D. falls to V_2. If such a lamp is connected as in Fig. 206 find an expression for the period of the electrical oscillations which occur when $E = V_1$. You may neglect the actual time occupied by the discharge. (C. S.)

28. Describe a simple A.C. generator and explain the principles on which its action depends.

What are the chief sources of energy loss in a practical A.C. generator and how are they reduced to a minimum? (C.)

29. Describe and explain the working of a simple permanent magnet dynamo such as is used for bicycle lighting. How does the current through the lamp change as the speed increases? (O. S.)

30. A brass disc is mounted on an axis parallel to a magnetic field H. A current i flows through the disc from the axis to a contact on its periphery. Calculate the torque and the power supplied if the disc is running as an electric motor at a speed of n revolutions per second. (O. S.)

31. Explain the construction and action of a simple form of series-wound D.C. dynamo.

Account for the fact that the machine will run as a motor if supplied with D.C. of suitable voltage.

Explain what would happen if you connected the machine to a source of A.C. of the same voltage. (O.)

32. State the laws of electromagnetic induction and explain how these apply to the action of an electric motor. What are the special features of series and shunt-wound motors?

A shunt motor running on 200-volt supply has an armature resistance of 0·1 ohm. When running unloaded at 250 r.p.m. the armature current is 10 amperes. If with a full load the current is 100 amperes, calculate the full-load speed, assuming the field remains constant. (C. S.)

33. The resistance of a certain D.C. motor is 0·25 ohm. When running with no load on a 100-volt supply it takes 1·5 amperes, whilst on full load the current rises to 35 amperes. What is the back E.M.F. in each case and the electrical efficiency in the second case? (D.)

34. What do you understand by the back E.M.F. of an electric motor?

A shunt-wound D.C. electric motor takes a current of 100 amperes from 200-volt mains. The shunt-field coils have a resistance of 40 ohms; the armature has a resistance of 0·5 ohm. Find (*a*) the back E.M.F. and hence (*b*) the electrical energy converted into mechanical work per second. Verify that (*b*) is the difference between the total power supplied and that turned into heat. (C.)

35. A shunt-wound D.C. motor has a variable resistance connected in series with its field winding, while the armature is connected directly to the supply mains. It is found that, as the resistance in the field circuit is *increased*, the speed of the motor also *increases*. Explain this phenomenon and suggest a reason why this might not be a good method of controlling the speed of the motor over a wide range of values. (C. S.)

CHAPTER 13

ALTERNATING CURRENT

A.C. in rotating coil

Alternating current is generated when a coil rotates in magnetic flux.

Consider a plane coil with N turns and of area A which is rotated uniformly at a speed of f revolutions per second about an axis at right angles to a uniform magnetic field* of flux density B. Suppose the plane of the coil is initially perpendicular to the field, represented by PQ in Fig. 207, and that after time t it has turned through an angle θ and reached $P'Q'$.

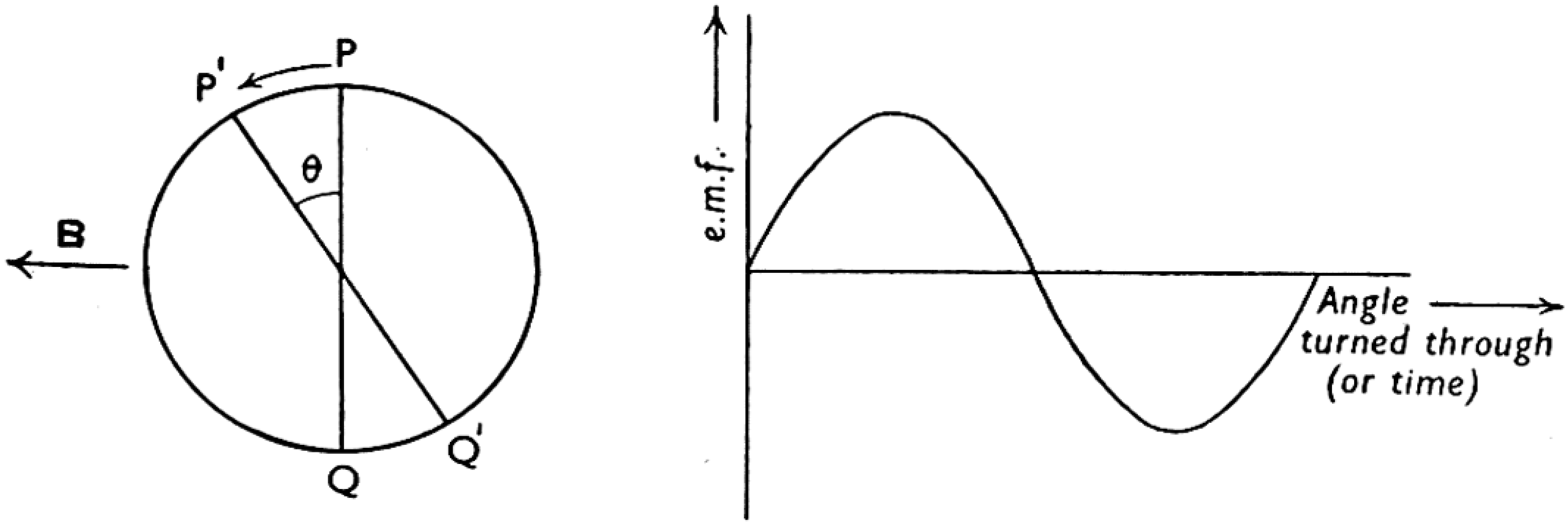


Fig. 207

Magnetic flux linkage through the coil in position $P'Q' = BAN \cos\theta$.

$$\therefore \text{ Induced E.M.F. in coil, } E = -\frac{d(BAN\cos\theta)}{dt}$$

$$= BAN \sin\theta \frac{d\theta}{dt} \text{ e.m.u.}$$

But $\theta = 2\pi ft$ (since the coil turns through $2\pi f$ radians in 1 second).

$$\therefore \ \frac{d\theta}{dt} = 2\pi f.$$

$$\therefore \ E = BAN\, 2\pi f \sin 2\pi ft.$$

---

* Unless the coil is rotating in a magnetic medium we could use H, the magnetic field strength, instead of B since $H = B$.

If E_0 is the maximum or peak value of E, i.e. the value when the coil is parallel to the field and $\sin 2\pi ft = 1$,

$$E_0 = BAN\, 2\pi f.$$

$$\therefore\ E = \mathrm{E}_0 \sin 2\pi ft.$$

Also, using similar nomenclature for current,

$$i = i_0 \sin 2\pi ft.$$

Thus the induced E.M.F. (and also the current) may be represented by a sine curve as in Fig. 207. The curve, known as the *wave form*, is only sinusoidal if the field and speed of rotation are uniform. In practice, the A.C. generated by turbo-alternators is very nearly sinusoidal. One complete wave, corresponding to one revolution of the coil, is called a cycle. The number of cycles per second, f, is called the *frequency* of the A.C. The angle measured along the horizontal axis in Fig. 207 is called the *phase angle*.

Root mean square current

To measure an A.C. an instrument must be selected whose indication is independent of the direction of the current, for example, a moving-iron or hot-wire instrument. The mean value of an A.C. is zero; what then does the instrument measure?

The main purpose of A.C. is to transfer energy from one point to another. The *effective* or *virtual* value of an A.C. is taken as *the value of a D.C. which produces the same amount of heat in the same time.*

Since the heat produced is proportional to the square of the current (heat $= i^2Rt/J$), the effective value of an A.C. is the square root of the mean value of i^2; this can be shown to be equal to $i_0/\sqrt{2}$ as follows, where i_0 is the peak value:

$$i^2 = i_0^2 \sin^2 2\pi ft$$
$$= \tfrac{1}{2} i_0^2 (1 - \cos 4\pi ft).$$

But the average value of $\cos 4\pi ft$, for any whole number of cycles, is zero.

$\therefore$ Mean value of $i^2 = \frac{1}{2} i_0^2$.

$\therefore$ Root mean square value of $i = i_0/\sqrt{2}$.

Thus a current with a peak value of 10 amperes has a r.m.s. value of 7·07 amperes.

Similarly, the virtual, effective or r.m.s. value of E is $E_0/\sqrt{2}$. A.C. supplied from the Grid at 230 volts has an effective P.D. of 230 volts; the peak value is $230 \times \sqrt{2} = 325$ volts.

Inductance in an A.C. circuit

On p. 219 it was shown that owing to self-induction an induced back E.M.F. is set up in a coil when the current through it is changing. In an A.C. circuit inductance is quite as important, and often enormously more important, than resistance.

Consider a coil, of inductance L and negligible resistance, through which an A.C., $i=i_0 \sin 2\pi ft$, is flowing (Fig. 208). Since there is no resistance in the circuit, the only opposition to the applied E.M.F. is the back E.M.F. due to self-induction, $L(di/dt)$. The current in the circuit adjusts itself at every instant so that the induced E.M.F. and applied E.M.F. are equal and opposite.

$$\therefore \text{ Applied E.M.F.} = L\frac{di}{dt}$$

$$= Li_0 2\pi f \cos 2\pi ft$$

$$= 2\pi fL . i_0 \cos 2\pi ft.$$

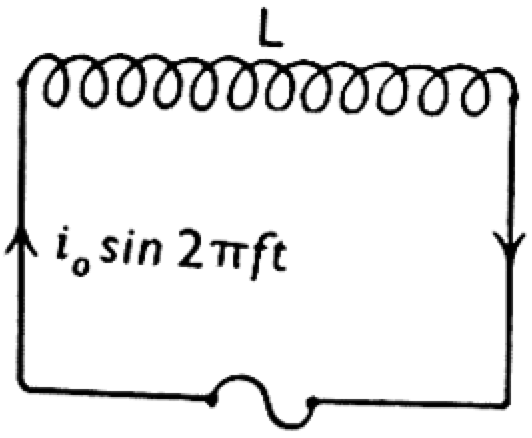


Fig. 208

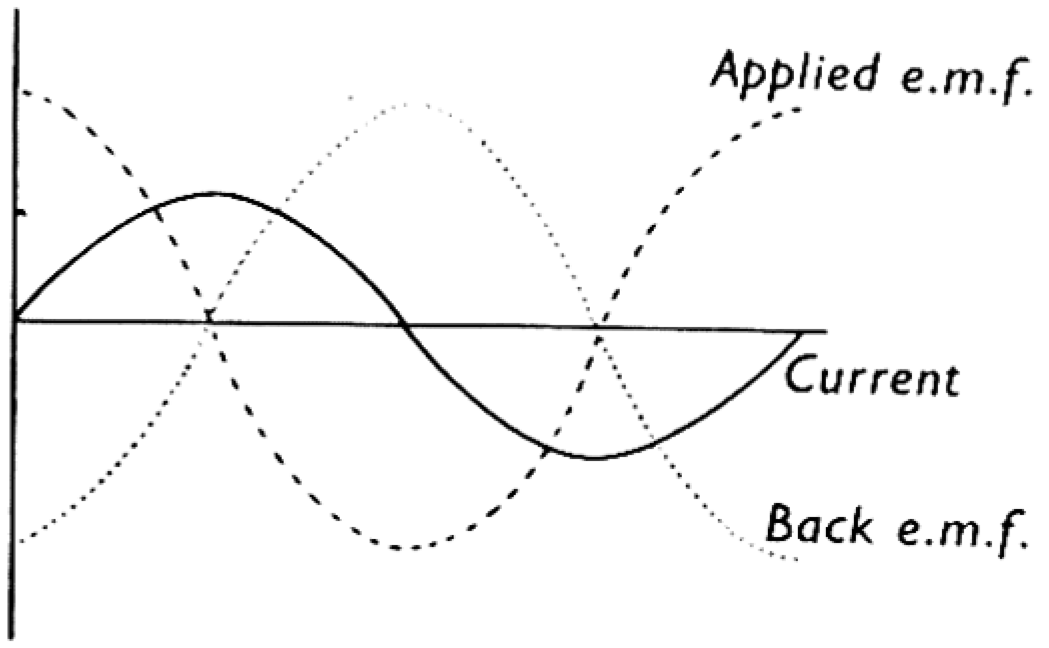


Fig. 209

$i_0 \cos 2\pi ft$ represents an A.C., differing in phase by 90° from $i_0 \sin 2\pi ft$, but otherwise similar. Hence the magnitude of the applied E.M.F. is $2\pi fL$ times the magnitude of the current.

In a circuit possessing resistance, R, only, applied E.M.F. would be $Ri_0 \sin 2\pi ft$.

Thus the term $2\pi fL$ is analogous to R; it is called the *inductive reactance*, usually abbreviated to reactance, and is measured in ohms. The reactance increases with the frequency f. The reactance of a coil of inductance 1 henry, for a frequency of 50 cycles per second, is $2\pi . 50 . 1 = 314$ ohms. For a frequency of 10^6 cycles per second, a typical radio frequency, the reactance is $2\pi . 10^6 . 1 = 6{\cdot}28$ megohms.

The term $\cos 2\pi ft$ in the above expression for the applied E.M.F. indicates that the current lags behind the applied E.M.F. by $\frac{1}{2}\pi$ radians (or 90° or $\frac{1}{4}$ cycle). The sine curve representing the variation of current with phase angle or time is displaced 90° behind that representing the applied E.M.F. (Fig. 209).

It may seem at first surprising that the current should be a maximum when the applied E.M.F. is zero. But, as already stated, the current continuously adjusts itself, so that the back E.M.F. due to self-induction is equal to the applied E.M.F. Since the back E.M.F. is proportional to the rate of change of the current, the applied E.M.F. is also proportional to the rate of change of the current. So long as the applied E.M.F. is positive, the current continues to increase; as soon as the applied E.M.F. becomes negative the current begins to decrease, thereby changing the direction of the induced E.M.F.

The power absorbed in the circuit, which is the product of the instantaneous values of the applied E.M.F., E, and the current, i, is represented in Fig. 210. Since E and i are both zero once in each half-cycle, the power curve has a frequency double that of E or i. The shaded areas represent the product of power and time and hence the energy absorbed in the circuit. In the lower shaded loops, marked −, the current is flowing in a direction opposite to the applied E.M.F. and hence is giving up energy to the source of supply. Thus there is a transference of equal amounts of energy to and fro between the source of supply and the coil in each half-cycle; no energy is lost.

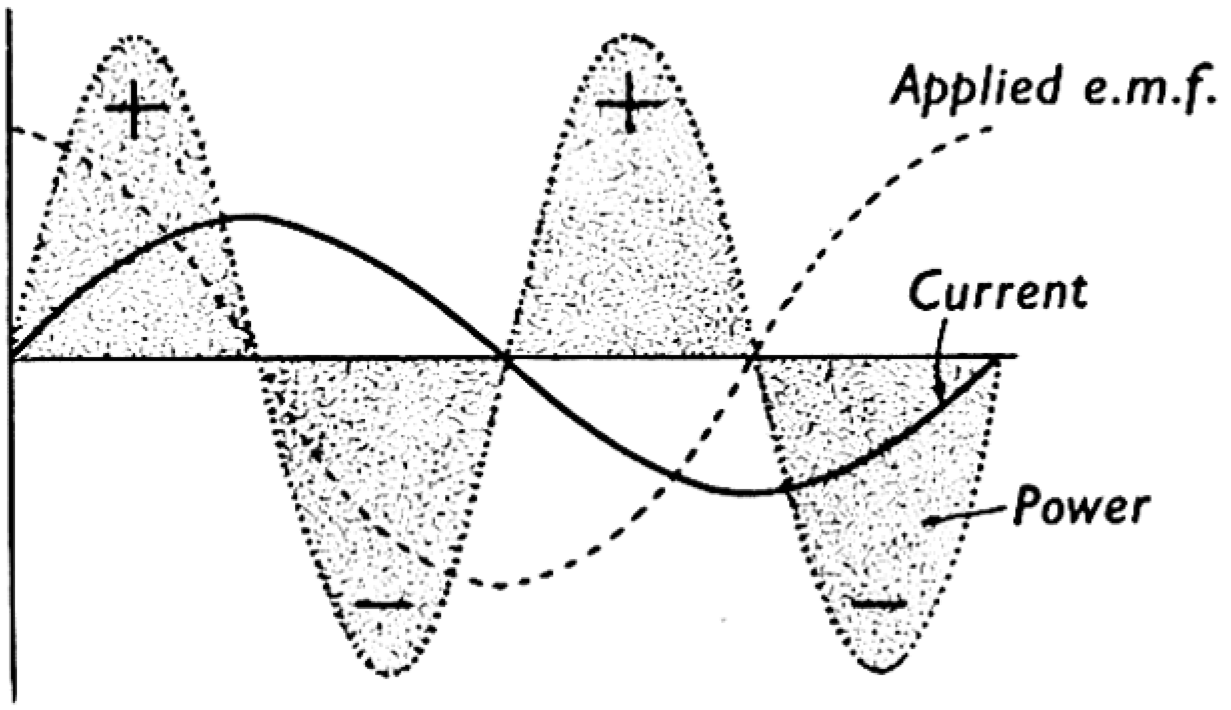


Fig. 210. Circuit possessing inductance only.

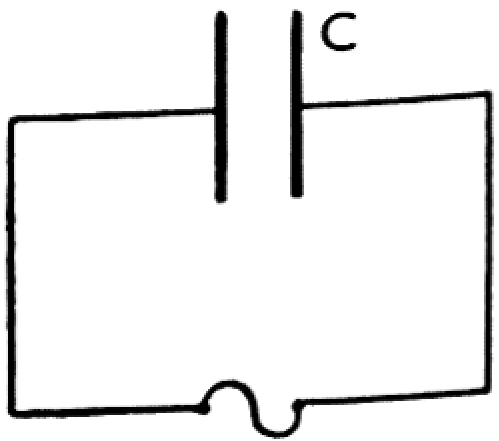


Fig. 211

Capacitance in an A.C. circuit

A.C. will flow in a circuit containing a condenser; the plates of the condenser are alternately charged and discharged, and the magnitude of the current depends on the capacitance of the condenser.

Suppose a condenser, of capacitance C, is connected by wires of negligible resistance to a source of alternating E.M.F. (Fig. 211). Let the charges on the plates at any instant be Q and $-Q$.

$$\text{Then P.D. between plates} = \frac{Q}{C} \quad \text{(see p. 10).}$$

$$\text{But P.D. between plates} = \text{applied E.M.F.}$$

$$\therefore \frac{Q}{C} = E_0 \sin 2\pi ft,$$

$$i = \frac{dQ}{dt} = 2\pi fCE_0 \cos 2\pi ft$$

$$= \frac{E_0}{1/2\pi fC} \cos 2\pi ft.$$

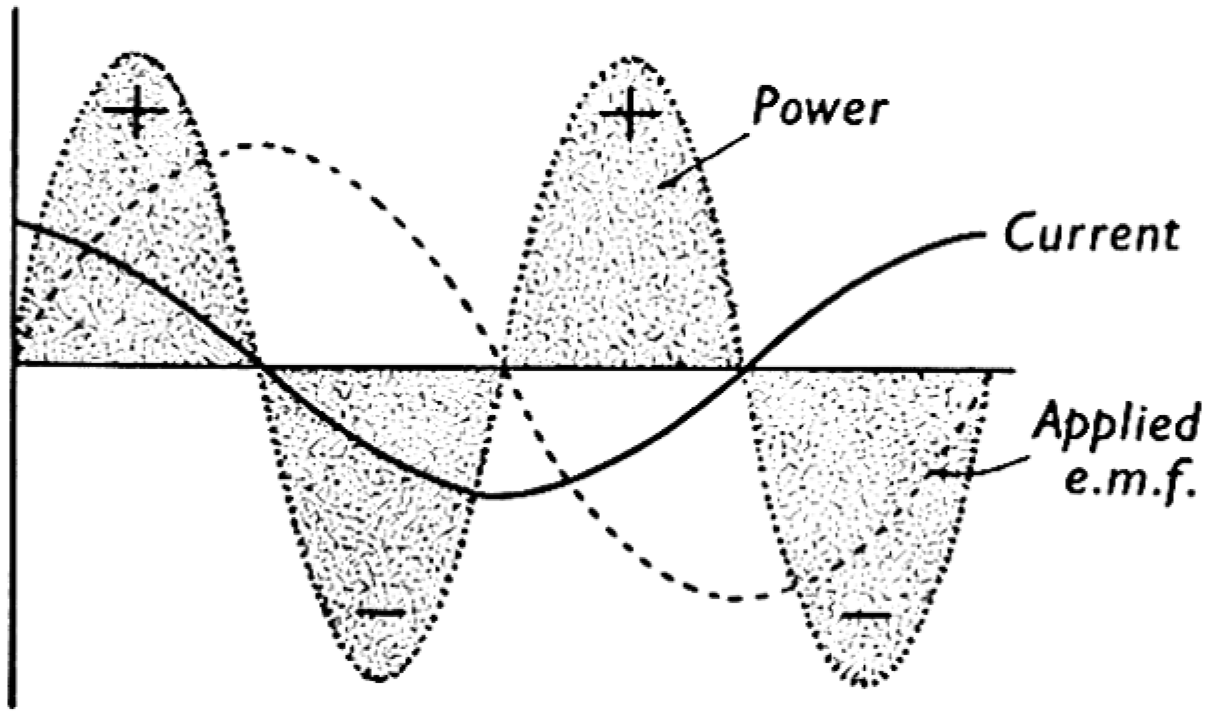


Fig. 212. Circuit possessing capacitance only.

The term $1/2\pi fC$, analogous to resistance, is called the *capacitive reactance*, usually abbreviated to reactance, of the condenser and is measured in ohms. The reactance decreases as the frequency, f, increases. The reactance of a condenser of capacitance 1 microfarad, at a frequency of 50 cycles per second, is $\frac{1}{2\pi . 50 . 10^{-6}} = 3180$ ohms. At a frequency of 10^6 cycles per second the reactance is $\frac{1}{2\pi . 10^6 . 10^{-6}} = 0{\cdot}159$ ohm.

The term $\cos 2\pi ft$ in the above expression for the current indicates that the current leads the applied E.M.F. by $\frac{1}{2}\pi$ radians. This phase lead is shown in Fig. 212. The curve representing the current is $\frac{1}{2}\pi$ or $\frac{1}{4}$ cycle ahead of the curve representing the applied E.M.F. There is a rush of current to the uncharged plates of the condenser when the E.M.F. is first applied; the current falls to zero when the E.M.F. reaches its maximum value. As the applied E.M.F. decreases the discharge from the condenser becomes increasingly great, reaching its maximum when the applied E.M.F. is zero.

As in the case of a purely inductive circuit, no energy is dissipated in the circuit; the energy expended by the source of supply in charging up the condenser is returned to it when the condenser discharges. The power curve is shown in Fig. 212.

Resistance and inductance in an A.C. circuit

Consider a circuit containing both resistance and inductance (Fig. 213). At first sight it might be thought that the total 'resistance' of the circuit is $R+2\pi fL$; this, however, does not allow for the phase lag of the current due to the inductance.

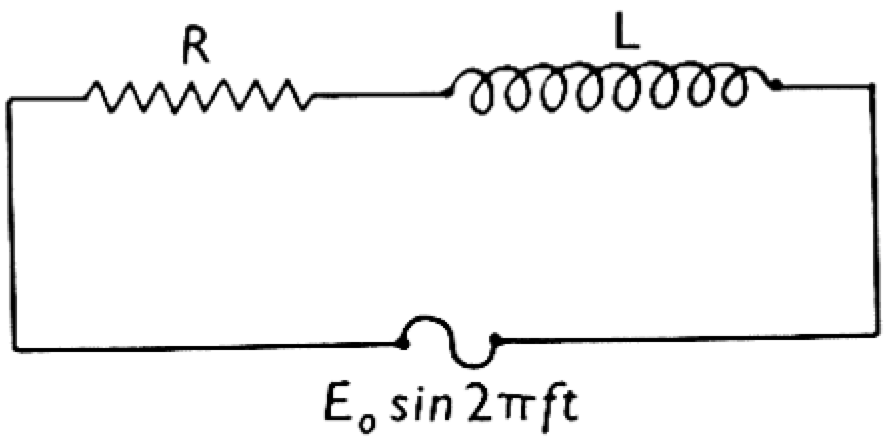


Fig. 213

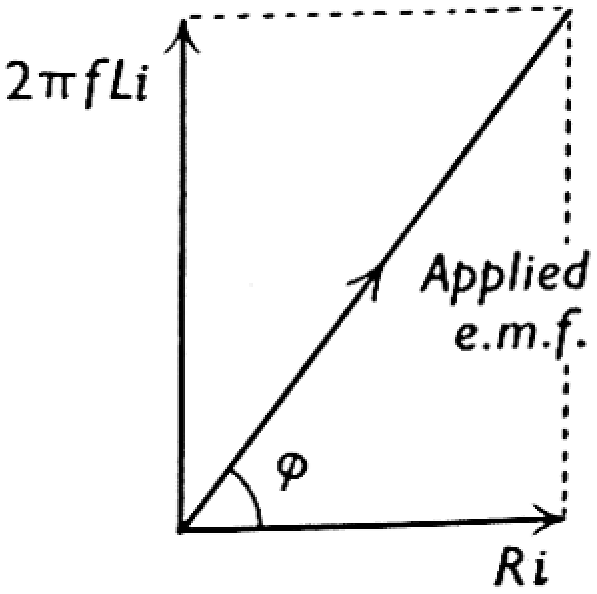


Fig. 214

The problem is solved most simply by means of vectors. The applied E.M.F. must supply, at any instant, Ri volts to drive the current through the resistance, in phase with the current; it must also supply $2\pi fLi$ volts to overcome the back E.M.F. due to self-induction, with a phase lead of 90° over the current. The applied E.M.F. is the resultant of these two vectors (Fig. 214); it must therefore lead the current by an angle ϕ, known as the phase difference or *angle of lag*, where

$$\tan\phi=\frac{2\pi fLi}{Ri}=\frac{2\pi fL}{R}.$$

From Fig. 214 Applied E.M.F. $=\sqrt{\{R^2+(2\pi fL)^2\}}i.$

The term $\sqrt{\{R^2+(2\pi fL)^2\}}$, analogous to resistance, is called the *impedance* of the circuit and is measured in ohms.

Power factor

The component of the applied E.M.F., $2\pi fLi$, is often called the wattless component because no power is dissipated in the inductance. Power is, however, dissipated, in the form of heat, in the resistance. Fig. 215

represents the power curve; the current lags behind the applied E.M.F. by less than 90°, and hence the positive loops of the power curve are larger than the negative loops.

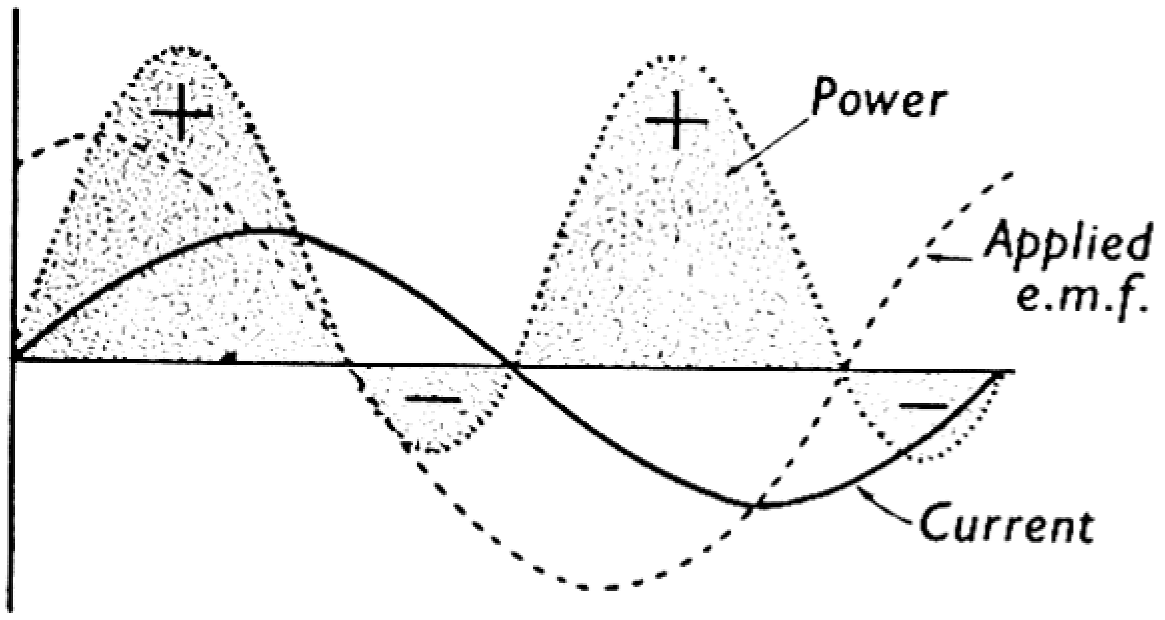


Fig. 215. Circuit possessing resistance and inductance.

The power dissipated in the resistance is i^2R. From Fig. 214,

$$iR = E\cos\phi, \quad \text{where } E = \text{applied E.M.F.}$$

$$\therefore \text{ Power dissipated} = iE\cos\phi.$$

The term $\cos\phi$ is called the *power factor.*

The true power dissipated in the circuit is thus $i_v E_v \cos\phi$ (where i_v and E_v are virtual or root mean square values): it is measured in watts. The apparent power is $i_v E_v$ and is measured in volt-amperes. The power factor is the ratio of the true power to the apparent power.

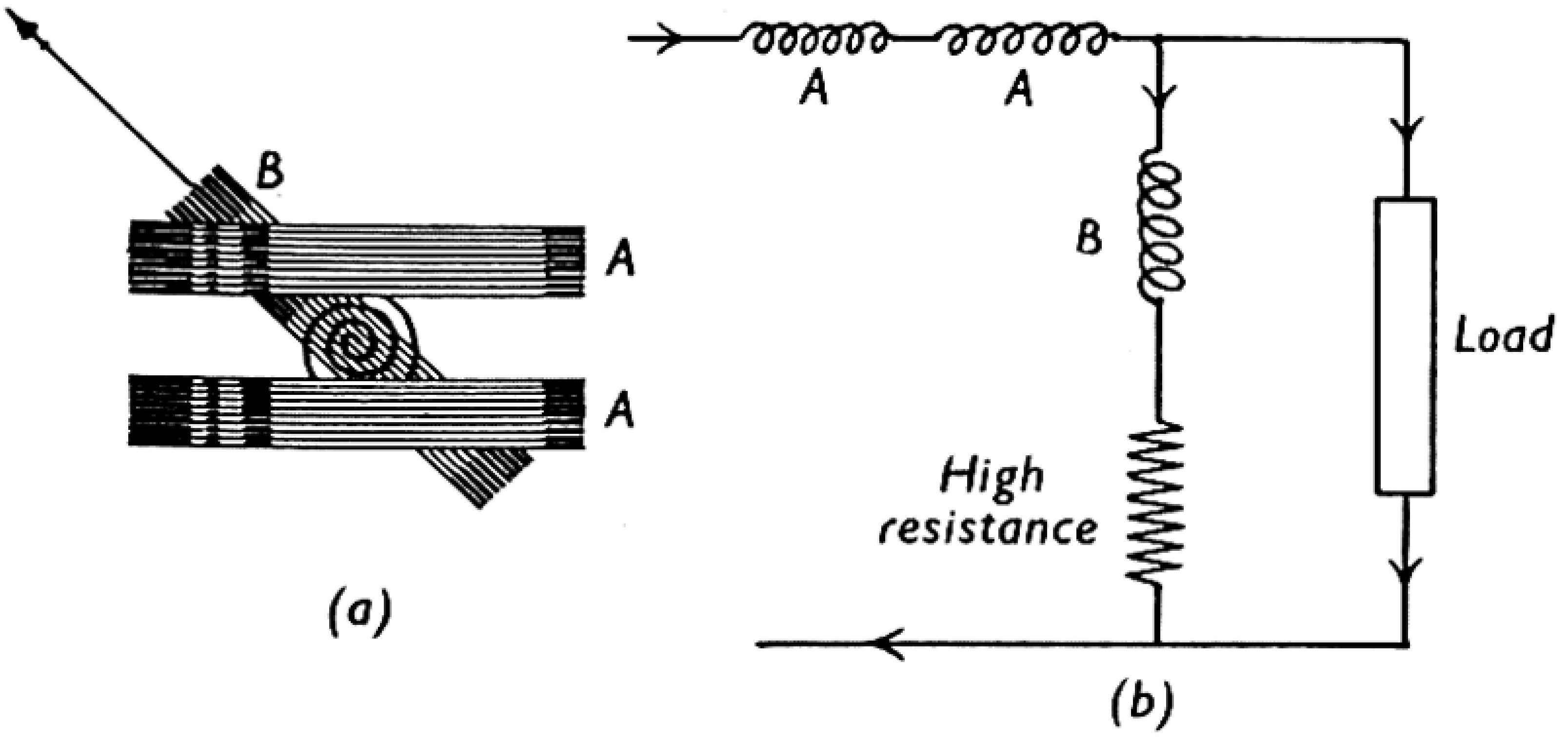


Fig. 216

The wattmeter

An instrument for measuring the true power in an A.C. circuit, known as a wattmeter, is shown in Fig. 216 (*a*). It consists of a coil, *B*, pivoted between two fixed coils *A*, *A*, and controlled by hair springs. The connections of the three coils to the load, in which the power is to be measured, are shown in Fig. 216 (*b*). The same current passes through the coils A, A, as through the load (since the current through B and the high resistance is very small); the same P.D. is applied across B and the high resistance as across the load. Thus the force on the moving coil is proportional to the product of the current through, and the P.D. across the load, i.e. the power in the load. By detaching the high resistance in series with B and replacing it by a different resistance the range of the instrument can be changed.

The apparent power can be obtained by using a separate ammeter and voltmeter. From the values of the true and apparent power the power factor can be calculated.

Circuit containing resistance, inductance and capacitance

Fig. 217 represents the components of the applied E.M.F. for a circuit containing resistance, inductance and capacitance. The impedance of the circuit must therefore be

$$\sqrt{\left\{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2\right\}}.$$

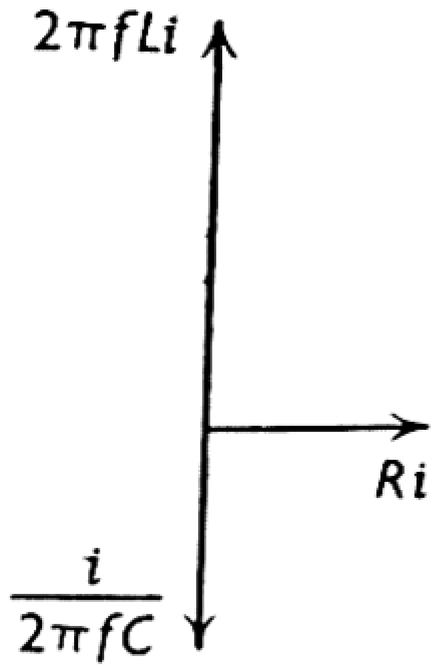


Fig. 217

Electrical resonance

Since inductance causes lag and capacitance causes lead it is possible to adjust their values in a circuit so that they neutralise each other. Only the resistance is then effective in limiting the current which is a maximum. Since inductance is proportional to frequency and capacitance is inversely proportional to frequency, the two can be equated only for a particular frequency. This is known as the *resonant frequency*, and at this frequency the circuit is said to be in a state of resonance. For resonance,

$$2\pi fL=\frac{1}{2\pi fC},$$

$$\therefore\ f=\frac{1}{2\pi\sqrt{(LC)}}.$$

If L is in henries and C in farads, then f is in cycles per second. The equation is of great importance in radio.

Acceptor and rejector circuits

Fig. 218 (*a*) and (*b*) represent the application of an alternating E.M.F. to an inductance and a capacitance connected in series and in parallel respectively. The resonant frequency is the same for both circuits but they behave differently.

In circuit (*a*) the current is a maximum and the impedance is a minimum at the resonant frequency; the circuit is called an acceptor circuit.

Suppose the resistance of circuit (*b*) is zero. The source of supply sends currents through L and C which lag and lead by 90° respectively; hence the effective current from the source is zero, since equal currents are flowing in opposite directions simultaneously through L and C. Thus the circuit acts as a complete barrier to currents at the resonant frequency and it is called a rejector circuit. In practice the circuit has some resistance, of course, and there is not complete rejection. It will, however, offer a very high impedance to currents at the resonant frequency.

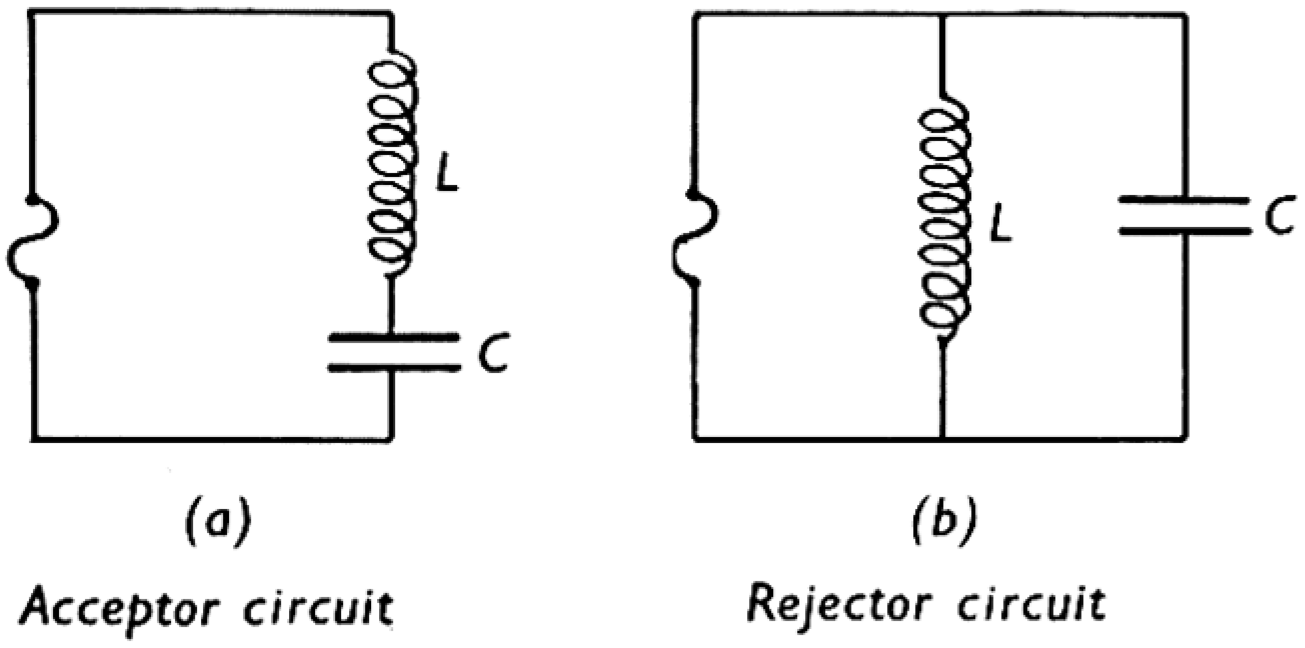


Fig. 218

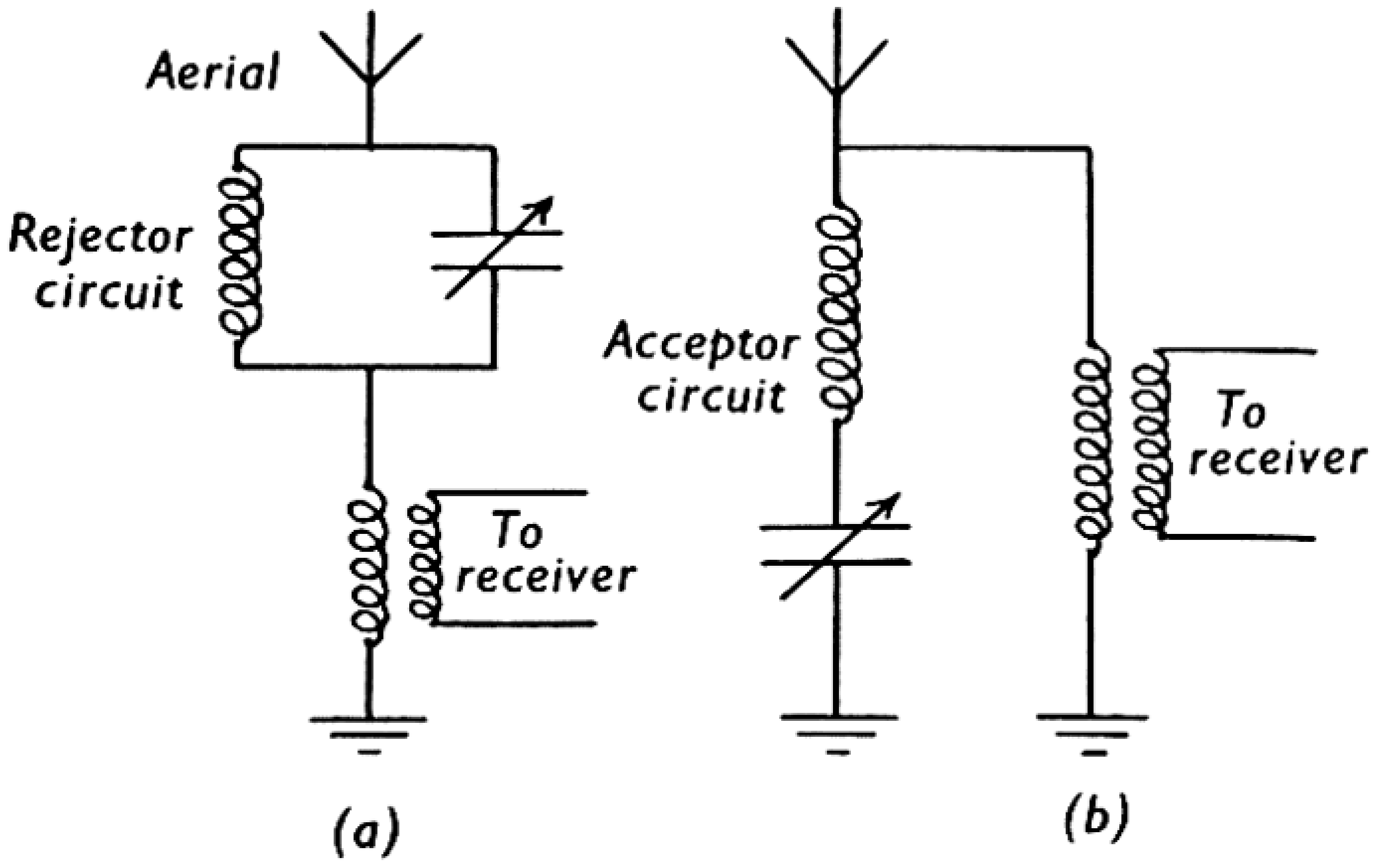


Fig. 219

As an example of the use of these circuits in radio two wave traps are shown in Fig. 219. The source of supply, the waves from the transmitting station, may be regarded as applied between aerial and earth. To eliminate an undesired signal, (1) the rejector circuit in (*a*) may be tuned to the signal, thus offering a very high impedance and stopping the signal from getting through to the receiver; (2) the acceptor circuit in (*b*) may be tuned to the signal, thus offering a low impedance path to earth and removing the signal in that way.

The metal rectifier

A rectifier is an instrument which suppresses one set of half-cycles of an A.C. so that the resulting current is in one direction only.

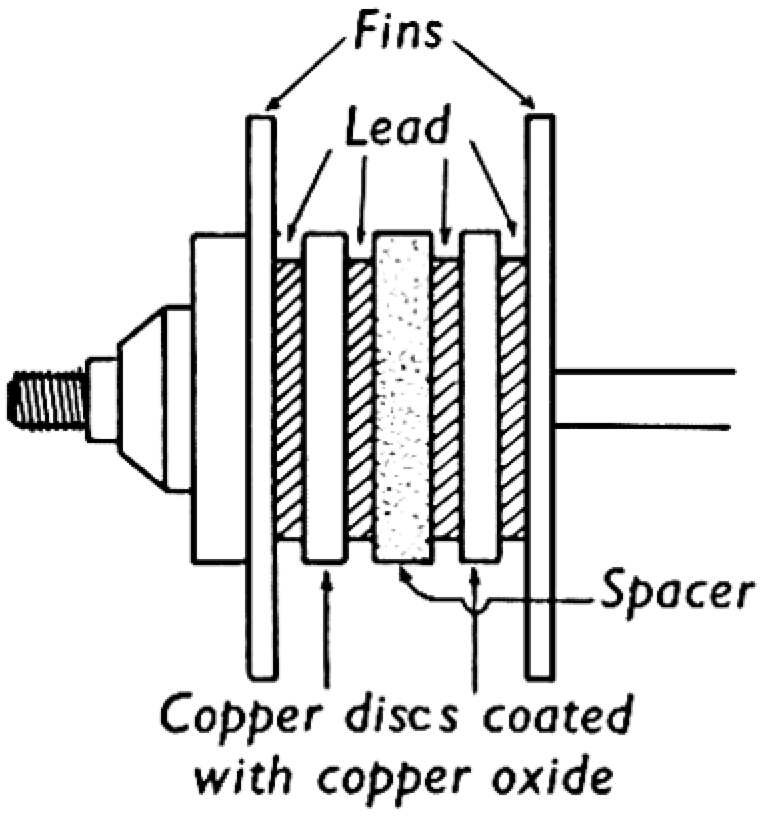


Fig. 220

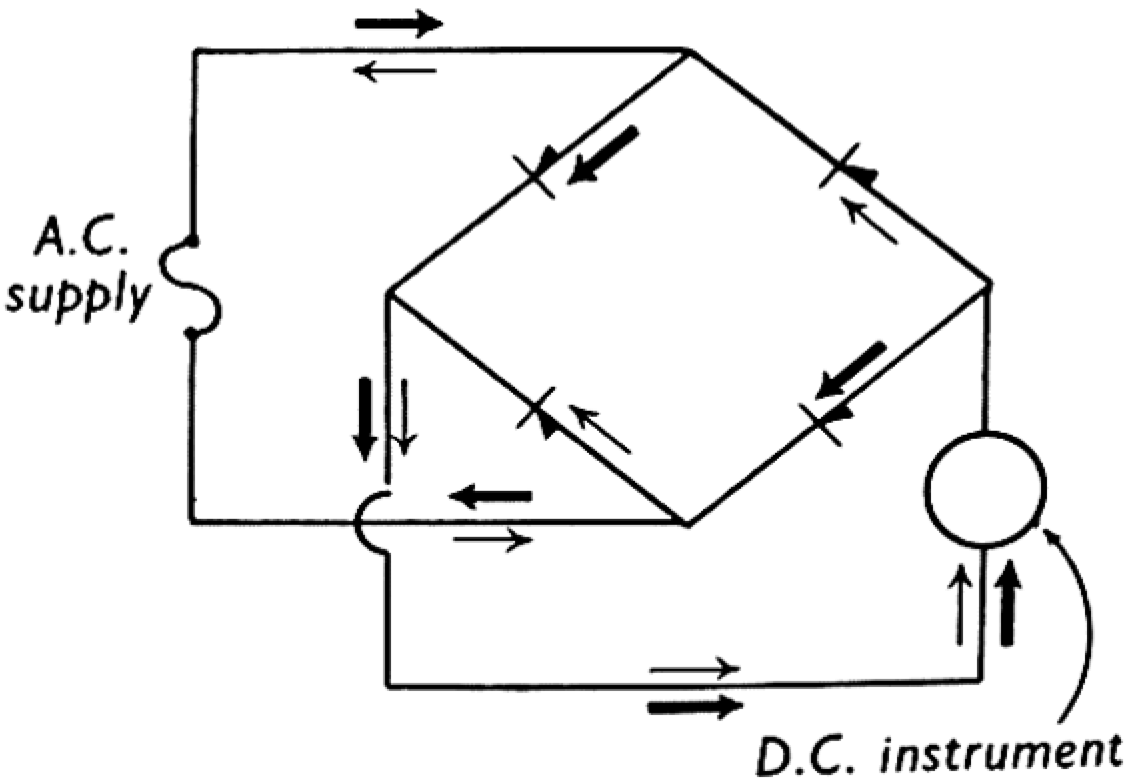


Fig. 221

One type of rectifier, shown in Fig. 220, depends on the fact that a current flowing from copper oxide to copper experiences a low resistance, but when flowing in the opposite direction it experiences a very high resistance. The rectifier consists of a number of specially treated copper discs, bolted together with lead washers, metal spacers and large metal fins to dissipate heat. One side of each of the copper discs has been subjected to special heat treatment to form a thin layer of copper oxide. This type of rectifier is often used for a high-tension supply to mains-operated radio receivers.

Bridge-rectifier method of adapting moving-coil instruments to measure A.C.

Fig. 221 represents a method of connecting four metal rectifiers to a D.C. moving-coil instrument to measure A.C. The arrangement allows both half-cycles of the A.C. to pass in the same direction through the instrument which, however, measures the rectified mean and not the r.m.s. value of the current.

QUESTIONS

1. Fig. 222 represents a method of finding the frequency of the A.C. mains. The A.C. is passed through a sonometer wire, *PQ*, which runs between the poles of a strong horseshoe magnet. The length of the wire is adjusted until it vibrates to the maximum extent. Explain fully why the wire vibrates.

Suppose the length of the wire resounding to the A.C. is 77·4 cm., while the length which resounds to a tuning fork of frequency 128 cycles per second is 30·3 cm. (no current then passing through the wire but the tension remaining unaltered). Calculate the frequency of the A.C.

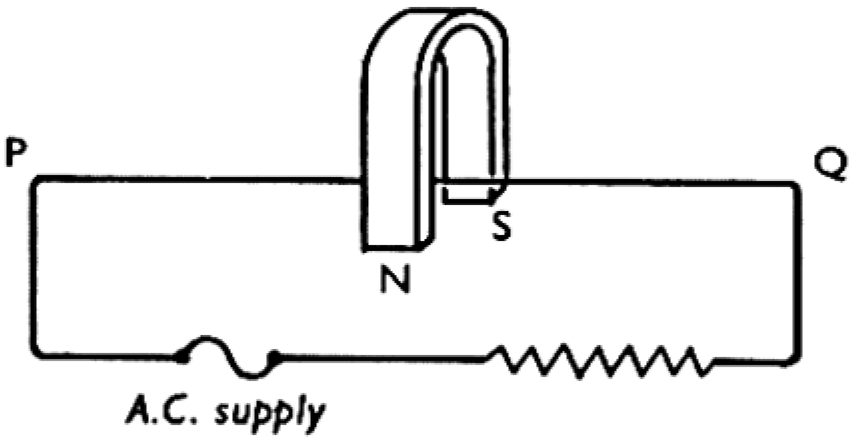


Fig. 222

2. A coil of 50 turns and of radius 10 cm. makes 1200 revolutions per minute in a magnetic field of intensity 200 dynes per unit pole. Calculate the average value of the E.M.F. in the coil in volts (irrespective of sign). (1 volt$=10^8$ e.m.u.) (C.)

3. A circular coil of 100 turns, each of radius 10 cm., is rotated 10 times per second about an axis at right angles to a magnetic field of 1000 e.m.u. Find the position of the coil when the E.M.F. across its ends is a maximum, and the value of this E.M.F.

If, when in this position, a current of 1 ampere is taken from the coil, what is the torque required to maintain its angular velocity? (O. & C.)

4. A small flat circular coil of radius r and of n turns is situated at the centre of a larger flat coil of radius R and N turns carrying a current of i amperes. The small coil is rotated at an angular velocity ω about an axis in its own plane and in that of the larger coil. Show that there is induced across the end of the small coil a single-phase alternating E.M.F., and find its peak value in volts, and its frequency. Explain the meaning of the r.m.s. value of an alternating E.M.F. and deduce its relation to the peak value.

If the resistance of the small coil is small, explain the relation between the current flowing at any time and the E.M.F. if (*a*) a condenser, (*b*) an inductance, is connected to it. (O. & C.)

5. Compare and contrast the effect in the case of D.C. and A.C. when a resistance, an inductance and a condenser are placed in the circuit. (O. & C.)

6. What is meant by an A.C.? How is such a current obtained? A coil containing inductance and resistance is supplied with an alternating E.M.F. of constant amplitude. Draw a rough graph showing how the power delivered alters as the frequency of alternation increases. (O. S.)

7. An 8-microfarad condenser is placed across the 200 volts r.m.s. A.C. 50-cycle electrical supply. Calculate the r.m.s. current which flows in the circuit. What is the peak value of the voltage across the condenser? (C. S.)

8. Write a short account of resonance, illustrating your answer by examples taken from light and electricity. (C. S.)

9. Describe a cathode-ray tube.

An alternating E.M.F. is applied to a resistance *AB*, a capacity *BC* and an inductance *CD* connected in series. The *X*-plates are connected across *AB*. What is the pattern on the screen when the *Y*-plates are connected (*a*) across *AB* also, (*b*) across *BC*, (*c*) across *CD*, (*d*) across *AD* and the capacity changed from a very small to a very high value? (O. S.)

10. Find the impedance of a telephone receiver of resistance 100 ohms and inductance 0·002 henry (*a*) at 1000 cycles, (*b*) at 100,000 cycles.

11. Find the magnitude and phase of the current which flows when an alternating P.D. of r.m.s. value 200 volts and frequency 50 cycles per second is applied to the ends of the following:

- (*a*) a coil of inductance 0·2 henry and negligible resistance;
- (*b*) a condenser of capacitance 80 microfarads;
- (*c*) a coil of inductance 0·2 henry and of resistance 20 ohms;
- (*d*) a condenser of capacitance 80 microfarads in series with a resistance of 20 ohms;
- (*e*) a coil of inductance 0·2 henry and resistance 20 ohms in series with a condenser of capacitance 80 microfarads.

12. A coil of inductance 0·2 millihenry and negligible resistance is in series with a condenser of capacitance 0·0005 microfarad. Find the resonant frequency of the circuit.

13. A P.D. with a frequency of 50 cycles per second is applied to a coil of inductance 4 henries and resistance 200 ohms. What is the power factor of the coil?

14. An A.C. of 2 amperes and frequency 100 cycles per second flows through a coil of inductance 2 millihenries and resistance 4 ohms. What is the P.D. across the coil, the power absorbed in the coil, and the power factor?

15. A coil, of resistance 20 ohms and inductance 0·5 henry, is connected in series with a condenser, of capacitance 40 microfarad, to a 230-volt, 50-cycle supply. Determine (*a*) the current, (*b*) the voltage drop across the coil and condenser.

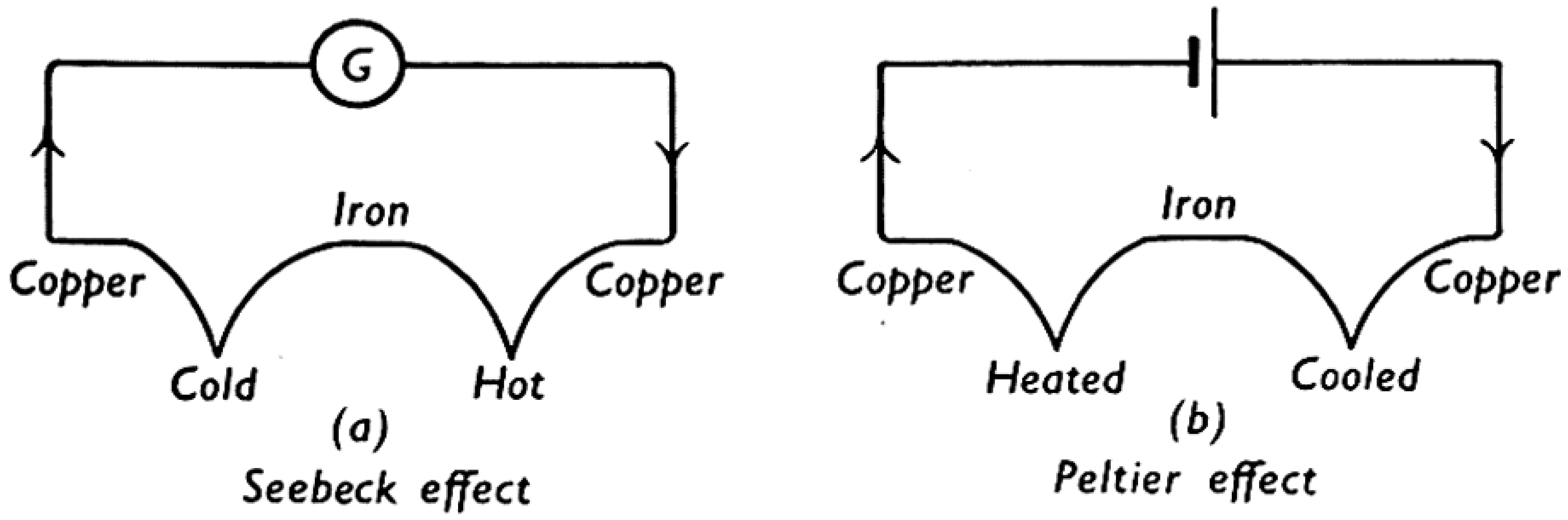
16. A circuit consists of a resistance, an inductance and a capacitance in series. The resistance of the circuit is 200 ohms, and the inductive and capacitive reactances are each 200 ohms at a frequency of 50 cycles per second. Taking different values of the frequency from 0 to 100 cycles per second, plot rough graphs of (1) resistance and frequency, (2) inductive reactance and frequency, (3) capacitive reactance and frequency, (4) impedance and frequency, (5) phase difference and frequency.

CHAPTER 14

THERMOELECTRIC, PHOTOELECTRIC AND THERMIONIC EFFECTS

Seebeck and Peltier effects

In 1821 Seebeck discovered that if the junctions of two different metals in a circuit are kept at different temperatures, a small current flows. The phenomenon is known as the thermoelectric or Seebeck effect. If the junctions of the iron–copper thermocouple in Fig. 223 (*a*) are maintained at 0 and 100° C. respectively, the thermo-E.M.F. is of the order of 1 millivolt; the current flows from copper to iron across the hot junction.



(*a*)
Seebeck effect

(*b*)
Peltier effect

Fig. 223

In 1834 Peltier discovered the inverse effect. If a current passes through a circuit containing two different metals, heat is generated at one junction and absorbed at the other (Fig. 223 (*b*)). The effect occurs whether the current is driven by an external cell or whether it is generated by the thermocouple itself. Thus, when the Seebeck effect occurs, heat is continually absorbed at the hot junction and developed at the cold junction; likewise when the Peltier effect occurs it tends to set up a Seebeck E.M.F. which opposes the current.

Experimental demonstration

The Seebeck effect is easily demonstrated. It is necessary only to heat the junction of pieces of copper and iron wire with a match, the other ends of the wires being connected to the terminals of a fairly sensitive galvanometer. The cold junction in this case is at the galvanometer.

It is not so easy to demonstrate the Peltier effect since it may be masked by the normal Joule heating effect. The Joule effect, of course, occurs all over the circuit, whereas the Peltier effect is confined to the junctions. One method of demonstration is shown in Fig. 224. An iron-constantan thermocouple is connected to a double-pole, double-throw switch, so that it may be connected either to a battery or to a sensitive mirror galvanometer. A current is first passed through the thermocouple by the battery for a short time, after which one junction should be hot and the other cool (Peltier effect). If the thermocouple is now connected to the mirror galvanometer a small current should flow (Seebeck effect). To ensure that it was the Peltier and not the Joule effect which gave rise to the Seebeck effect, the thermocouple is again connected to the battery with the current reversed by means of the commutator. After a minute or two the thermocouple is once more connected to the galvanometer and a deflection in the opposite direction to the previous one should result.

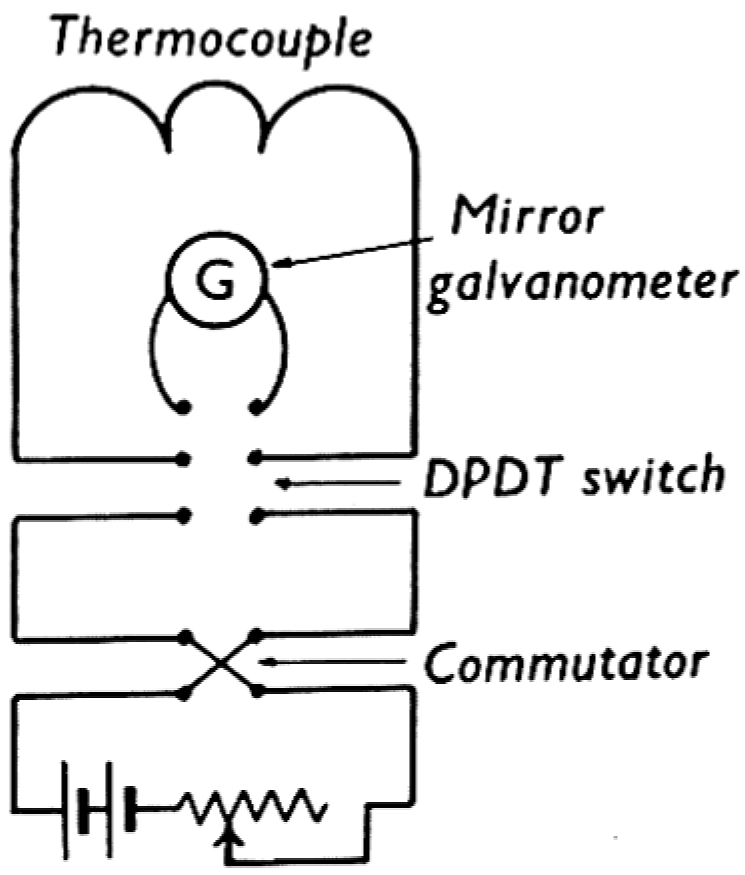


Fig. 224

Laws of intermediate metals and of intermediate temperatures

The following two laws have been established experimentally:

(1) If A, B and C are three different metals the thermoelectric E.M.F. of the couple AC is equal to the sum of the E.M.F.'s of the couples AB and BC over the same temperature range.

It follows that the junctions of a thermocouple may be soldered without affecting the E.M.F.

(2) The E.M.F. of a thermocouple, with junctions at temperatures θ_1 and θ_3, is the sum of the E.M.F.'s of two couples of the same metals with junctions at θ_1 and θ_2, and at θ_2 and θ_3, respectively.

It follows that when a galvanometer is connected to a thermocouple, as in Fig. 223 (*a*), the E.M.F. is independent of the temperature of the galvanometer.

Measurement of thermoelectric E.M.F.

The E.M.F. of a thermocouple may be measured by means of a potentiometer. Since the thermo-E.M.F. is of the order of a millivolt it is necessary to arrange that the potential drop across the potentiometer wire is only slightly more than this by putting a resistance R in series (see Fig. 225). Then if E is the E.M.F. of the cell, and r is the resistance of the potentiometer wire, the potential drop across potentiometer wire $=\frac{r}{R+r}E$. The internal resistance of the cell has been ignored, but R must be several hundred ohms and hence no appreciable error is introduced.

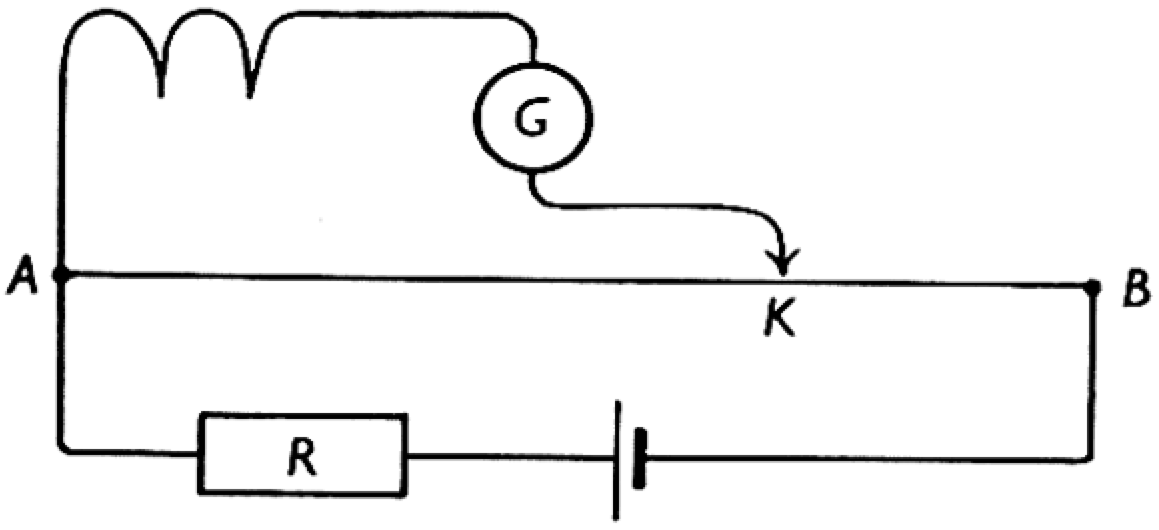


Fig. 225

The junctions of the thermocouple are maintained at the desired temperatures, for example, those of melting ice and boiling water, while the balance point K is found. Then E.M.F. of thermocouple $=\frac{AK}{AB}\cdot\frac{rE}{R+r}$.

The value of E may be found with moderate accuracy by means of a voltmeter. Alternatively, the potentiometer can be calibrated with a standard cell as described on p. 150.

Variation of thermo-E.M.F.'s with the temperatures of the junctions

The variation of the E.M.F. of a thermocouple when the temperature of one junction is kept constant and the temperature of the other junction is varied, may be investigated by the method just described.

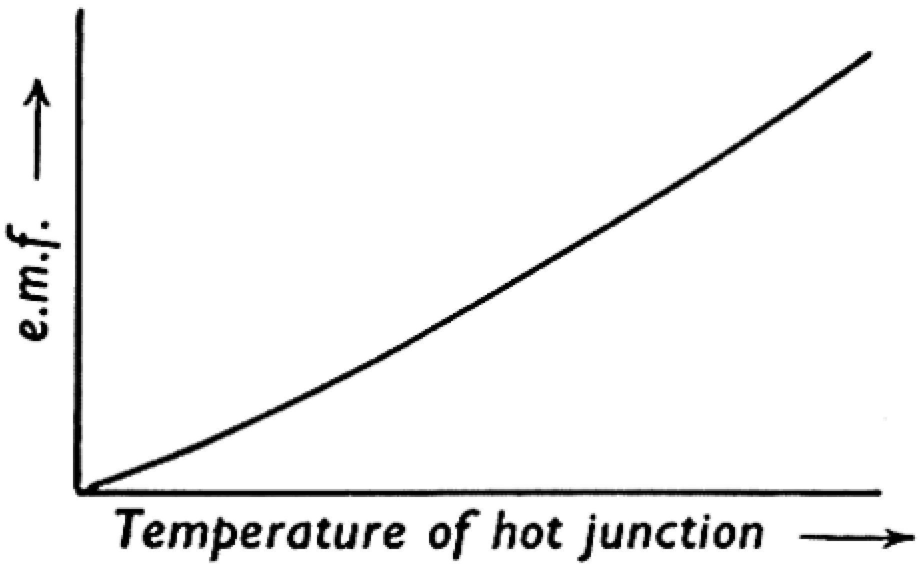


Fig. 226. Iron-constantan thermocouple.

Figs. 226 and 227 show graphs of E.M.F. and hot-junction temperature, the cold junction being maintained at 0° C., for iron-constantan and iron-copper thermocouples. The former is part of a parabola but is appreciably straight for the range 0–100° C., and is therefore suitable for measuring temperature. The latter is an almost perfect parabola; the temperatures of the hot junction at which the E.M.F. is a maximum and afterwards zero are called, respectively, the neutral temperature and the temperature of inversion.

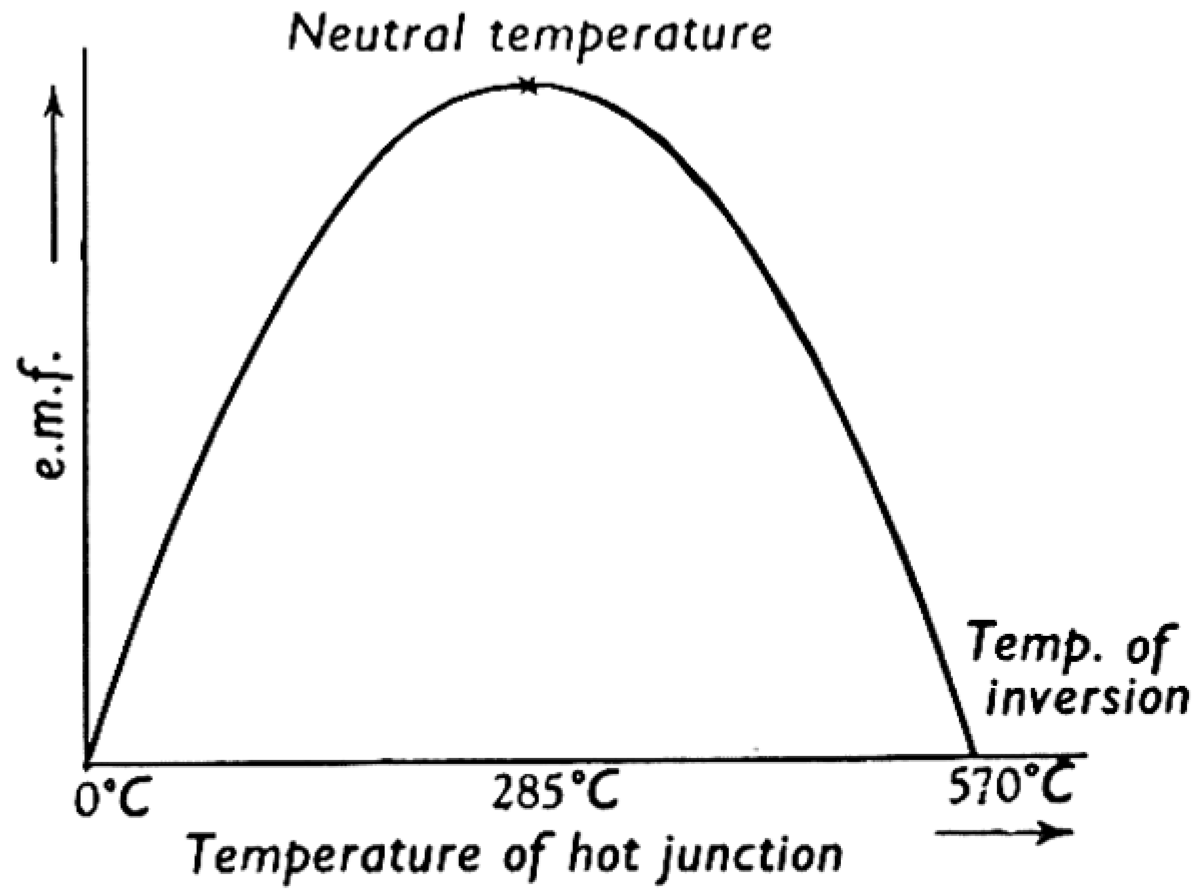


Fig. 227. Iron-copper thermocouple.

Thermoelectric series

For small values of temperature difference between the junctions, the metals can be arranged in a thermoelectric series as follows: antimony, iron, zinc, copper, silver, lead, aluminium, mercury, platinum-rhodium, platinum, nickel, constantan (60% copper, 40% nickel), bismuth.

When two of these metals are combined to form a thermocouple, the current flows from the one earlier in the list to the other across the cold junction; thus the current flows from *a*ntimony to *b*ismuth across the *c*old junction (note the italicising of *a*, *b*, *c*, which suggests a method of memorising the fact). Again, the thermo-E.M.F. is greater the farther apart the metals are in the list. The following are several thermo-E.M.F.'s in millivolts when the junctions are maintained at 0 and 100° C.: antimony-bismuth 12, iron-constantan 5, copper-constantan 4, platinum with 10% rhodium-platinum 0·64.

Thermopiles designed to measure heat radiation and consisting of a number of thermocouples in series are usually constructed of antimony and bismuth; iron and constantan also give a comparatively high E.M.F. and they are sometimes used. For measuring high temperatures the alloy platinum-rhodium and platinum are employed because of their high melting-points.

The Thomson effect

A thermocouple may be regarded as a reversible heat engine, absorbing heat at one junction, rejecting it at the other, and generating electrical energy equivalent to the difference in these two quantities of heat. William Thomson (later Lord Kelvin) applied the theory of heat engines to a thermocouple and deduced that the law of variation between the E.M.F. and the temperatures of the junction should be the same whatever the nature of the metals used. This is contrary to experiment. He therefore predicted another thermal effect, known as the *Thomson effect*, which can be stated as follows: *if a temperature gradient exists in a conductor an electric potential gradient also exists.* As an analogy, the pressure of the air in a glass tube heated at one end and cooled at the other is the same throughout, but the density of the air is greater at the cool end than at the hot end. Similarly, the pressure of the electron gas in an unevenly heated conductor is the same throughout, but its density varies; hence there is a P.D. in the conductor.

The Seebeck effect can be explained fully in terms of the Peltier and Thomson effects. By Peltier effect is here meant the reversible effect at the junction. An absorption or evolution of heat equivalent to π joules, when 1 coulomb of electricity flows, is equivalent to an E.M.F. of π volts (π is called the Peltier coefficient). At the two junctions of a thermocouple there are Peltier E.M.F.'s in opposite directions, the E.M.F. at the hot junction being greater than that at the cold. In the two metals there are temperature gradients and hence Thomson E.M.F.'s which are small compared with the Peltier E.M.F.'s. The algebraic sum of the Peltier and Thomson E.M.F.'s is equal to the Seebeck E.M.F.

It should be noted that a thermoelectric E.M.F., which is of the order of a millivolt, is a phenomenon quite distinct from a contact potential which is of the order of a volt. The former is a transport effect and the latter an equilibrium effect. The contact P.D.'s at the two junctions of a thermocouple act in opposite directions.

PHOTOELECTRIC EFFECT

When light falls on certain metals electrons are emitted from them; this is called the photoelectric effect.

The effect was discovered accidentally by Hertz in 1887 while performing his experiments on electromagnetic waves. He found that a spark jumped more readily across the gap between the metal knobs of his receivers when the knobs were illuminated by ultra-violet radiation. Hallwachs studied the phenomenon; he found that when light from an arc falls on a negatively charged zinc plate connected to a gold-leaf electroscope, the plate and electroscope become discharged. If the plate and electroscope are positively charged there is no leakage of the charge. Subsequently it was shown by J. J. Thomson and Lenard, independently, that the action of the light caused the metal to emit electrons; a positive charge on the metal prevents the electrons from escaping.

It is now known that most metals emit electrons when illuminated by ultra-violet light of suitable frequency; the alkali metals, lithium, sodium, potassium, rubidium and caesium emit electrons under the action of visible light as well as of ultra-violet light.

Einstein's photoelectric equation

An experimental study of the photoelectric effect revealed a surprising fact. The velocity of the emitted electrons is rigorously independent of the intensity of the light. If the intensity of the light is increased the number of electrons emitted is increased but their velocity is unchanged. The velocity of the electrons does depend, however, on the frequency of the light—the greater the frequency, the greater the velocity. Thus ultra-violet light is more effective than visible light in causing electrons to be emitted, because it has a greater frequency.

In 1901 Planck had put forward a revolutionary theory to account for the observed radiation of heat and light from hot bodies. He assumed that energy is radiated in the form of discrete units called quanta, and that each quantum has energy $h\nu$, where h is a universal constant now called Planck's constant, and ν is the frequency of the radiation. The quantum theory was revolutionary because it challenged, and seemed completely to contradict, the well-established wave theory of radiation.

In 1905 Einstein proposed an explanation of the photoelectric effect on the quantum theory. He suggested that when a quantum of energy $h\nu$ (now also called a photon) strikes a metal, part of its energy is used in liberating an electron, and the rest in giving the electron kinetic energy; the photon is annihilated. The work required to liberate an electron is ϕe, where ϕ is the electronic work function (see p. 204), and e is the charge of the electron.

Suppose m is the mass of the electron and v its velocity of emission; then

$$h\nu = \phi e + \tfrac{1}{2}mv^2.$$

This is one of the most famous and significant equations of the physics of this century, and it gained for Einstein a Nobel prize. Its significance lies in the fact that it is one of the main foundations on which the quantum theory rests.

Millikan's experimental verification of Einstein's equation

Fig. 228 represents diagrammatically the apparatus used by Millikan in a classic series of experiments by which he verified Einstein's equation and made determinations of h and ϕ.

The plate A is made of the metal whose photoelectric emission is to be investigated. It is illuminated by light of measured frequency ν, from a source L. The plate B is given a negative potential V by means of the potentiometer P, which is just sufficient to prevent the electrons emitted by A from reaching it. When no electrons reach B from A the electrometer E will register zero deflection.

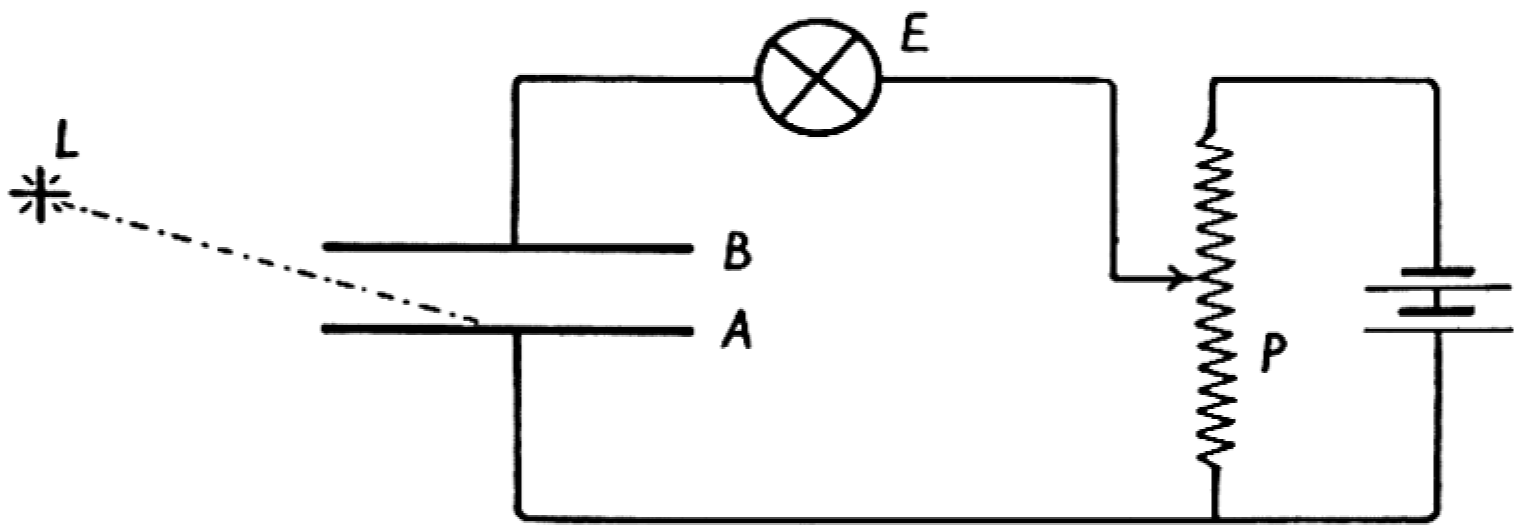


Fig. 228

The work done against the stopping potential is equal to the kinetic energy of the electrons, i.e.

$$Ve = \tfrac{1}{2}mv^2.$$

Thus $$Ve = h\nu - \phi e \quad \text{(from Einstein's equation).}$$

If ν_0 = frequency of the radiation which just enables the electron to escape with zero velocity, called the threshold frequency,

$$h\nu_0 = \phi e,$$

$$\therefore\ Ve = h\nu - h\nu_0.$$

Thus, if Einstein's theory is correct, V should be a linear function of ν. Millikan found that this was the case (Fig. 229). The value of h can be determined from the slope of the graph, knowing e, since $h = \dfrac{Ve}{\nu - \nu_0}$; Millikan obtained $h = 6{\cdot}56 \times 10^{-27}$ erg second. The value of ϕ can be found from the intercept on the axis of ν since $\phi = \dfrac{h\nu_0}{e}$.

A series of values of ϕ for different metals is given in the table below.

Since a contamination of the surface by a thin film of oxide has a considerable effect on the emission of electrons, Millikan enclosed the plates B and A in a vacuum and, just before illuminating A, removed a thin shaving from its surface with a knife which could be operated from outside the vacuum. B was made of copper and an allowance had to be made for the considerable contact potential difference between A and B; the stopping potential was taken as the difference between the potentiometer reading and the contact potential difference.

Electronic work functions (*volts*)

Caesium	1·87–1·96	Zinc	3·57
Rubidium	2·16–2·19	Copper	4·16
Potassium	2·24	Tungsten	4·54
Lithium	2·28	Silver	4·74
Sodium	2·46	Platinum	6·30

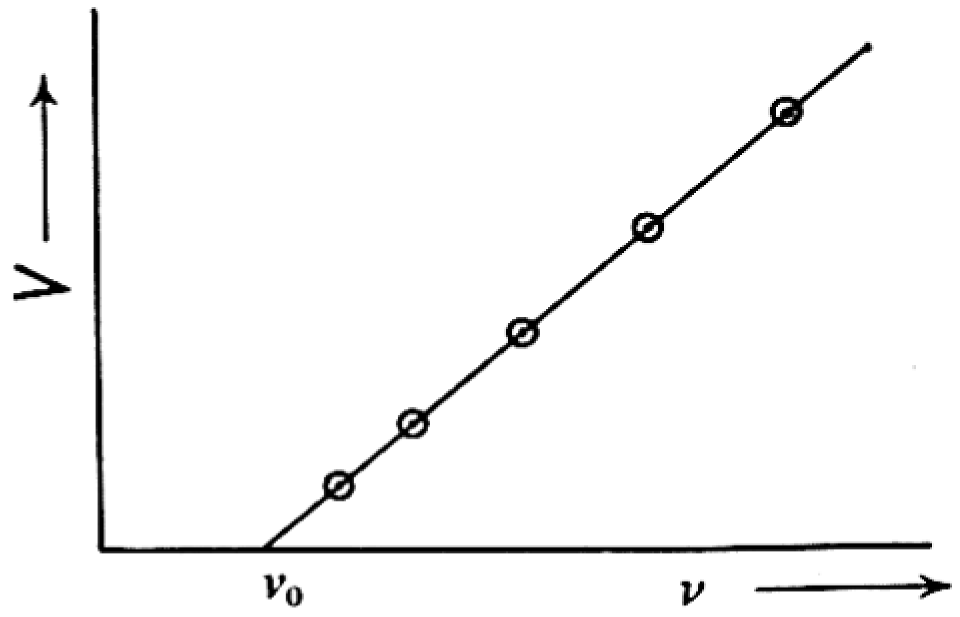


Fig. 229

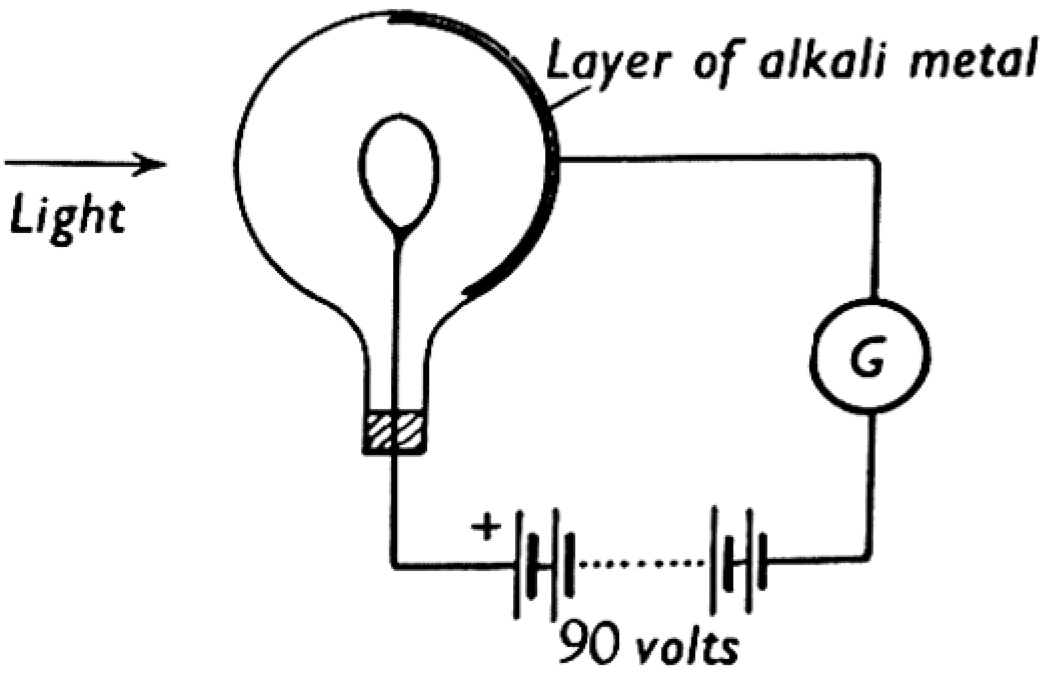


Fig. 230

The photoelectric cell

The photoelectric cell consists of a glass bulb provided with two electrodes. The cathode consists of a coating of an alkali metal such as sodium, potassium or caesium on part of the inside surface of the glass; the anode is a loop of platinum wire (Fig. 230). When light falls on the coating of alkali metal, electrons are emitted and these can be attracted to the anode if the latter is raised to a suitable positive potential. Hence a small current flows through the circuit in Fig. 230. The bulb may be evacuated or contain an inert gas such as argon. It is essential that oxygen should be excluded since once the photoelectric film has become oxidised it loses much of its emissive power.

In conjunction with a valve amplifier (described in Chapter 16), the photoelectric cell has a multiplicity of applications. It can be used to count objects as they interrupt the beam of light falling on it, to switch street lights on and off automatically at dusk and sunrise, to ensure the precise levelling off at floors of lifts, to act as a burglar alarm when an intruder interrupts the light; and it is an essential component of sound film and television apparatus.

Sound films

The sound track of a film is made as follows. The sound vibrations are converted by a microphone (similar in principle to a telephone transmitter) into corresponding, fluctuating, electric currents. The currents are amplified and made to open the jaws of a narrow slit through which light shines on to a strip at the edge of the film. Loud sounds open the slit wider than soft sounds, and hence the sound track has a variable density (Fig. 231 (*a*)).

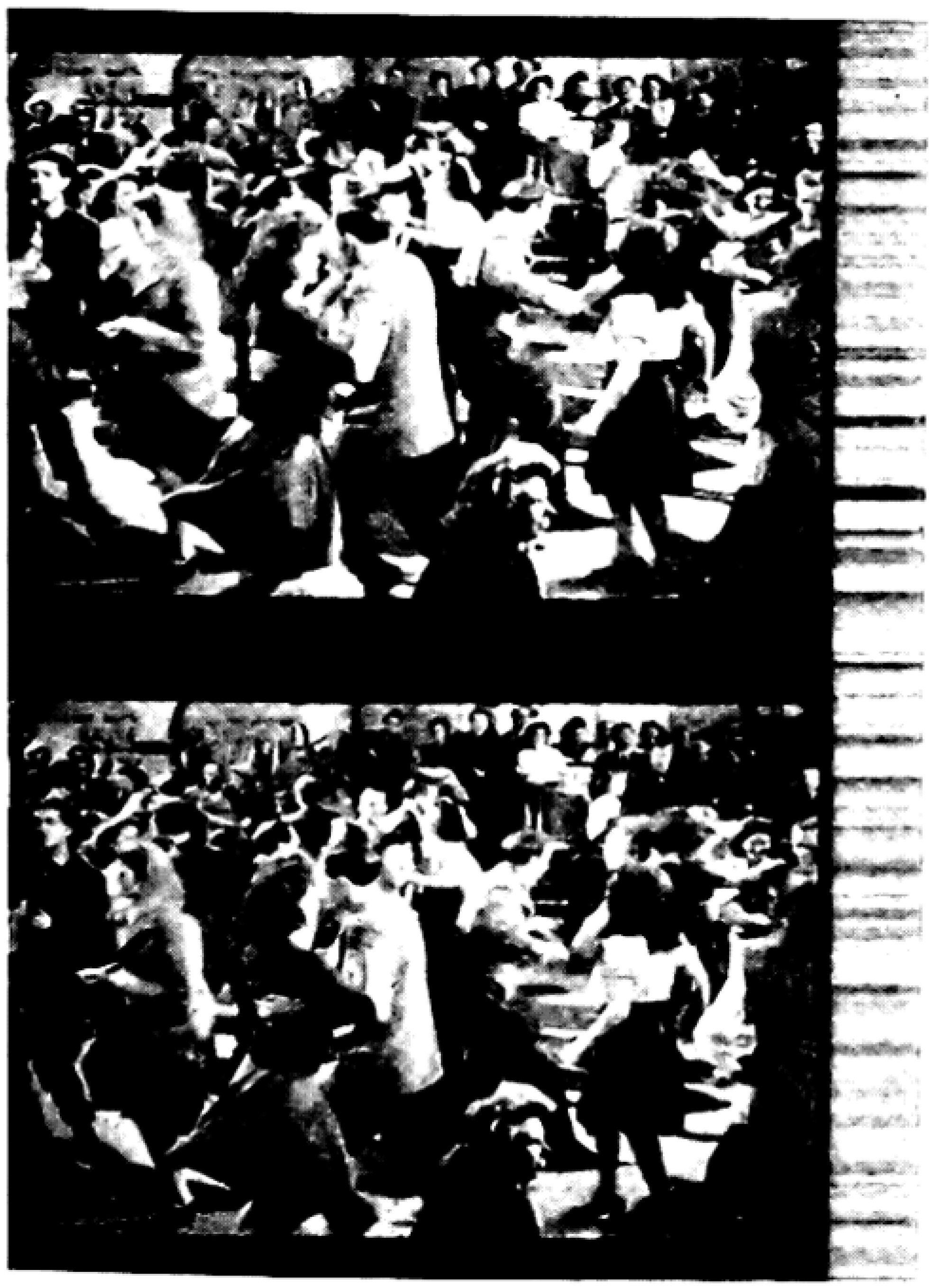


Fig. 231 *a*. Variable density sound track.

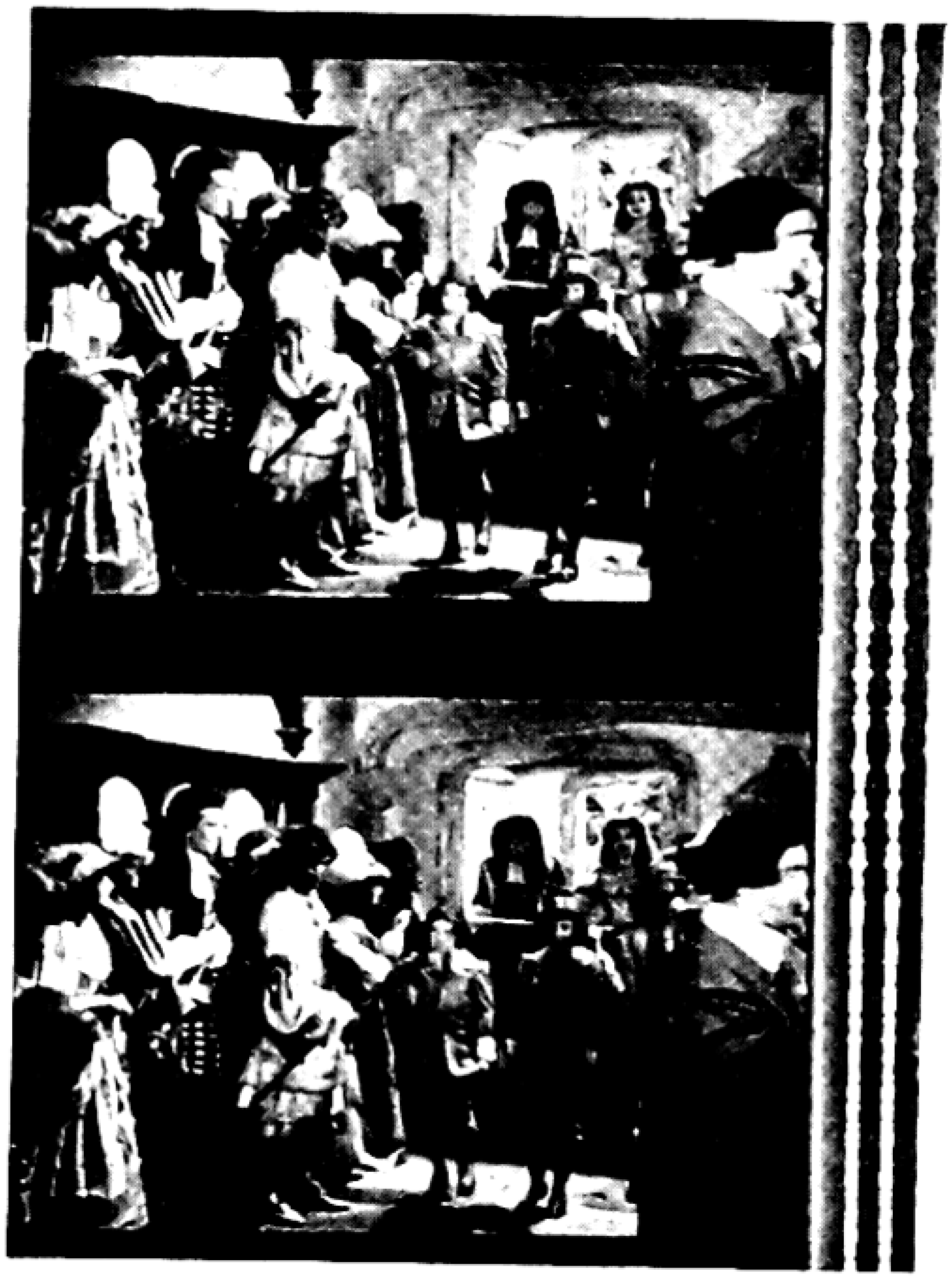


Fig. 231 *b*. Variable area sound track.

In another method the varying currents cause the mirror of a delicate galvanometer to vibrate, hence giving a variable area sound track as opposed to the variable density track (Fig. 231 (*b*)).

When the film is projected, light from an 'exciter' lamp is passed through the sound track into a photoelectric cell (see Fig. 232), thus giving rise to varying currents which are converted back to sound waves by a loud speaker. For the picture projection it is necessary to jerk the film forward at the rate of 24 frames per second, but the sound apparatus requires perfectly steady motion past the photoelectric cell. To enable these conflicting requirements to be satisfied there must be a free length of film between the two parts of the machine and, in consequence, the sound track is printed 25 frames ahead of the pictures to which it corresponds.

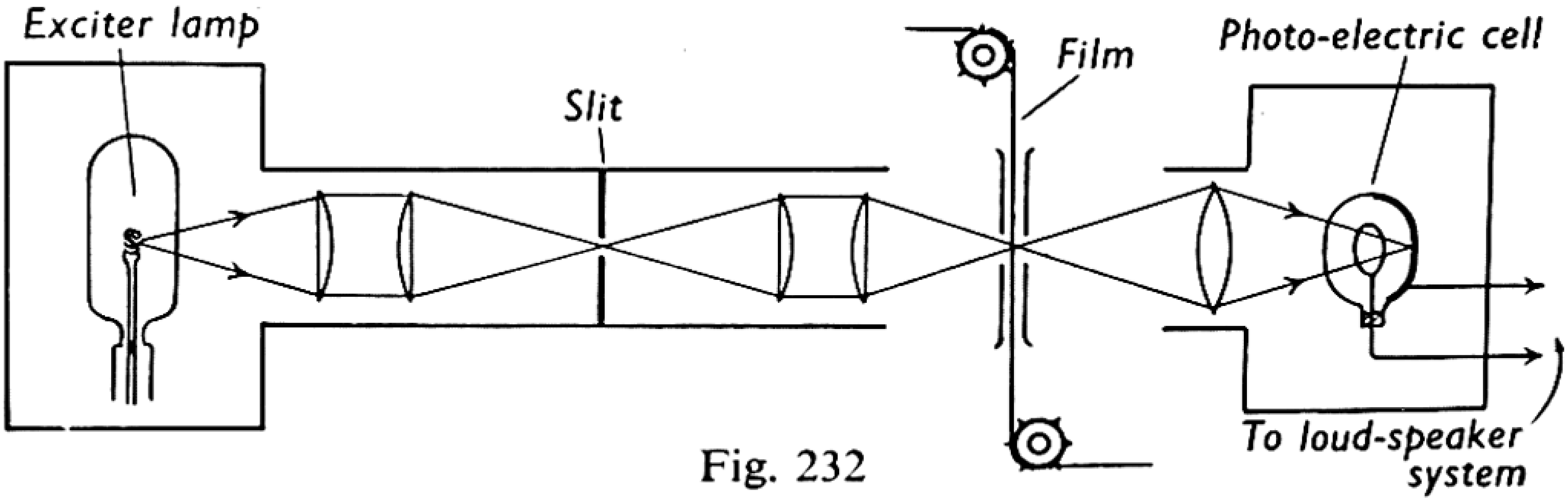


Fig. 232

THERMIONIC EFFECT

When a metal is heated it emits electrons; this is called the thermionic effect.

An electron cannot escape from the surface of a metal unless its kinetic energy, $\frac{1}{2}mv^2$, exceeds a certain value: viz. $\frac{1}{2}mv^2 \geqslant \phi e$, where ϕ is the electronic work function, and e is the charge of the electron. As the temperature of the metal is raised, the kinetic energies of the electrons increase and hence more electrons can escape.

The effect is similar to the increase in the evaporation of a liquid with temperature. The heat of evaporation of the electrons, which is ϕe electron-volts per electron, is analogous to the latent heat of vaporisation of a liquid. This suggests a method for the determination of ϕ which was employed successfully by Davisson and Germer in 1922.

The electrons emitted by a hot metal accumulate in the space round the metal until there is a dynamic equilibrium between the electrons outside and inside the metal, similar to the equilibrium of a vapour over a liquid. If, however, the electrons are drawn away from the metal by a plate at a positive potential, electrons will continue to escape from the metal. Davisson and Germer measured the electrical energy which it was necessary to supply to a tungsten filament to maintain its temperature at a fixed value, (1) when a plate near to the filament was kept at a positive potential to draw off the electrons, and (2) when the plate was kept at a negative potential to prevent the emission of electrons. The difference in the energy inputs to the filament in the two cases gave the heat of evaporation of the electrons; while the number of electrons escaping in the former case could be calculated from the current flowing between the filament and the positive plate. The values of ϕ obtained were in good agreement with those obtained from the photoelectric effect.

The diode

In 1883 Edison inserted another electrode into an ordinary carbon filament lamp, an arrangement similar to Fig. 233, and discovered that a current will flow through the vacuum inside the bulb if the extra electrode is made positive with respect to the filament, but not if it is negative. The phenomenon is called the Edison effect, and it is now realised that the hot filament of the lamp is emitting electrons; the extra electrode attracts the electrons if it has a positive potential, but repels them if it has a negative potential.

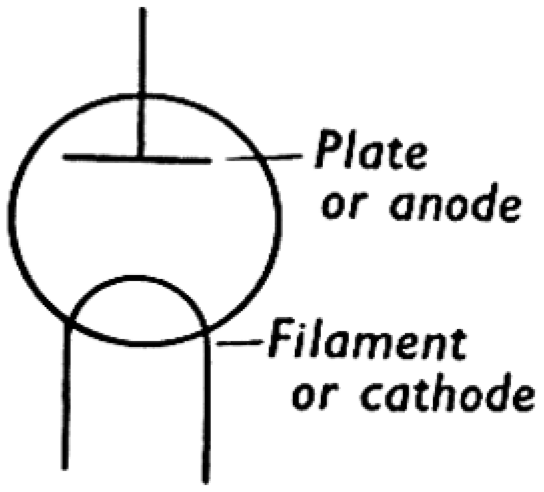


Fig. 233

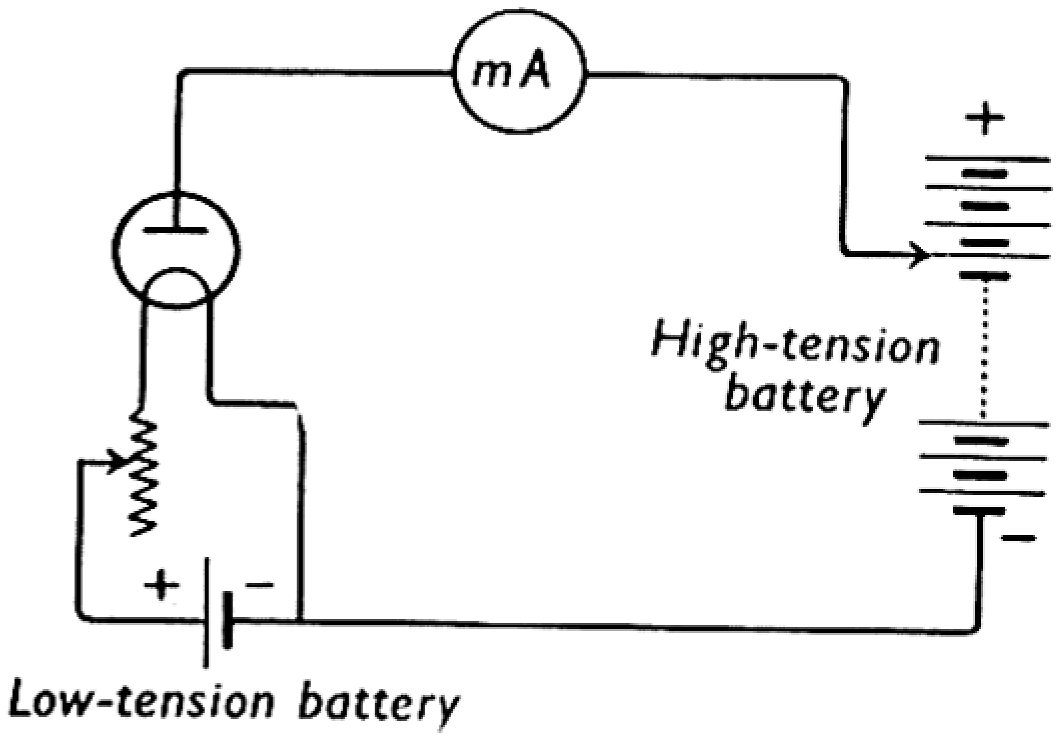


Fig. 234

In 1904 Dr (afterwards Sir Ambrose) Fleming used the device for radio reception and hence invented the first wireless valve. It is called a valve because it allows a current to pass through it in one direction only; alternatively it is called a diode because it has two electrodes.

The way in which the current through a diode varies with the P.D. between the anode and the filament can be investigated by the circuit in Fig. 234. The filament is heated by the 'low-tension' battery and the positive potential of the anode is varied by means of a 'high-tension' battery with tappings at suitable intervals. The current, measured with a milliammeter, flows through the diode, through the milliammeter, through the lower part of the high-tension battery, and through the circuit of the low-tension battery.

The graph of the current against anode potential is called the characteristic of the diode and is shown in Fig. 235. The graph flattens out at the top and the maximum current is called the *saturation current*; all the electrons emitted by the filament are then proceeding to the anode. It is natural to ask why all the electrons are not attracted to the anode as soon as a small positive potential is applied to the anode. The reason is that the space round the cathode is filled with a cloud of electrons called the *space charge*; these set up an electric field in opposition to that of the anode. The middle part of the graph is comparatively straight and over this portion the change in current is proportional to the change in anode potential. The effect of raising the temperature of the filament, by slightly increasing the heating current, is shown in the dotted graph in Fig. 235.

The filament of a valve is made of tungsten, which is usually coated with oxides of thorium or of barium and strontium, thereby increasing its electron emission by as much as twenty times; an oxide-coated filament is run at a much lower temperature than a non-coated filament and continues to emit for a long period. Usually, in an A.C. mains operated wireless receiver, valves with indirectly heated cathodes are used. The cathode takes the form of an oxide-coated metal tube through the middle of which, insulated from it, runs a heating filament. This construction gives a cathode system which has a rather high thermal capacity and so prevents any fluctuation in the emission of electrons due to a variation in the heating current.

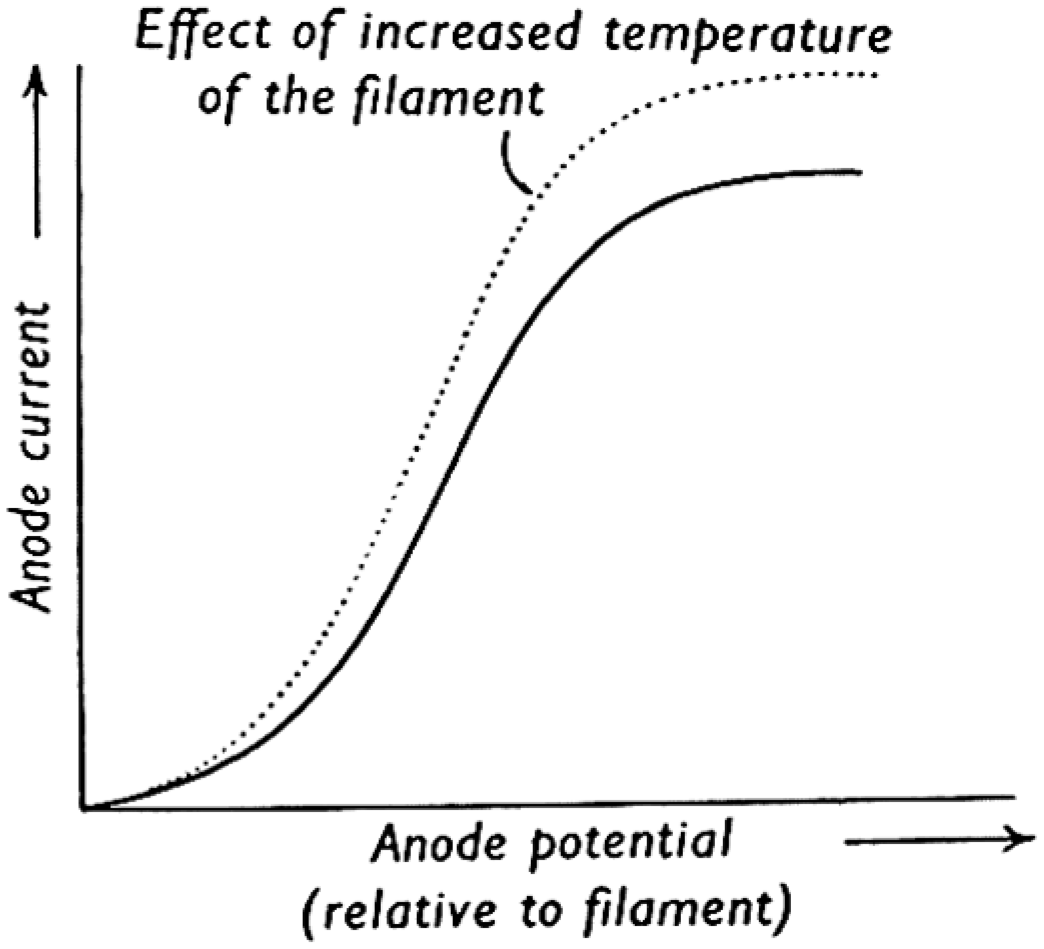


Fig. 235

The diode as a rectifier

The diode may be used to rectify alternating current. A single diode produces half-wave rectification in which half of the A.C. is not used (Fig. 236). An alternating P.D. is applied to the anode of the diode by means of a transformer. The method of heating the filament is not shown in the figure; a battery may be employed or, as is more usual, an extra small secondary coil on the transformer. A positive, conventional current can flow through the diode in one direction only, from anode to filament (actually electrons flow in the opposite direction).

Thus the output current must flow from the terminal connected to the filament, marked + in the figure.

Fig. 237 represents full-wave rectification in which both halves of each cycle of the A.C. are utilised. The valve is a double-diode containing one filament and two anodes. The secondary of the transformer has a centre-tap which is connected to the negative output terminal. As in half-wave rectification the filament must be connected to the positive output terminal since a positive conventional current can flow through the valve only from one of the anodes to the filament, and not in the reverse direction.

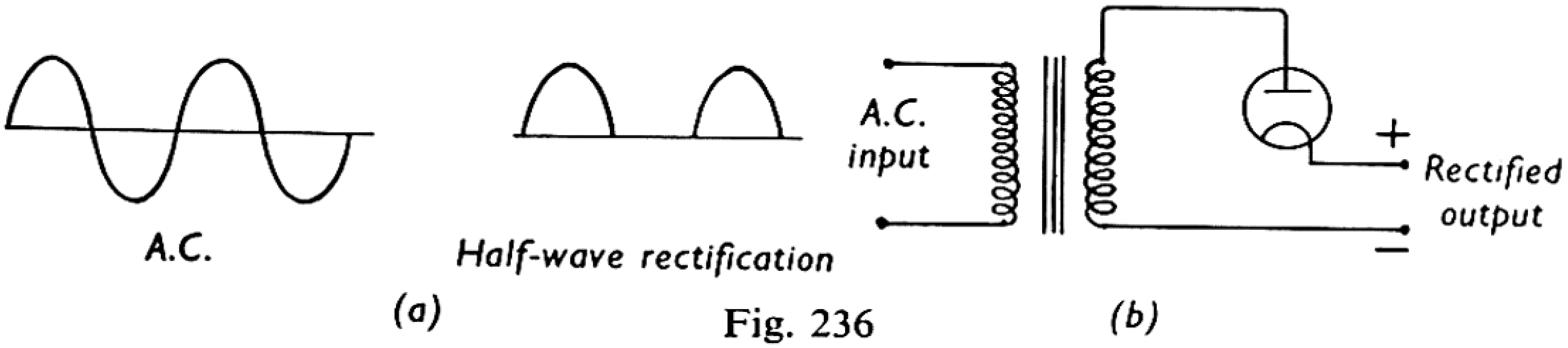


Fig. 236

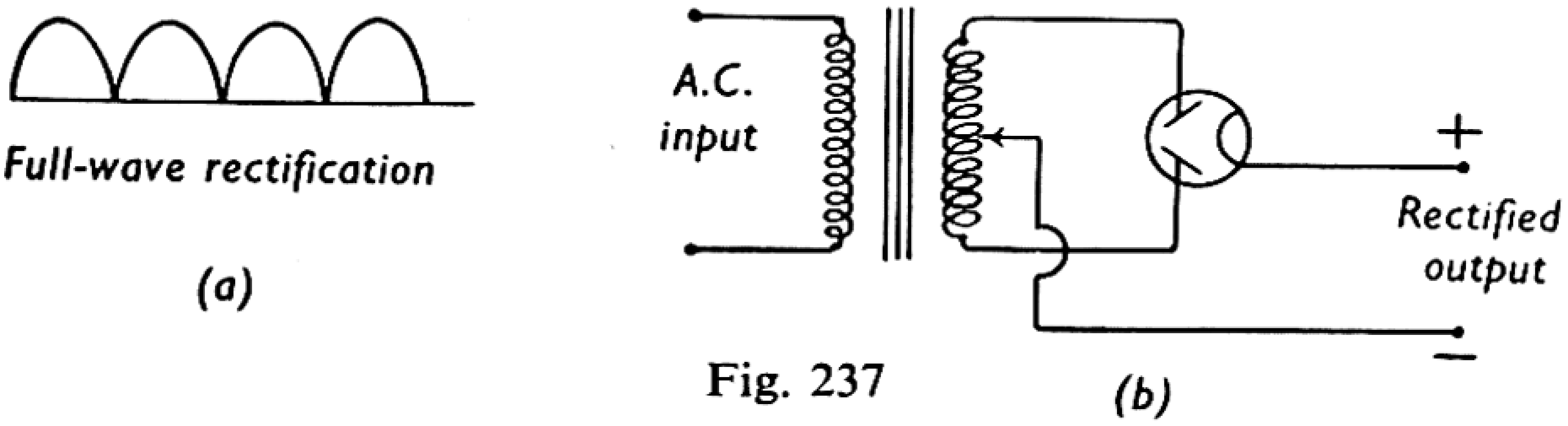


Fig. 237

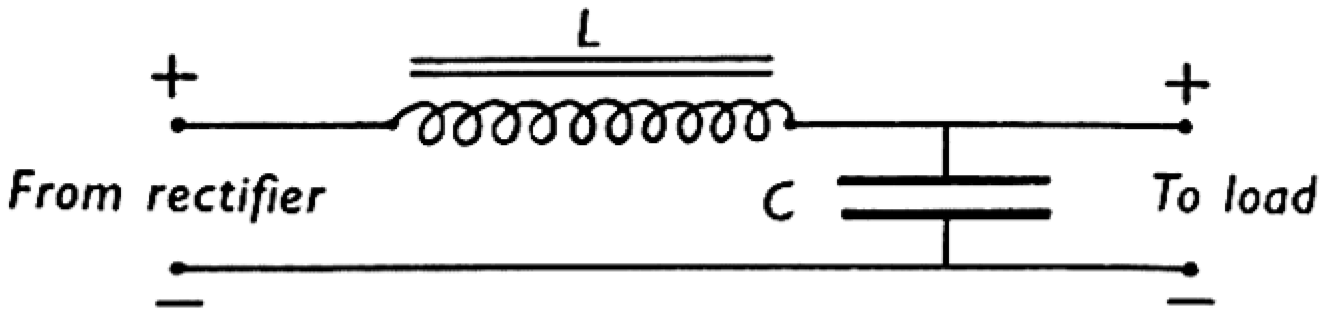


Fig. 238

The secondary of the transformer may be regarded as two separate coils, each of which is used only for alternate half-cycles. The two circuits operate like Fig. 236 (*b*).

The output current of a rectifier is pulsating; it can be smoothed by a filter similar to that in Fig. 238 composed of a choke *L* and a condenser *C*. The back E.M.F. induced in *L* tends to oppose the voltage changes, and thus reduce the variations in current, while *C* acts as a reservoir to steady the remaining fluctuations.

The triode

In 1907 Lee de Forest introduced a third electrode into the valve and so produced the triode (Fig. 239), which has become as important and fundamental in wireless transmission and reception as the telescope in astronomy or the microscope in biology. There are now about 1000 types of valve in use, but the triode is the prototype because it embodies the fundamental idea of a control electrode or grid, *G*.

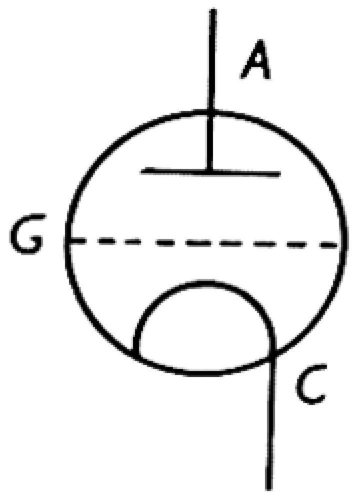


Fig. 239

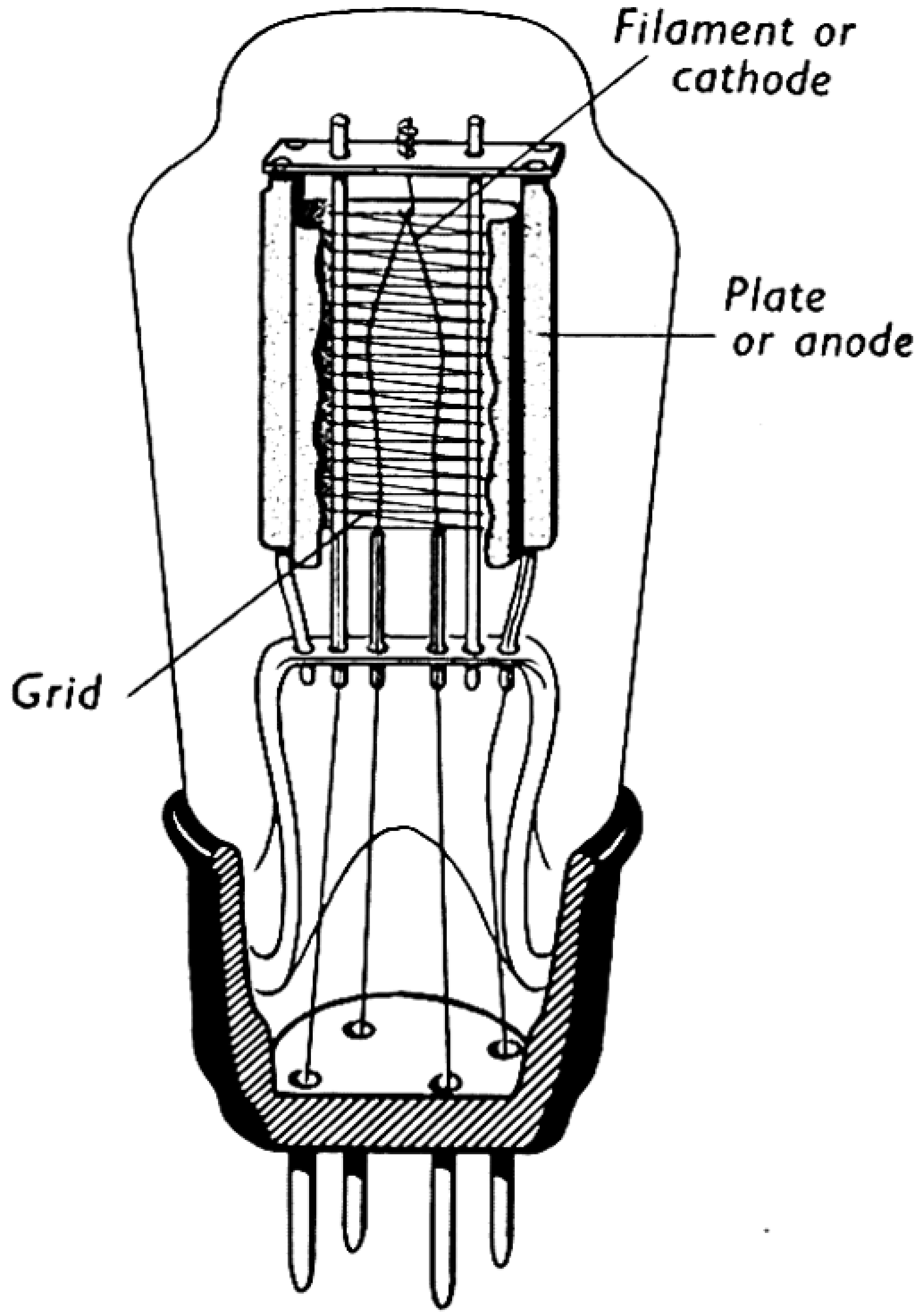


Fig. 240

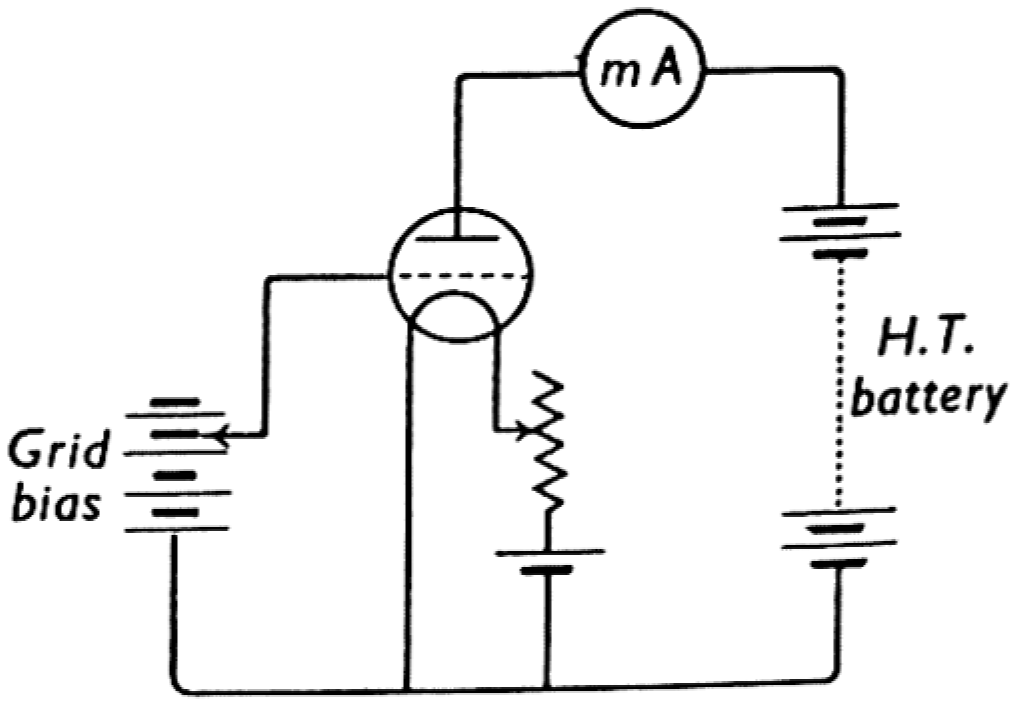


Fig. 241

The interior of a triode is shown in Fig. 240. The cathode or filament is a coated tungsten wire and round it is the grid in the form of an open spiral of nickel wire; the anode or plate surrounds both. The grid is much more effective than the anode in controlling the current through the valve because it is nearer to the filament; a change of voltage on the grid causes a bigger change in the electrostatic field near to the filament than an equal change in the voltage of the anode. Because of their high velocity the electrons do not follow the lines of force but shoot straight through the grid spaces which cover a much greater area than the wires.

If the grid voltage is kept constant and the anode voltage, V_a, is varied, the graph of V_a and of current between filament and anode, i_a, is similar to that for a diode (Fig. 235).

The most interesting and useful graphs are those for V_g, the voltage of the grid, and i_a, when V_a is kept constant. These are called the mutual characteristics of the valve.

Fig. 241 shows the circuit by which these characteristics may be obtained. A constant potential difference, say 100 volts, is maintained between the anode and filament by the high-tension battery and the current in the milliammeter is read as the voltage on the grid is varied by means of the grid-bias battery.

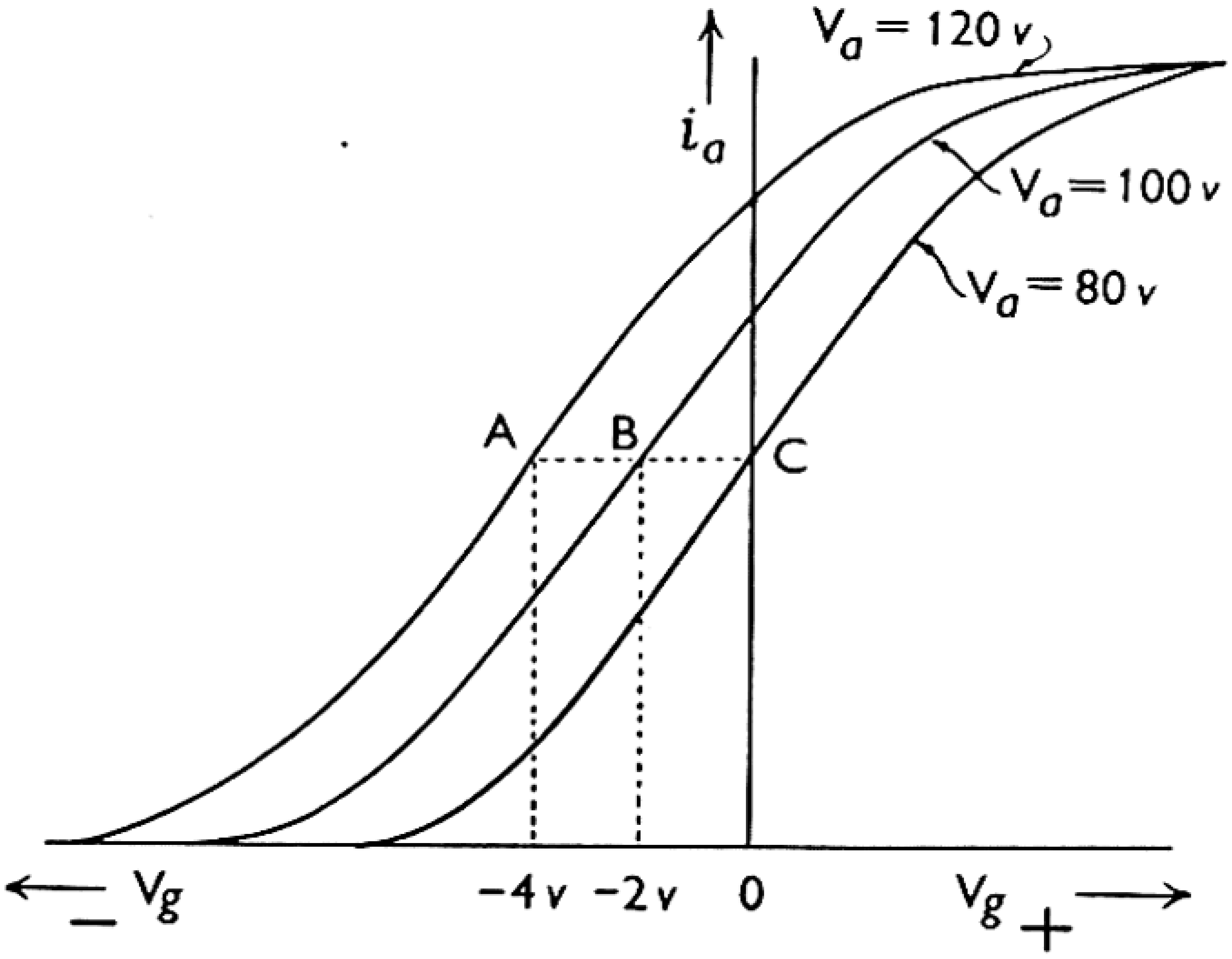


Fig. 242

Three characteristics, for anode voltages of 80, 100 and 120, are shown in Fig. 242. Consider points *A*, *B*, *C* on the three characteristics, and suppose that the grid voltages at these points are -4, -2 and 0 volts respectively. A change can be made from *A* to *B*, or from *B* to *C*, without altering the current, by a simultaneous increase in the grid voltage of 2 and a decrease in the anode voltage of 20. Thus a change of 2 volts on the grid has the same effect on the current as a change of 20 volts on the anode. The valve is said to have an *amplification factor* of $\frac{20}{2} = 10$.

Alternatively we may define the amplification factor, μ, as

$$\frac{\text{Change in anode voltage to produce a small change in anode current}}{\text{Change in grid voltage to produce the same change in anode current}},$$

i.e.

$$\mu = \frac{\delta V_a}{\delta V_g}.$$

The use of the triode in wireless receivers and oscillators will be discussed in Chapter 16.

The cathode-ray oscillograph

The cathode-ray oscillograph (Fig. 243) is an instrument whose fundamental principle is the deflection of an electron beam by an electric (or magnetic) field and it may be used for showing the wave-form of A.C. and of sound waves, for radiolocation, for television and for numerous other purposes. It has become, in recent years, an extremely valuable and important instrument.

A stream of electrons from a heated filament, *C* (the cathode), is focused on to a fluorescent screen, *S*, in an evacuated glass bulb, and may be deflected horizontally and vertically by P.D.'s between the plates *HH* and *VV* respectively. Since the inertia of the electrons is very small they respond instantly to rapid changes in voltage between the plates *HH* and *VV*. The perforated metal cylinder, *G*, is maintained at a negative potential with respect to *C* and hence repels the electrons. By varying its potential the flow of electrons, and hence the brightness of the spot on the screen, can be controlled; as it is a control electrode it is called a grid. The hollow cylindrical anode *A* is maintained at a positive potential of 500 volts or more with respect to *C* and serves to accelerate and focus the stream. *A* is connected to *S* so that the electrons travel at uniform speed after leaving *A*. The screen *S* is coated with a thin opalescent layer of a salt such as calcium tungstate, zinc phosphate, or zinc orthosilicate, which fluoresces a blue, green or bluish white colour when struck by fast-moving, charged particles.

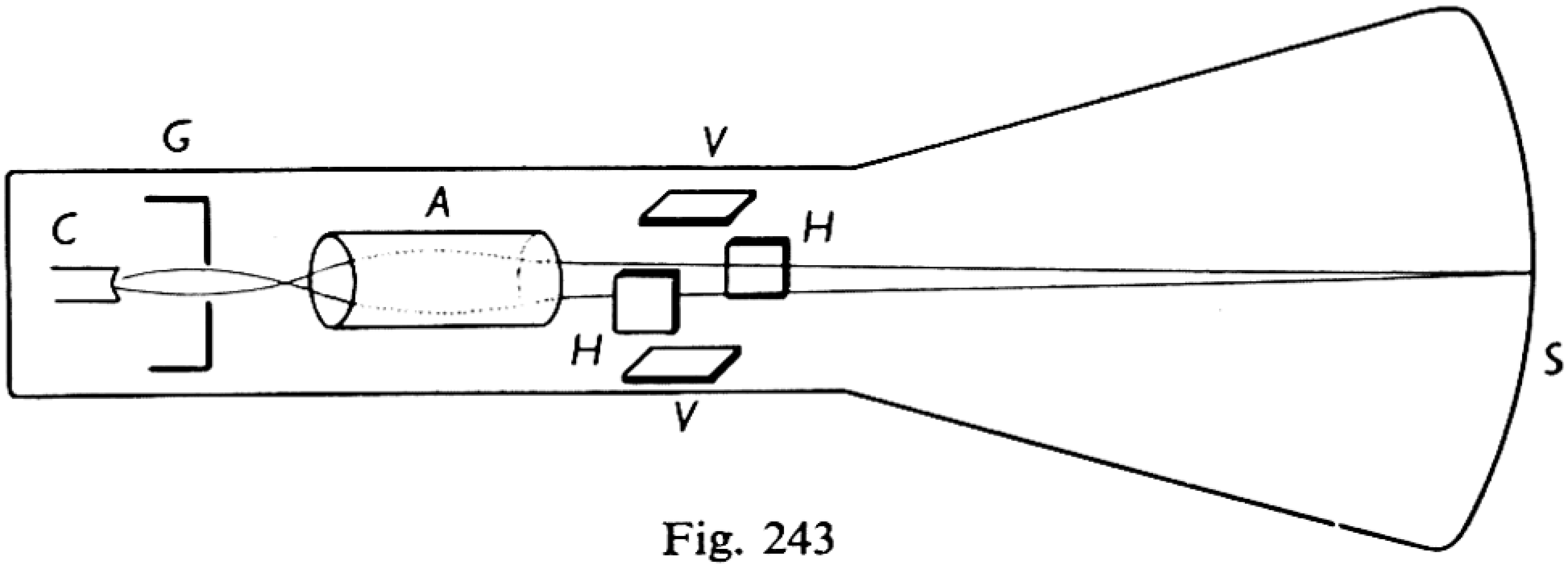


Fig. 243

The time-base

To show the wave-form of an alternating voltage as in Fig. 244 the spot of light on the screen of the cathode-ray oscillograph must be moved horizontally at a uniform rate while the varying P.D. of the A.C. is applied to the plates *VV* to cause vertical deflections.

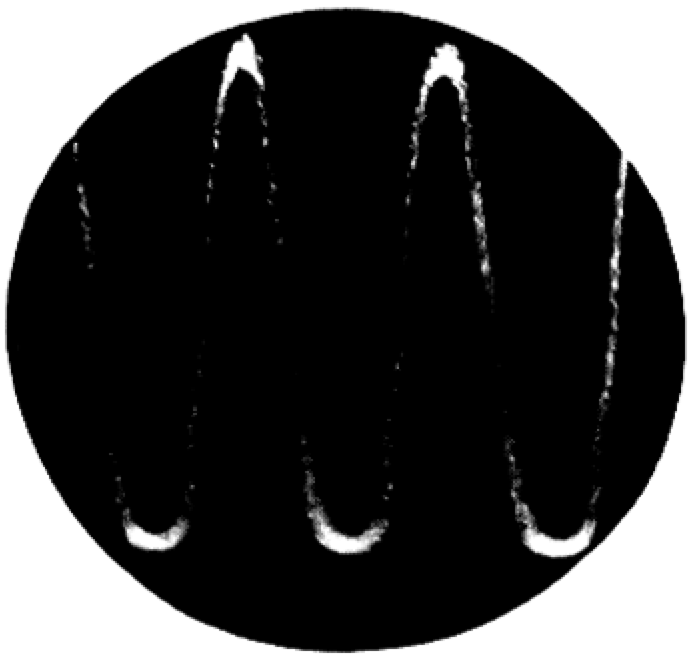


Fig. 244. Photograph of a trace obtained with a cathode ray oscillograph showing the wave form of A.C. mains.

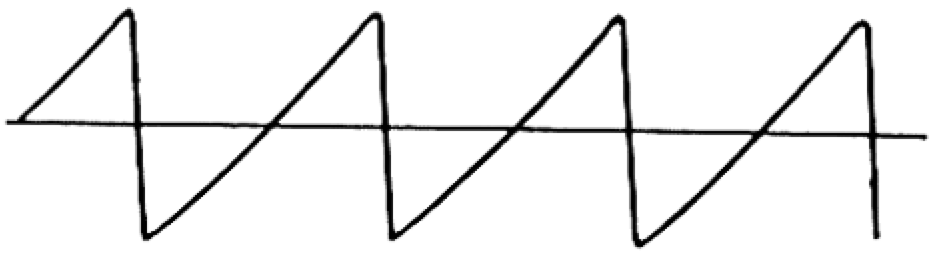


Fig. 245

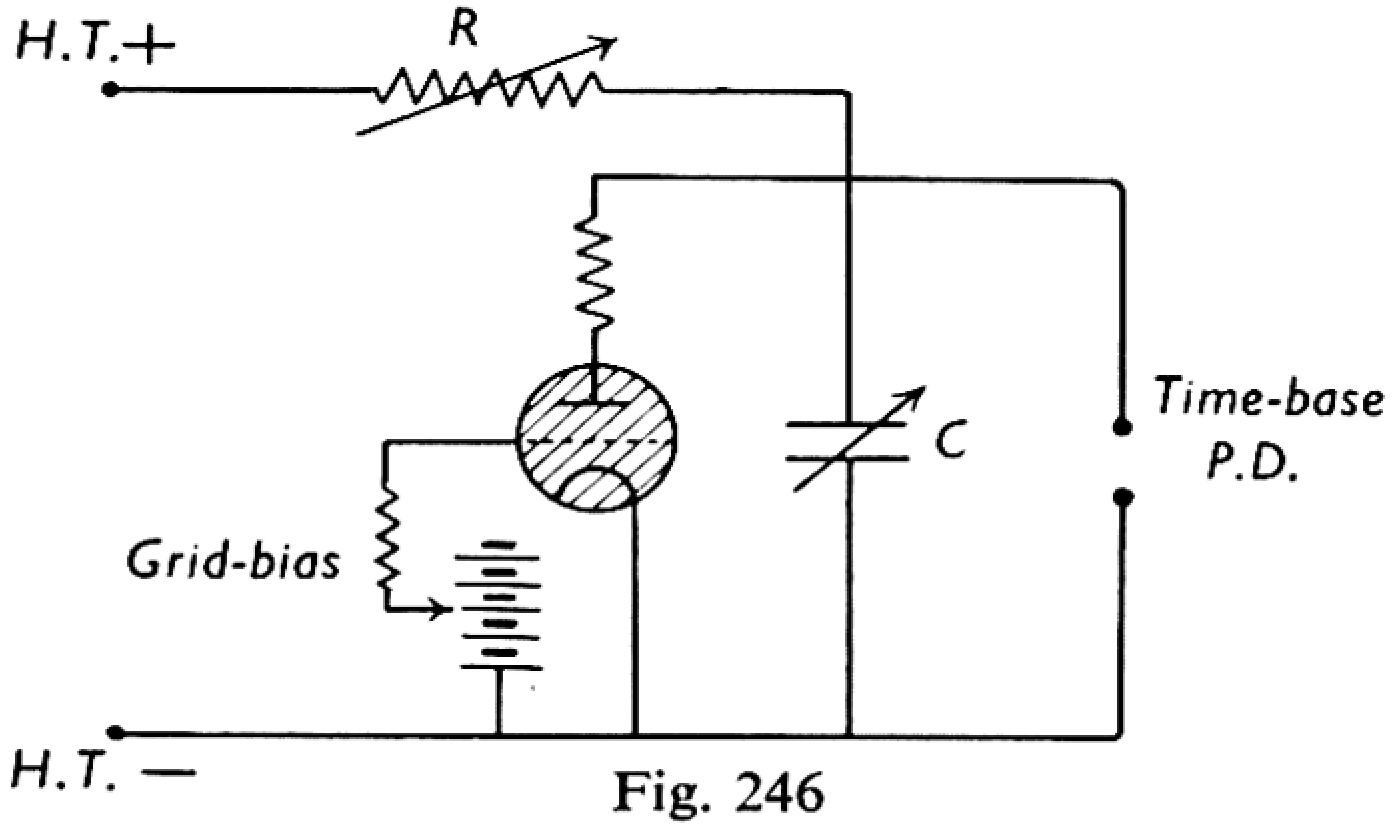


Fig. 246

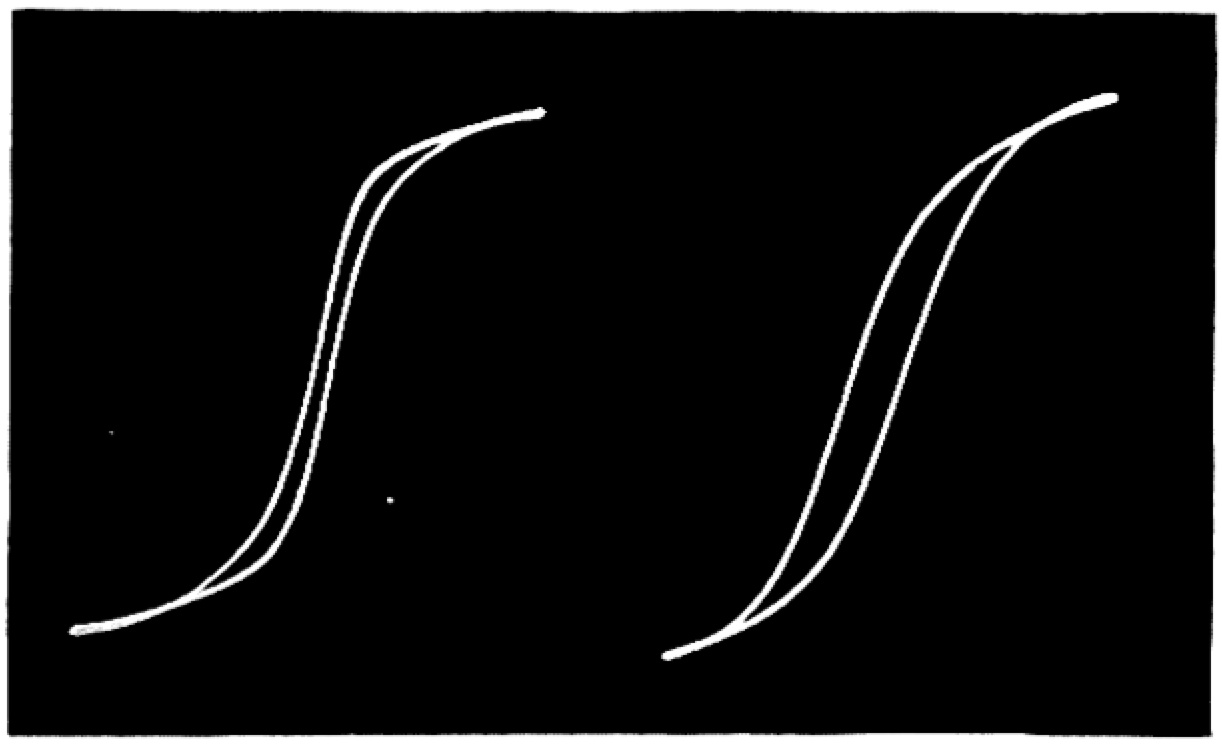


Fig. 247. *B*/*H* curves of mild steel (left) and stalloy (right) obtained with a cathode-ray oscillograph (compare with Fig. 116). The method employed is as follows. The iron under test is in the form of a closed ring; round it are wrapped a primary coil which carries the magnetising A.C. (proportional to *H*) and a secondary coil in which the induced E.M.F. is proportional to the rate of change of the flux, i.e. to dB/dt. The primary and secondary coils are connected through a suitable circuit to the two pairs of plates of the cathode-ray oscillograph so that the varying P.D.'s are proportional to *H* and *B*.

The horizontal motion of the spot of light is controlled by a time-base, the output of which must have a saw-tooth wave-form as in Fig. 245. The spot of light must move uniformly in one direction and then fly back as rapidly as possible.

A simple time-base circuit is shown in Fig. 246. The condenser *C* is charged by a constant current through a high resistance, *R*, so that there is a linear increase of P.D. across *C* with time. To ensure that this is approximately the case the H.T. voltage must be at least ten times the peak P.D. across *C*, say 1000 volts. Variation of the value of *C* or of *R* enables the time-base frequency to be varied. The valve is a gas-filled triode which passes no anode current until the anode potential reaches a critical value, depending on the grid bias. When the critical potential is reached the condenser is very rapidly discharged through the valve.

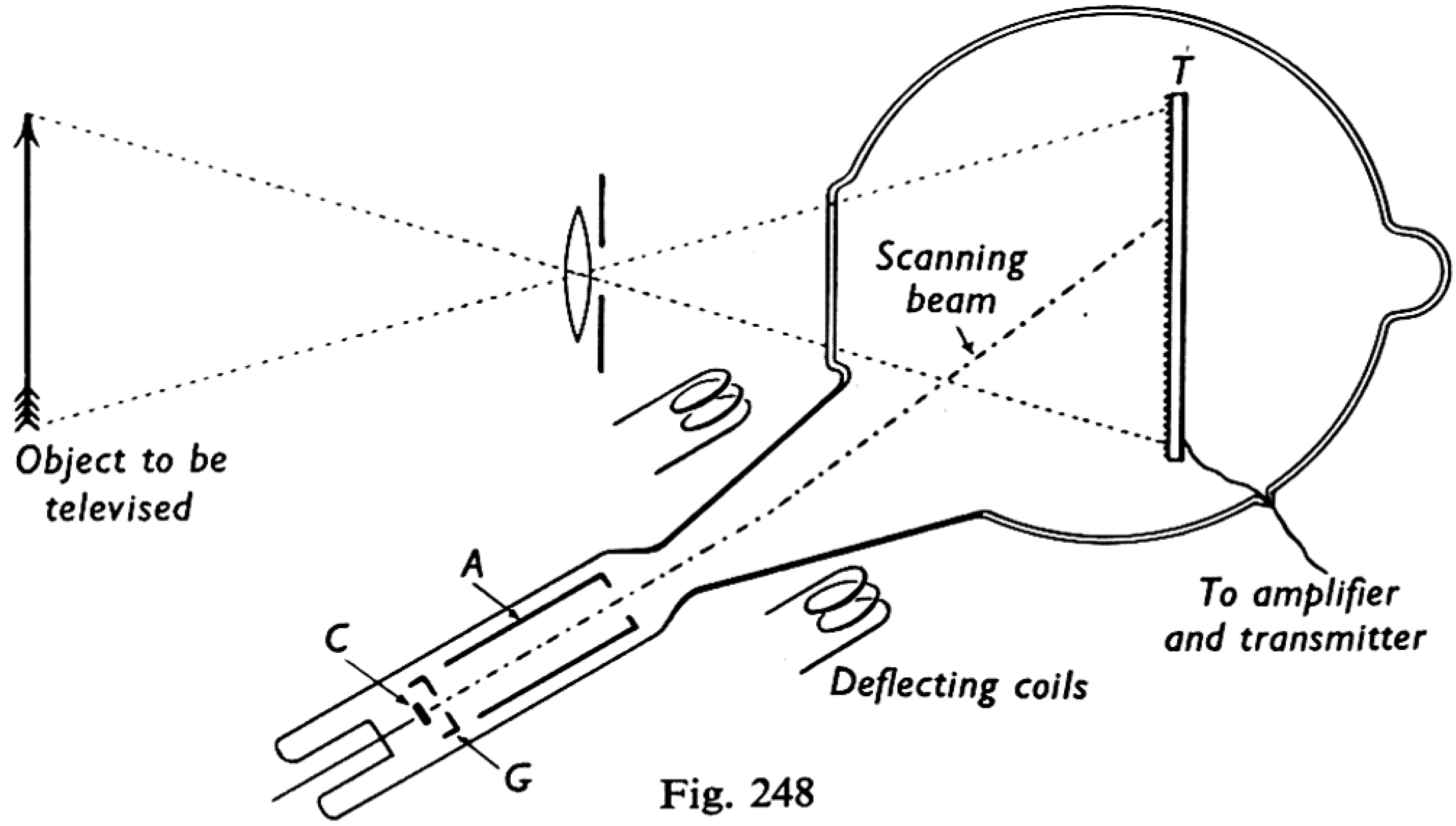


Fig. 248

Television

Television transmission consists in transforming the variations in brightness of a picture into corresponding fluctuating electric currents, amplifying the currents and broadcasting them as electromagnetic waves; reception consists of amplifying the fluctuating currents from the receiving aerial and converting them back into a picture.

Fig. 248 shows the principle of the Emitron camera used by the B.B.C. An image of the object to be televised is focused by means of a lens on to a target plate, *T*, in an evacuated glass bulb. *T* consists of a sheet of mica on the front of which are millions of tiny photoelectric cells in the form of caesium-silver nodules, insulated from each other. On the back of the mica is a thin metal coating. When the image falls on T electrons are emitted, more electrons being emitted from the bright parts than from the dark parts.

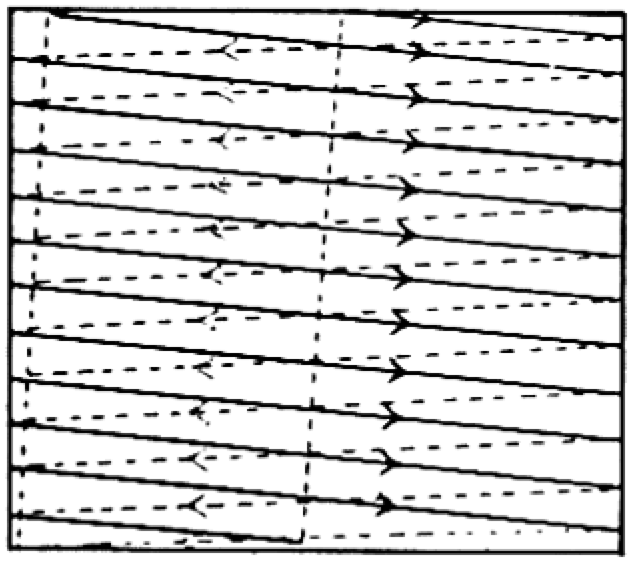


Fig. 249

T is now 'scanned' by a beam of electrons. The process of scanning is something like reading a page of a book; the electron beam runs from side to side and works down the plate. The B.B.C. scan the target in 405 horizontal lines, the process being repeated 25 times per second. It has been found that flicker can be reduced by 'interlaced scanning' (Fig. 249); the electron beam scans alternate lines, say lines 1, 3, 5, etc. first and then returns to scan the even lines 2, 4, 6, etc. The return strokes of the scanning beam, represented by discontinuous lines in Fig. 249, must be very rapid compared with the scanning strokes.

The scanning beam is produced by a cathode-ray tube very similar to the oscillograph already described. C is a heated filament which emits electrons and A is an anode which accelerates them. Clearly two time-bases are required to produce the necessary horizontal and vertical motion of the electron beam; instead of electrostatic deflection, however, by plates similar to HH and VV in Fig. 243, magnetic deflection is provided by means of two pairs of deflecting coils, one pair of which is indicated diagrammatically in Fig. 248.

Each photoelectric nodule on T, together with the mica and the metal plate at the back, acts as a tiny condenser. When the photoelectrons are discharged as a result of the light image falling on T, the nodules are left with a positive charge. The scanning electron beam replaces the lost electrons and so enables the back plate of the condenser to send a discharge to the transmitting circuit. Thus during the process of scanning, a series of discharges, varying in magnitude according to the variations in brightness of the image, pass to the transmitter.

For reception a cathode-ray oscillograph is used. The fluctuating P.D.'s from the receiving aerial, after amplification, are applied between G and C in Fig. 243, and hence the brightness of the spot of light is varied. Two time-bases, connected to the plates HH and VV, cause a scanning motion of the electron beam which must correspond exactly to the motion of the scanning beam at the transmitter. The picture is then reproduced on the screen S.

The electron microscope

The resolving power (or ability to show detail) of an optical microscope is limited by the wave-length of light. The image of a point formed by a lens consists of a bright spot surrounded by dark rings. The image systems of two points very close together overlap and it can be shown that two points on the object, which are closer than about half a wave-length, cannot be separately distinguished in the image however great the magnification produced by a microscope.

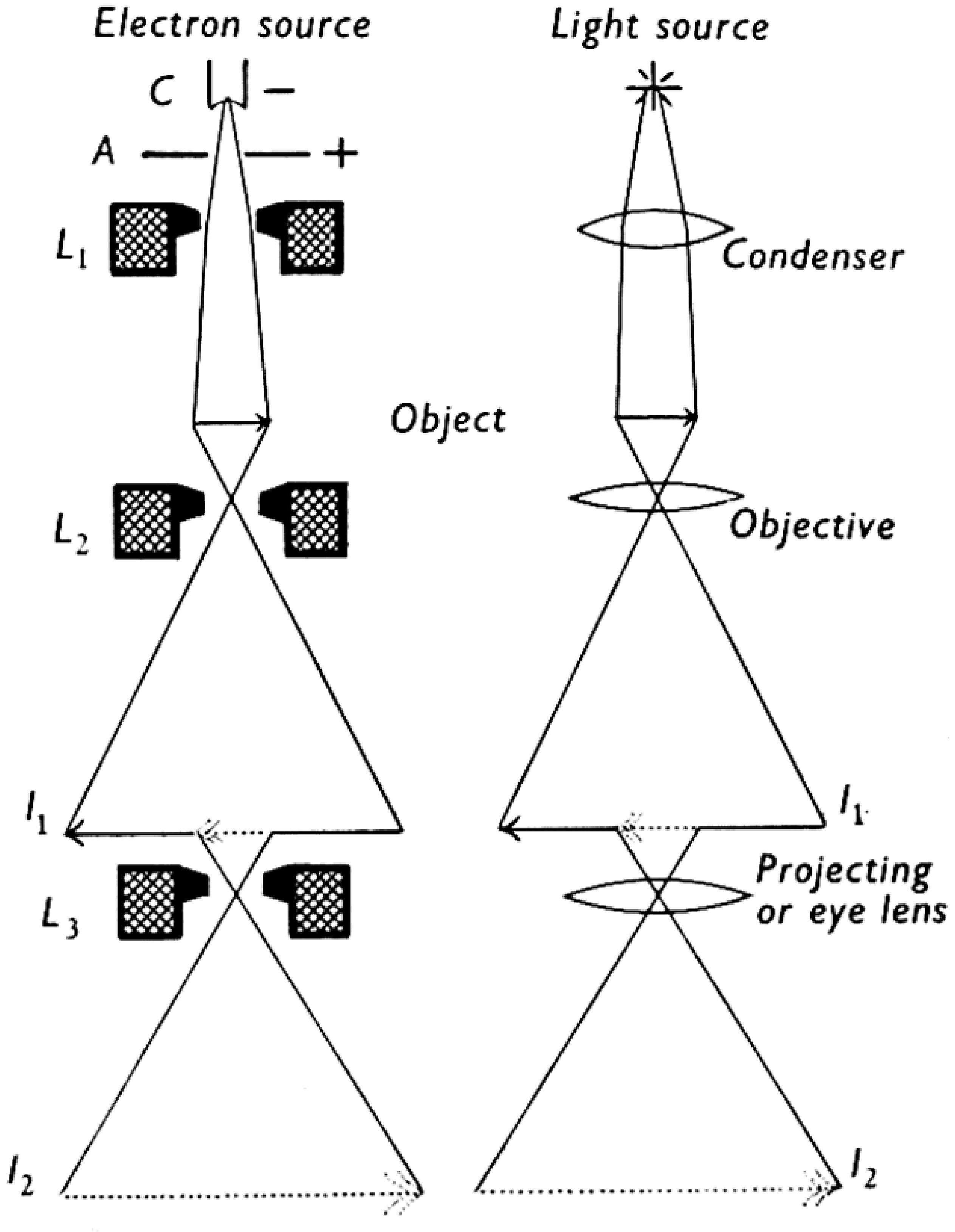


Fig. 250

Electrons may be focused by electric or magnetic fields, and hence they may be used instead of light. Because of the minute size of electrons, an electron microscope might be thought capable of revealing atoms. As far as resolution is concerned, however, electrons behave like waves, and the wave-length is shorter the greater their speed. When accelerated under a P.D. of 60 kV.—a P.D. commonly used in electron microscopes—the electrons have a wave-length only $\frac{1}{100,000}$th that of light. Thus much greater resolving powers are possible with an electron microscope than can be obtained with an optical one.

The general arrangements of the two instruments are identical, as can be seen from Fig. 250. The source of electrons is a heated filament, C, acting as a cathode, and the electrons are given a high speed by an anode, A, maintained at a P.D. with respect to C of about 60 kV. L_1 is a condenser lens which serves to concentrate the electron beam on to the object to be viewed. L_2 is the objective forming an image I_1, and L_3 is the projecting lens throwing a final image I_2 upon a fluorescent screen or a photographic plate. Since electrons are scattered by air the microscope must be evacuated as completely as possible.

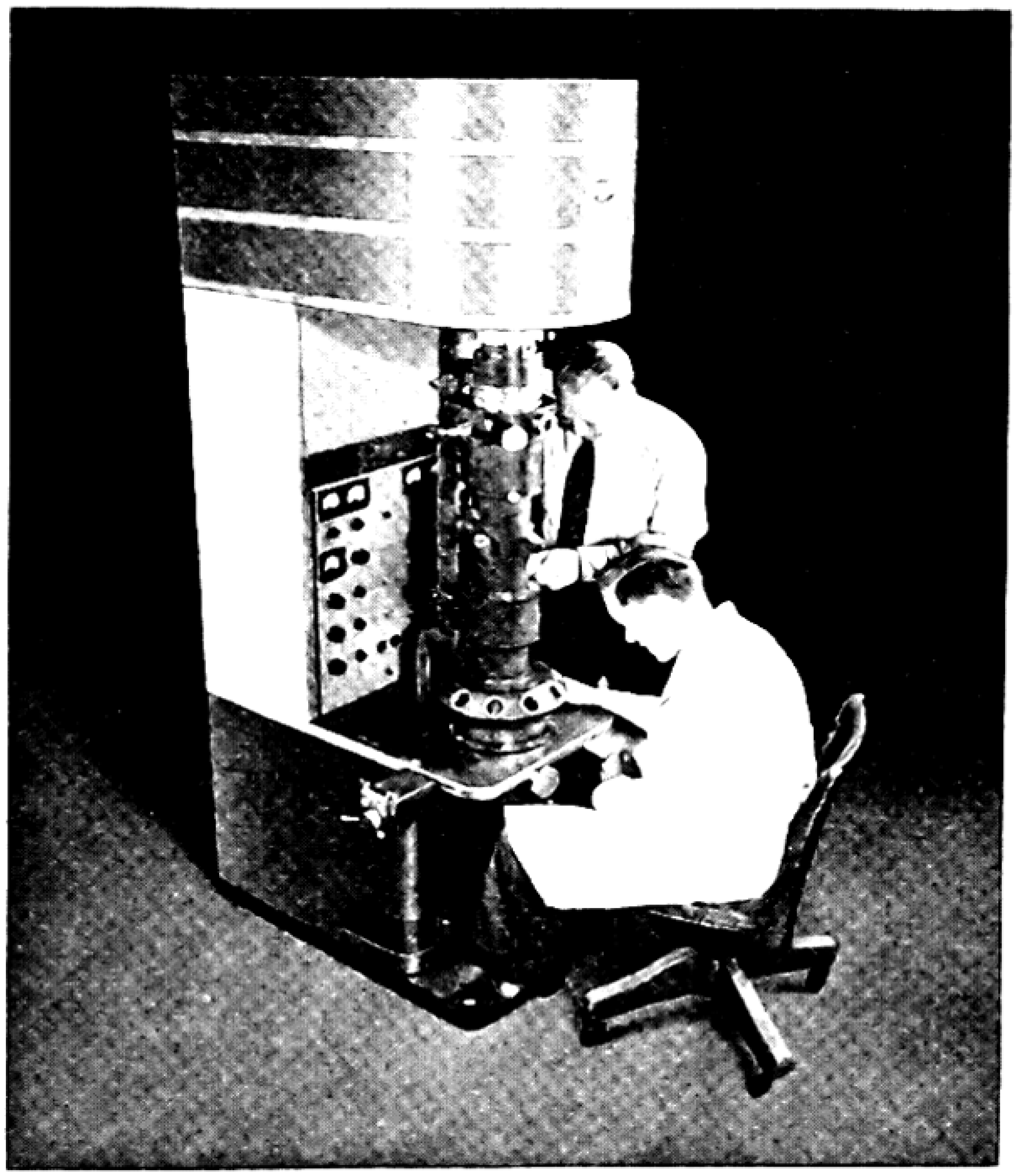


Fig. 251. The electron microscope. The man sitting down is observing the image through the small circular windows while the man standing up is focusing the image by altering the focal length of the projector lens.

A typical form of magnetic lens, used in preference to an electrostatic lens, is shown in Fig. 253. It consists of a short solenoid entirely encased in steel, except for a brass ring in the middle of the circular pole piece. The magnetic flux would be confined largely to the steel were it not for the brass ring, which causes a leakage of flux into the space in the middle of the pole piece.

Consider an electron moving down through the magnetic lens (Fig. 254). There is a component of the flux from left to right at right angles

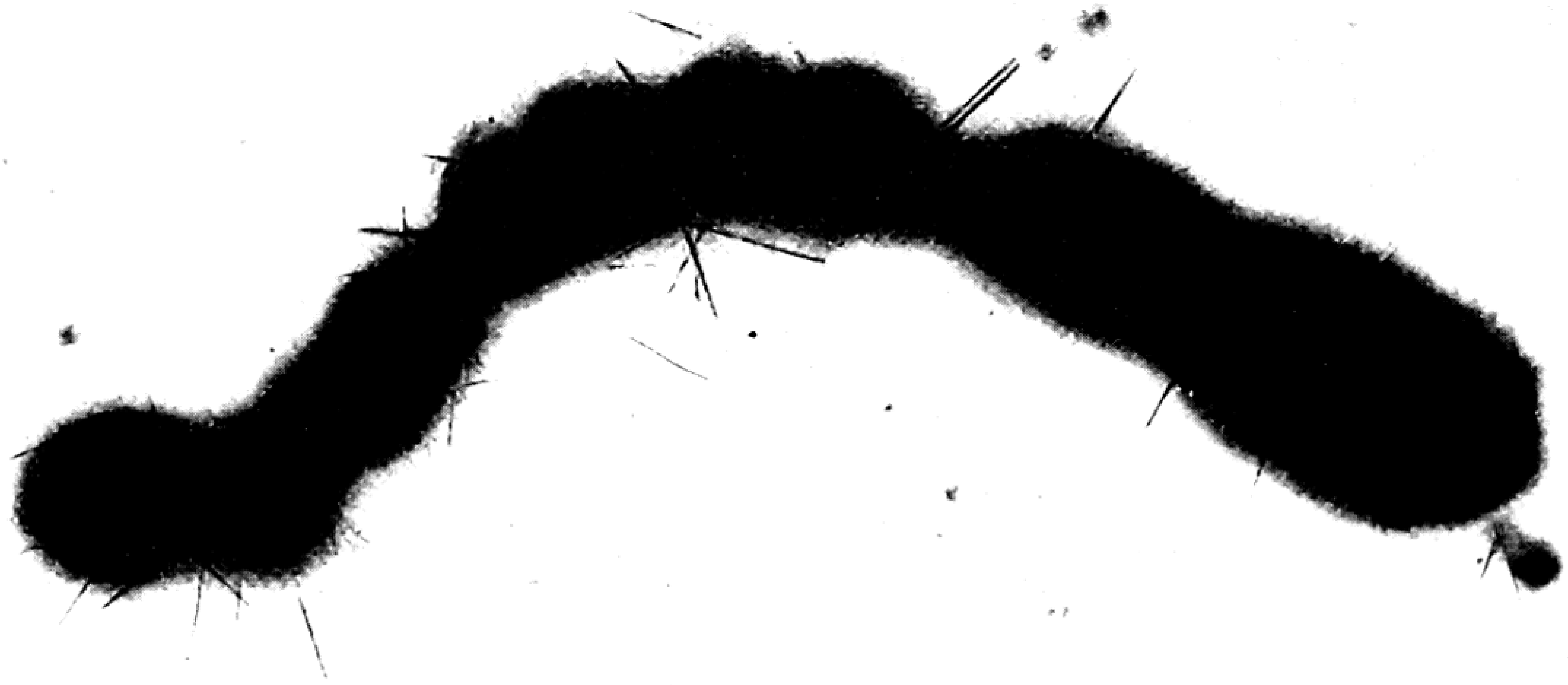


Fig. 252. Microbe of common sore throat (21,500 ×). This has been reduced in size from the original photograph, taken with an electron microscope, which showed a magnification of 43,000 × .

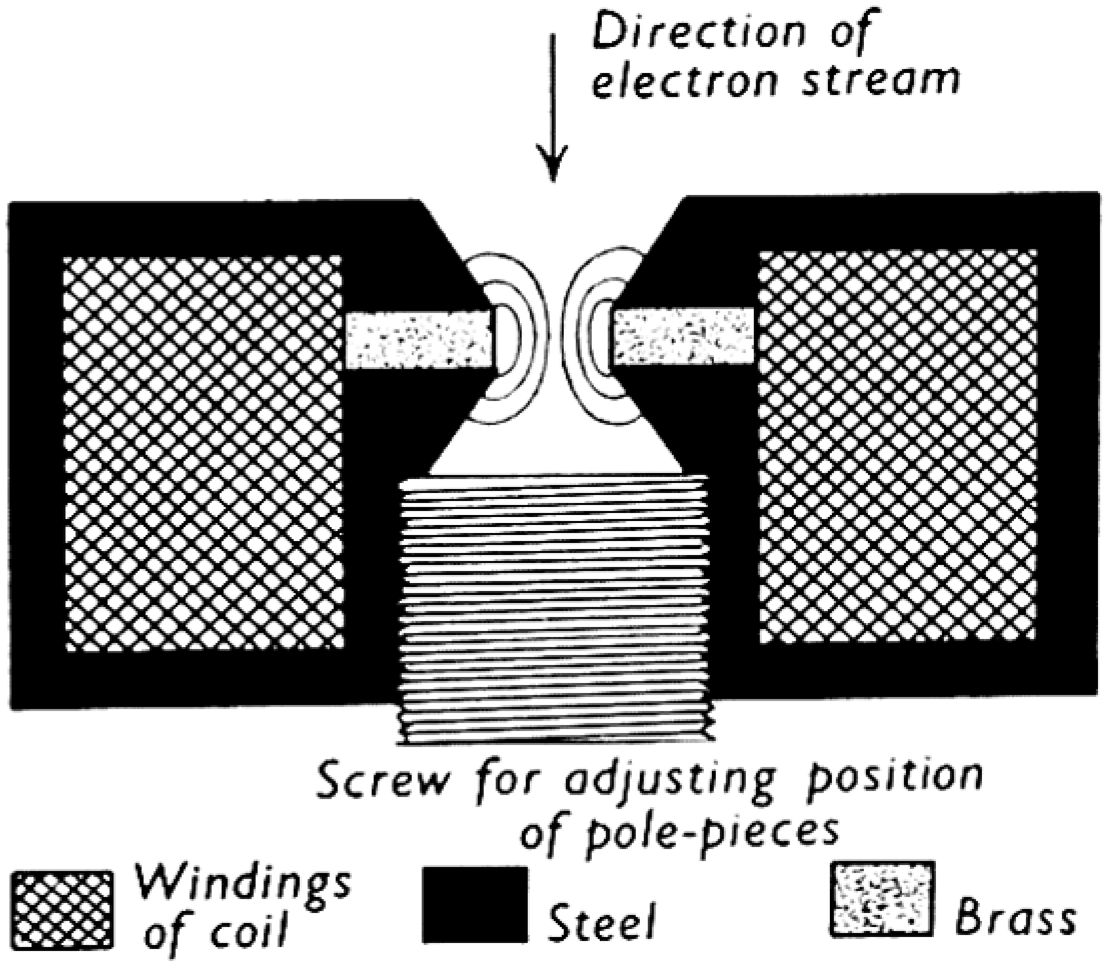

Direction of electron stream
Screw for adjusting position of pole-pieces
Windings of coil
Steel
Brass

Fig. 253

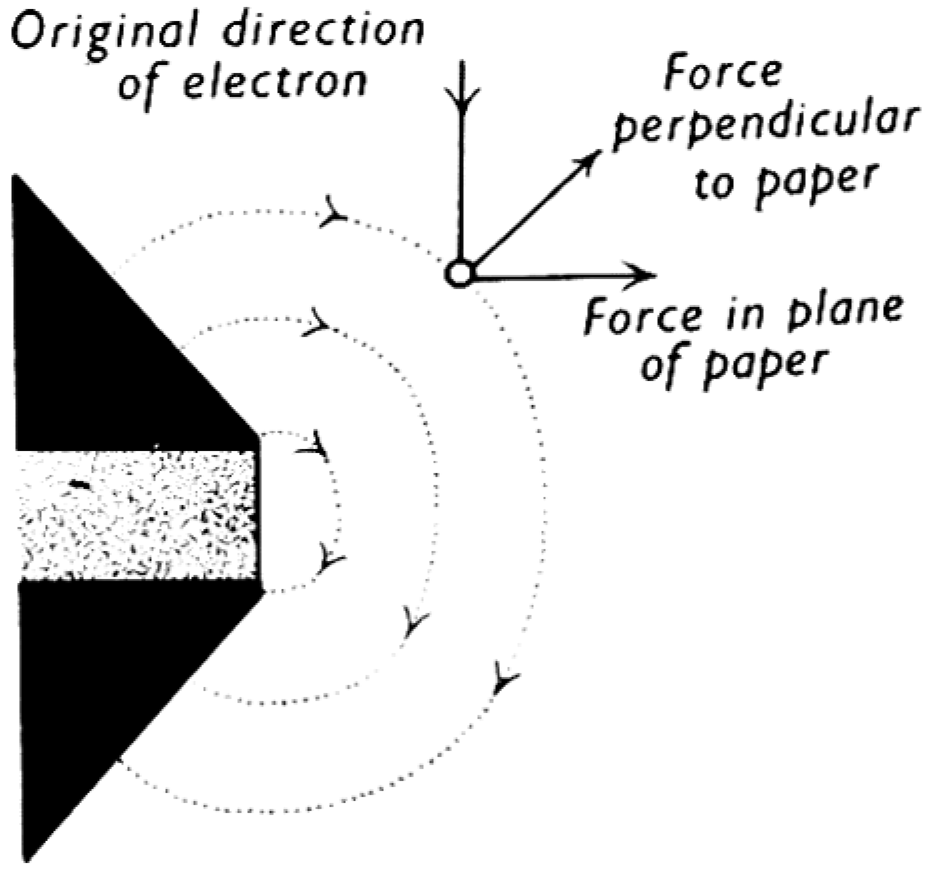


Fig. 254

to its direction of motion and applying Fleming's left-hand rule and remembering that the direction of the positive current is opposite to that of the electron, the electron will be found to be urged into the paper. When it has a motion at right angles to the paper, because of the component of the field downwards, it will be urged to the right. An electron nearer to the pole piece, being in a stronger field, will experience a stronger force to the right. Thus the electrons can be brought to a focus, but at the same time the image is rotated through an angle. (The final image is rotated through an angle of about 60° with respect to the object.)

The lenses in the electron microscope are not moved up and down as in the optical microscope; their focal lengths are adjusted by varying the currents through the coils.

Of the electrons which reach the object to be viewed some pass freely while others are stopped by the denser parts. The final image is thus a shadow-graph, similar to that obtained with X-rays. It is necessary that the object should be in the form of a very thin slice or section—a few millionths of an inch thick if the image is to show light and shade. This makes the use of the microscope difficult and restricted although there is very considerable scope for its use, particularly in chemistry, metallurgy and biology.

QUESTIONS

1. Give an account of the Seebeck and the Peltier effects showing how you would demonstrate them.

These two effects are said to be reversible effects. What is meant by such a statement? (O.)

2. Explain, with diagrams, how you would set up a potentiometer circuit suitable for measuring the E.M.F. of a thermocouple. Indicate the approximate voltages and resistances of the parts of the apparatus if the E.M.F. to be measured is about 0·1 volt and the potentiometer wire has a resistance of 5 ohms. (O.)

3. A copper-eureka thermocouple has a resistance of 1 ohm and is connected to a millivoltmeter of resistance 10 ohms. When one junction is in ice and the other in steam at 100° C. the instrument records 4·17 volts. The value calculated from the known constants of the metals is 4·596 millivolts. How do you account for the discrepancy? (B.)

4. Give an account of the Seebeck and Peltier effects, and explain how they are related.

A thermocouple is made up in the usual way, with 2 metres of iron wire of 1 sq. mm. cross-section, and 2 metres of constantan of the same cross-section. The average specific resistance over the range of temperature concerned is 15×10^{-6} ohm cm. units for iron, and 50×10^{-6} ohm cm. units for constantan. The E.M.F. of this couple is 0·000063 volt per degree difference in temperature between the two junctions. It is connected in series with a sensitive galvanometer of resistance 5 ohm, and the galvanometer records 0·005 amp. Find the temperature difference between the junctions of the thermocouple. (O.)

5. A thermojunction of iron and copper connected to a galvanometer cannot be used to measure temperatures above about 180° C., whereas one of iron-constantan may be used up to at least 600° C. What is the explanation? (C.)

6. Explain in outline the method you would use to verify (*a*) the law of intermediate metals, (*b*) the law of intermediate temperatures, as applied to thermocouples. Give full experimental details for one method.

7. An iron–copper thermocouple gives an E.M.F. of 15 microvolts per degree difference in temperature between hot and cold junctions. What arrangement of six such iron–copper thermocouples, each of resistance 1 ohm, will give the maximum deflection in a galvanometer of resistance $1\frac{1}{2}$ ohms? If the galvanometer has a sensitivity of 2/3 division per microampere, what is this maximum deflection when the difference in temperature of the junctions is 10° C.? (C. S.)

8. Le Chatelier's principle is as follows: 'If one of the factors of any system in equilibrium is changed, thus disturbing the equilibrium, the effect produced tends to restore that factor to its original value.'

Discuss the application of the principle to the phenomena of thermoelectricity, the thermionic effect and self-induction.

9. Describe a form of photoelectric cell and give a short account of the uses to which such a cell can be put. (C. S.)

10. The following facts are known about emission of photoelectrons from a metal surface: (*a*) the rate of emission is proportional to the intensity of incident light, (*b*) the ratio $\frac{\text{emission}}{\text{intensity}}$ varies with the wave-length, (*c*) the maximum velocity of emission is given by $v^2 \propto \left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$, where λ is the wave-length and λ_0 a constant characteristic of the metal. Devise a means of checking these facts. (O. S.)

11. Explain the construction and mode of action of a *diode valve*. If the filament current is kept constant, how will the current through the valve vary when the plate voltage is gradually increased? What would occur if the experiment were repeated with a slightly higher filament current? (B.)

12. A neon lamp has two electrodes. When connected to D.C. the positive electrode glows, and when connected to A.C. both electrodes glow. Explain which electrodes glow in the neon lamps *A* and *B* in the circuit of Fig. 255.

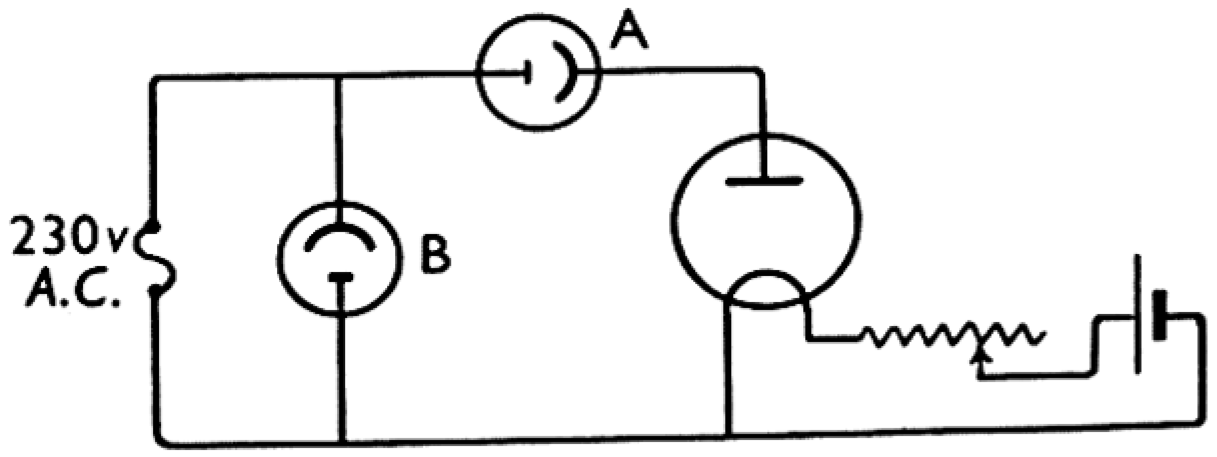


Fig. 255

13. How may the diode be employed to rectify an alternating current? (B.)

14. A triode consists of a filament, grid and plate in an evacuated bulb. Regarding the filament as a plane metallic sheet emitting electrons with negligible velocities, the grid as a plane mesh parallel to the filament and 2 mm. from it, the plate as a plane solid metallic sheet 12 mm. from the grid, and assuming that uniform electric fields

exist between filament and grid and between grid and plate, what must be the relative potentials of the grid and plate with respect to the filament for electrons to reach the plate? What important factor do you ignore?

15. Describe a cathode-ray oscillograph and give an account of some of its applications.
(O. S.)

16. If 100 volts D.C. are applied between the *Y*-plates of a cathode-ray tube, the spot is deflected vertically 1 cm. A sinusoidal alternating voltage is applied to the same plates, and a suitable time-scale to the *X*-plates, so as to display the form of the wave. If the vertical distance between the crest and the trough of the sine wave is 4 cm. how much power will be dissipated when the same voltage is applied to a resistance of 100 ohms?
(C. S.)

17. Write a short account of the electron microscope.

CHAPTER 15

ELECTROMAGNETIC WAVES

In 1865 Clerk Maxwell put forward the theory that electromagnetic waves should exist in the aether and that they should travel with the velocity of light. This is now recognised as the culminating triumph of electromagnetic theory.

Maxwell's mathematical theory is too advanced to be given here, but his main ideas will be presented in simple form. He made two fundamental assumptions: (1) that a moving (or changing) magnetic field sets up an electric field, (2) that a moving (or changing) electric field sets up a magnetic field. The two together may be regarded as providing the potential and kinetic energy necessary for wave propagation.

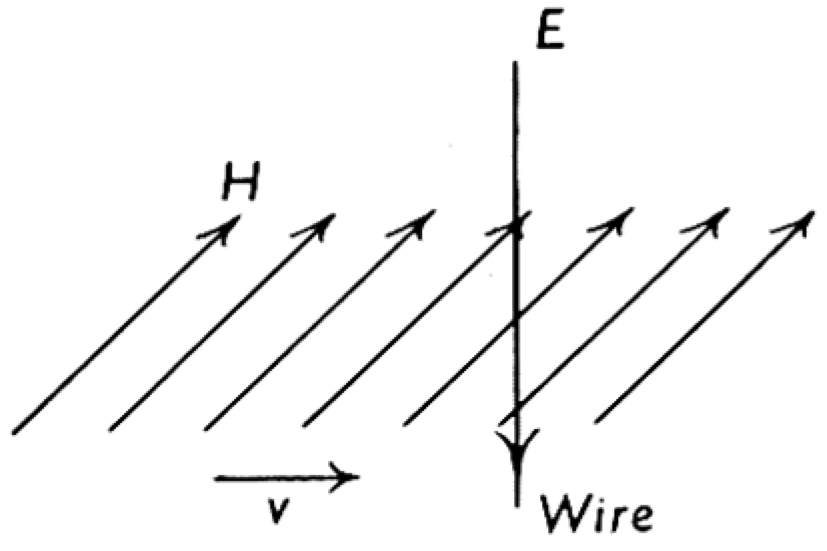


Fig. 256

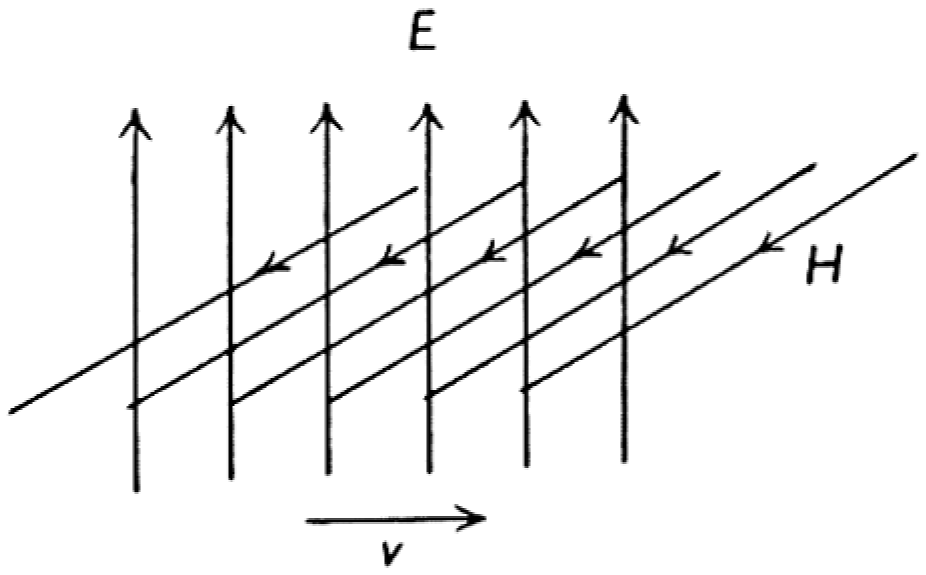


Fig. 257

Deduction of the velocity of electromagnetic waves

Consider a horizontal magnetic field H e.m.u. (Fig. 256), moving with velocity v at right angles to itself as shown, and cutting a vertical wire of unit length. An E.M.F. will be induced in the wire whose direction can be found from Fleming's right-hand rule, and whose value is equal to the number of lines of force cut per second, which is Hv e.m.u. The E.M.F. causes electrons to accumulate at the upper end of the wire. Maxwell assumed that if no wire is present there is still an effect; the aether is polarised and the consequent electric field, E, measured in e.m.u. of potential per unit length, is given by

$$E = Hv. \qquad (1)$$

Now consider a vertical electric field, E, moving with velocity v

(Fig. 257). It can be shown that it gives rise to a horizontal magnetic field H, such that $H=Ev$, H and E being measured in e.s.u.

In e.m.u.

$$H=\frac{1}{c^2}Ev,$$

where c is the ratio of the e.m.u. and e.s.u.

To derive this equation, imagine that the lines of electric force terminate on a charge q e.m.u. which also moves with velocity v. This moving charge constitutes an electric current. It was explained on p. 85 that the magnetic field due to a current is given by the equation $H=il/r^2$ (and in this case r must be assumed to be very great compared with l since the formula was derived for a circular current, and not a straight one). The time, t, taken by the charge to travel a distance l is l/v.

But

$$i=q/t.$$

Substituting for t

$$i=q\frac{v}{l} \quad \text{i.e. } il=qv.$$

Thus the magnetic field H set up by the charge q moving a distance l, at a distance r away, is given by

$$H=\frac{il}{r^2}=\frac{qv}{r^2}.$$

But the electric field, E e.m.u., due to a charge q e.m.u., at a distance r, is $c^2\frac{q}{r^2}$ e.m.u. (see p. 178).

$$\therefore\ H=\frac{1}{c^2}Ev. \qquad (2)$$

Now if the magnetic and electric fields move with the same velocity, each may be regarded as generating the other and as complying with the condition for the propagation of an electromagnetic wave, the form of which will be discussed later.

Combining equations (1) and (2)

$$v=c.$$

The value of c as obtained by experiment is 3×10^{10} (see p. 173), which is the value of the velocity of light in cm. per sec. Thus electromagnetic waves must travel with the velocity of light, and it may be assumed that light itself is electromagnetic in nature.

Relation between dielectric constant and refractive index

Twenty years elapsed before Maxwell's prediction of electromagnetic waves was confirmed experimentally by Hertz. However, there was a piece of evidence deduced by Maxwell by which his theory could be tested.

Suppose the waves are travelling not through a vacuum but through a medium of dielectric constant k and permeability μ. The number of lines of magnetic and electric induction are μH and kE respectively. Thus equation (1) becomes $E=\mu Hv$ and equation (2) becomes

$$H=\frac{1}{c^2}kEv.$$

Thus $$v=\frac{c}{\sqrt{\mu k}}.$$

Since for most transparent substances such as glass $\mu=1$, therefore $v=c/\sqrt{k}$. The refractive index of a medium is the ratio of the velocity of light in a vacuum to its velocity in the medium. Thus refractive index$=c/v=\sqrt{k}$.

In certain cases there is excellent agreement between theory and experiment; for carbon disulphide $\sqrt{k}$ is 1·63 and the refractive index is 1·64. In other cases, such as water, there is violent disagreement, $\sqrt{k}$ for water being 9 and the refractive index 1·33. It has since been found, however, that the dielectric constant of water decreases considerably with frequency, so that values of k and the refractive index must be used which relate to the same frequency.

Further evidence for the electromagnetic nature of light are the Faraday effect, the rotation of the plane of polarisation of light by a magnetic field, and the Kerr effect (not discovered until 1875), the rendering of a medium doubly refracting by an intense electric field.

Clerk Maxwell

James Clerk Maxwell (1831–1879), was one of the greatest of mathematical physicists; his treatise on *Electricity and Magnetism* published in 1873 has been compared with Newton's *Principia*.

He went to school in Edinburgh and was called 'Dafty' by his companions, probably as a result of that unaffected, impish streak in his character which persisted in manhood. He delighted to the end of his days in writing parodies introducing scientific topics, for example, on 'impact':

Rigid Body (sings): Gin a body meet a body
Flyin' through the air,
Gin a body hit a body,
Will it fly? and where?
Ilka impact has its measure
Ne'er a ane hae I,
Yet a' the lads they measure me,
Or, at least, they try.

At the age of sixteen he went to Edinburgh University and three years later to Cambridge. 'He brought to Cambridge', said Professor Tait, 'a fund of knowledge which was really immense for so young a man, but in a state of disorder appalling to his methodical private tutor.'

A fellowship at Trinity College, Cambridge, professorships at Aberdeen, King's College London, and then at Cambridge form the main events of his career. He left his mark at Cambridge quite apart from his scientific discoveries, in the erection of the Cavendish Laboratory. Electricity, Magnetism and Heat were new subjects in the nineteenth century, created by such men as Maxwell and William Thomson, and their importance in an industrialised community was becoming recognised. In 1869 a Syndicate of the University reported 'No reason can be assigned why Cambridge should not become a great school of experimental, as it is already of mathematical and classical instruction.' The seventh Duke of Devonshire, Chancellor of the University, himself a second wrangler and Smith's prizeman, offered the necessary money, and the laboratory was completed in 1874. Maxwell was the first Professor of Experimental Physics at Cambridge—to be followed by Lord Rayleigh, Sir J. J. Thomson, Lord Rutherford and Sir Lawrence Bragg.

It is natural to compare Maxwell with the other great Scottish physicist of his day, William Thomson (Lord Kelvin). Maxwell, who died at the age of 48, was undoubtedly underestimated in his lifetime while William Thomson, whose life extended over 83 years, was very highly regarded. Thomson enjoyed the fruits of this regard in a peerage and a private yacht. He had a powerful intellect which imposed order on a wide field in physics and enabled him to make a fortune from his patents; but he lacked that intuitive flexibility of mind which Maxwell possessed.

Maxwell's main contributions to physics were his electromagnetic theory of light, the mathematical development of the kinetic theory of gases and his work on colour vision. Many people would rank the first-named, but Sir James Jeans (well qualified to judge) considered the second, to be his most brilliant achievement.

Maxwell's electromagnetic theory of light was a mathematical development of Faraday's ideas. Faraday's fundamental contention, with which many of his contemporaries disagreed, was that since induction takes place in curved lines it must be an action of the contiguous particles of the aether and not action at a distance. Maxwell developed this in his papers on 'Physical Lines of Force' published in 1861 and 1862. The then Astronomer Royal said of these ideas, 'I can hardly imagine anyone who knows the agreement between observation and calculation based on action at a distance to hesitate an instant between this simple and precise action on the one hand and anything so vague and varying as lines of force on the other.'

Maxwell contributed the idea that changes in an electric field in the aether must produce a magnetic field. He arrived at this conclusion after devising mechanical models, such as the representation of lines of magnetic force by cylinders rotating round the lines as axes, and imagining the aether to be filled with interlocking gear wheels.

Faraday and Maxwell tended to make the aether the seat of all electrification. The following quotations show how cautious they were in making statements about what happens in a wire carrying a current, despite their brilliant conceptions of the action in the surrounding field. 'By a current', wrote Faraday, 'I mean anything progressive, whether it be a fluid of electricity or two fluids moving in opposite directions, or merely vibrations, or speaking more generally, progressive force.' And Maxwell: 'As long as we do not know whether positive electricity or negative or both should be called a substance or the absence of a substance, and so long as we do not know whether the velocity of an electric current is to be measured by hundreds of thousands of miles in a second or by an hundredth of an inch in an hour, or even whether current flows from positive to negative or in the reverse direction, we must avoid speaking of an electric fluid.'

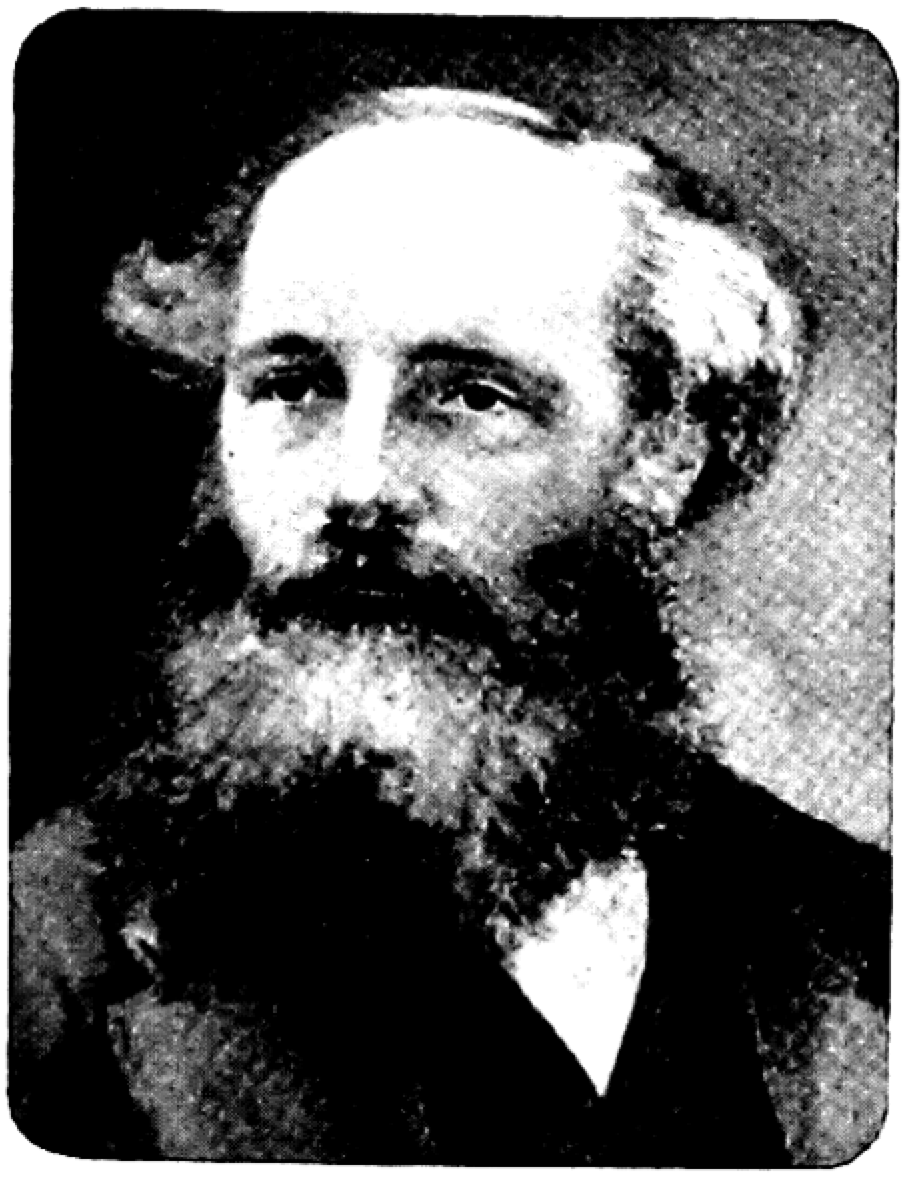
The shock of the discovery of the electron to the followers of Faraday and Maxwell is well illustrated by the remarks of Schuster: 'The separate existence of a detached atom of electricity never occurred to me as possible: and if it had and I had openly expressed such heterodox opinions, I should hardly have been considered as a serious physicist, for the limits to allowable heterodoxy in science are soon reached.'

Belief in the objective existence of magnetic and electric fields has been undermined by the inability of such experiments as that of Michelson and Morley to detect the existence of the aether. There can be little doubt that the ideas of Faraday and Maxwell, which for the past sixty years have overshadowed electromagnetic theory, require reinterpretation in the light of the electron theory.

The experiments of Hertz

In 1888 Heinrich Hertz produced electromagnetic waves. His oscillator (Fig. 258) consisted of an induction coil, to the secondary terminals of which were connected two brass plates 40 cm. square joined by copper wires 30 cm. in length to two balls forming a spark gap 2–3 cm. wide.

His receiver was a loop of wire of about 35 cm. radius with a small, adjustable, spark gap. When placed near the oscillator, the waves



JAMES CLERK MAXWELL

HEINRICH HERTZ

JAMES CLERK MAXWELL (1831–1879) *is best remembered for his famous equations which summarized electromagnetic theory, and predicted the existence of electromagnetic waves. 'So magnificent a summary in a few equations', wrote Lenard, 'had hitherto never been known.' Maxwell's ideas were based on his concept of the aether, 'that supreme paradox of Victorian science and yet a triumph of the scientific imagination'. A brief account of Maxwell's life and work is given on pp.* 278–280.

HEINRICH HERTZ (1857–1894) *was the first to establish the existence of electromagnetic waves. Maxwell's theory had shown that such waves should be generated by electrical oscillations, and that their velocity should be equal to that of light. It was realised that the oscillations must be very rapid, otherwise the wavelength would be of the order of miles, and that very rapid oscillations were produced by the spark discharge from condensers. Several investigators were trying to detect electromagnetic waves, including Sir Oliver Lodge, who said later, 'Hertz stepped in before the English physicists, and brilliantly carried off the prize'. Hertz's great achievement brought him the professorship at Bonn at the early age of 32, but his life was cut short by blood poisoning only five years later.*

from the latter induced currents in it and small sparks jumped across the gap.

Hertz showed that the velocity of the waves was equal to that of light and that, like light, they underwent reflection, refraction, interference, and polarisation. 'Hertz's researches were one of the most marvellous triumphs of experimental skill, of ingenuity, of caution in drawing conclusions, in the whole history of physics', wrote Sir J. J. Thomson. 'Younger physicists, with the very efficient means of detecting electrical waves now at their command will not realise the difficulties of these experiments, but older ones, who like myself, began by using Hertz's method, and had to observe whether tiny sparks only a fraction of a millimetre long waxed or waned when the detector was moved from one position to another, will remember how arduous and harassing these experiments were and how long it took to make sure that the effects observed were not spurious.'

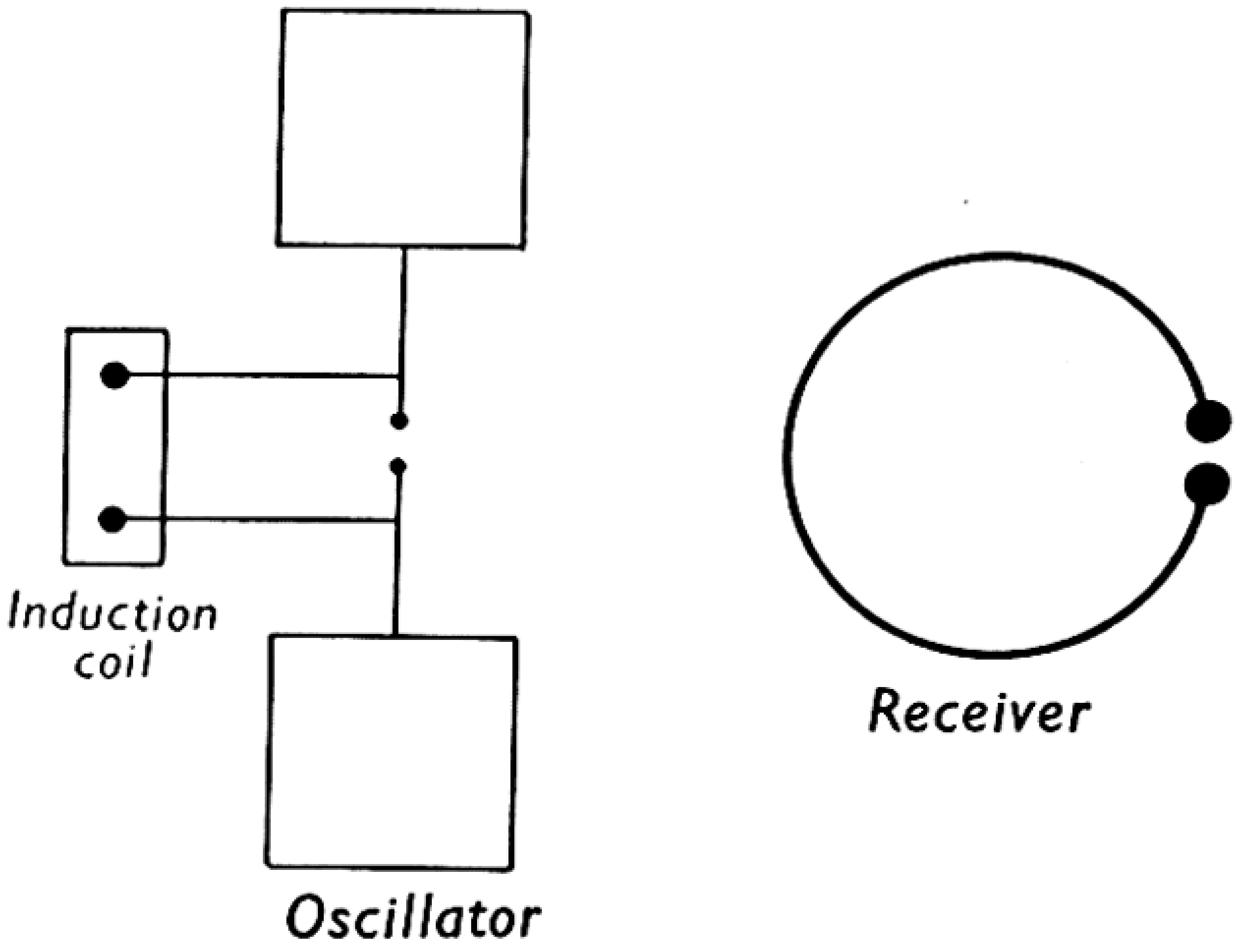


Fig. 258

The oscillatory circuit

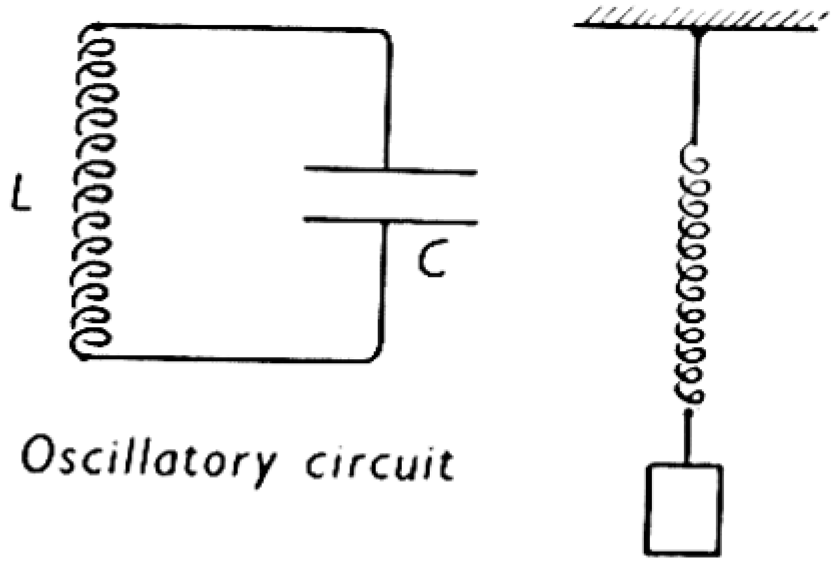
The circuit of Hertz's oscillator is essentially the same as Fig. 259, which is called an oscillatory circuit. The two brass plates in Hertz's oscillator act as the plates of a condenser. Each spark that passes across the spark gap sets up a train of very high frequency electrical oscillations in the circuit. Each spark itself is therefore an oscillatory discharge.

To produce electrical oscillations, a circuit must possess inductance, L, and capacitance, C. The frequency of the oscillations, f, is given by

$$f=\frac{1}{2\pi\sqrt{LC}} \quad \text{(see pp. 224, 246).}$$

If the inductance is expressed in henries and the capacitance in farads, the frequency is given in cycles per second.

It is, perhaps, helpful to compare electrical oscillations with the mechanical oscillations of a mass on the end of a spiral spring. The alternate charging of the condenser in opposite directions is comparable to the stretching and compressing of the spiral spring; the inertia of the electric current due to the inductance is comparable to the mass. Each discharge of the condenser builds up round the inductance a magnetic field which then collapses. While the current and the magnetic field are increasing, the E.M.F. due to the inductance opposes the current; but while the current is decreasing and the field is collapsing, the induced E.M.F. is in the same direction as the current and so causes the current to be prolonged and the condenser to be charged in the opposite direction. Similarly the mass, when the spiral spring is fully extended, opposes the shortening of the spring at first, but when the natural length has been reached, the kinetic energy causes the upward motion to continue so that the spring is compressed.



Oscillatory circuit

Mechanical analogy

Fig. 259

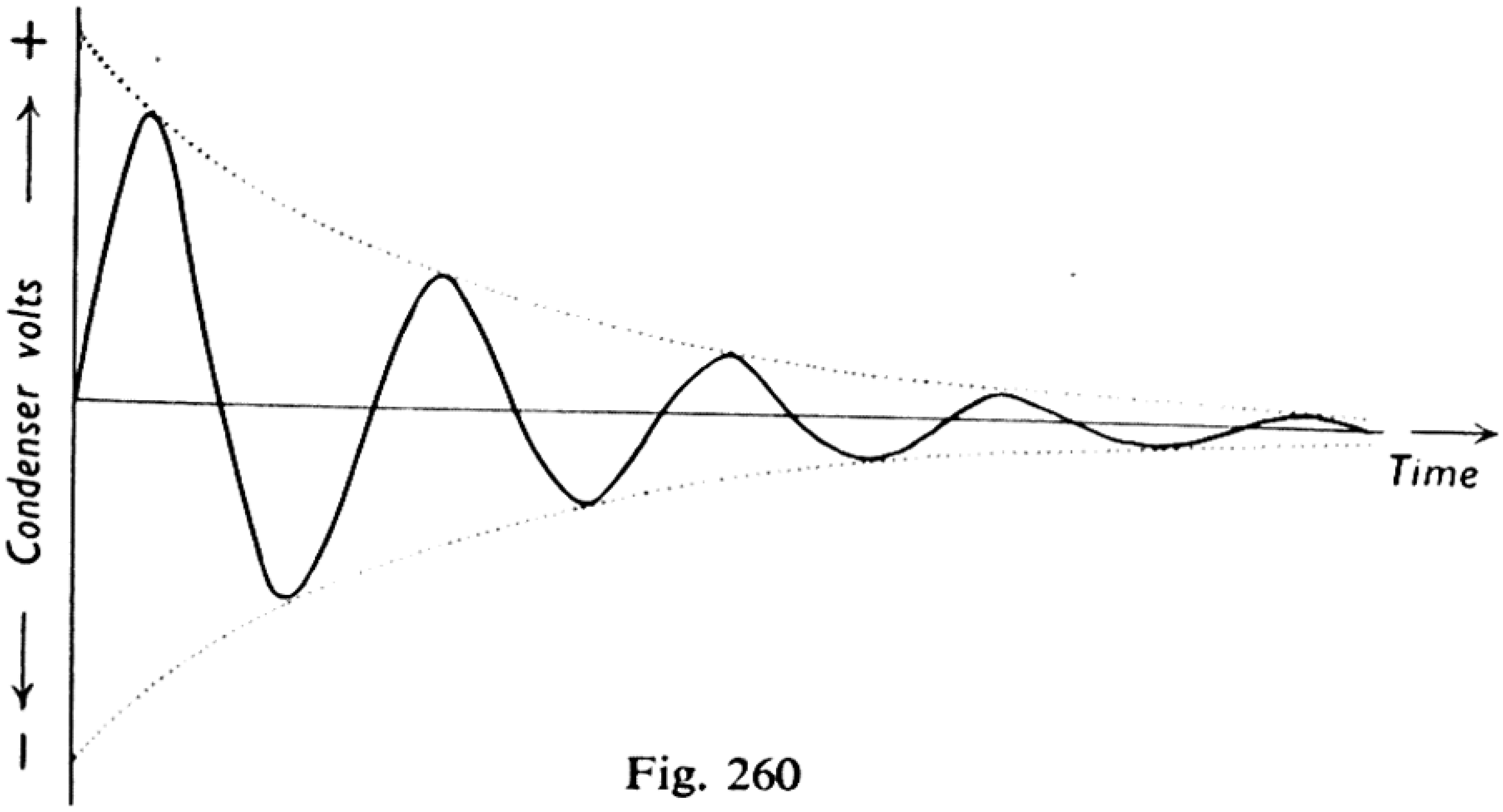


Fig. 260

The mechanical oscillations gradually die away owing to loss of energy through air resistance; similarly, the electrical oscillations decrease owing to loss of energy in the form of heat in the resistance of the circuit. A train of damped oscillations is shown in Figs. 260 and 261. The frequency of the exciting sparks is usually sufficiently low to ensure that one train of waves dies away before the next is initiated.

Hertz's circuit differs from that in Fig. 259 in that the plates of his condenser are not parallel and close together but are opened out. His circuit is called an 'open' oscillatory circuit and is a much better radiator of electromagnetic waves than the 'closed' oscillatory circuit of Fig. 259.

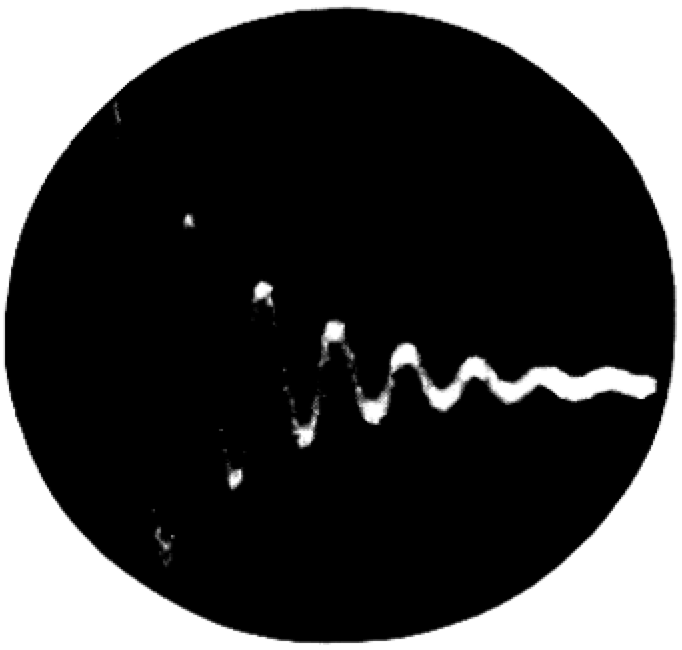


Fig. 261. Trace showing the oscillatory discharge of a condenser, taken with a cathode-ray oscillograph. Note how the brightness of the trace varies with the writing speed.

The mechanism of radiation

Many people find it helpful to devise a model or some concrete picture of a physical phenomenon; this is perhaps more characteristic of British than of continental physicists. A picture of the way in which

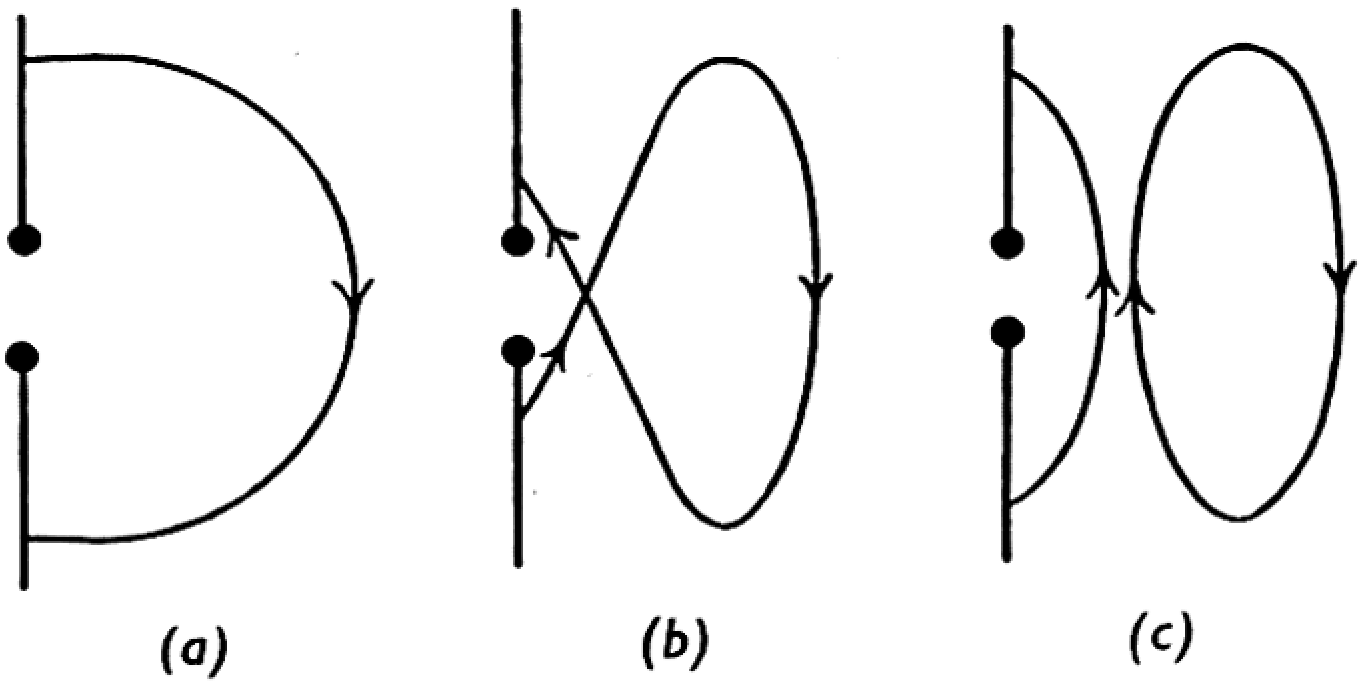


Fig. 262

the radiation takes place from Hertz's oscillator has been devised in terms of tubes of electric force. A tube of force, as was stated on p. 21, may be regarded as a collection of, or bounded by, lines of force.

Suppose the upper half of the spark gap in Hertz's oscillator is charged positively and the lower half is charged negatively. Tubes of force, starting on a positive charge and ending on an equal negative charge, will lie around the spark gap, like the tube shown in Fig. 262 (*a*).

When the rapid discharge across the spark gap commences, the ends of the tubes of force will be jerked across the gap and a loop will be formed rather like the loop in a skipping rope when its ends are jerked across each other. This loop breaks away; it is no longer anchored to positive and negative charges which must be dragged along with it and it moves outwards at right angles to itself, with the velocity of light, continually expanding as it goes. It creates a magnetic field perpendicular both to itself and to its direction of motion.

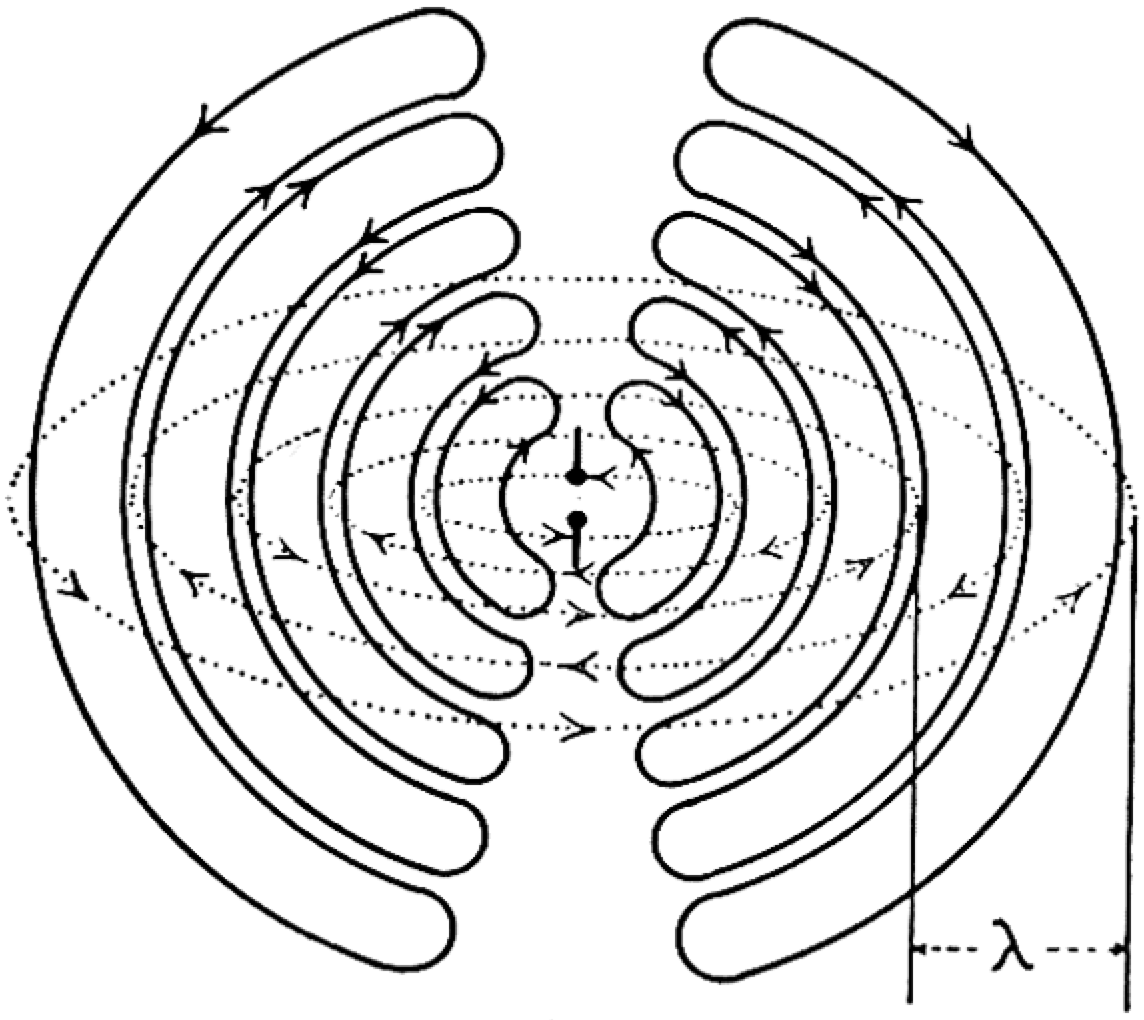


Fig. 263

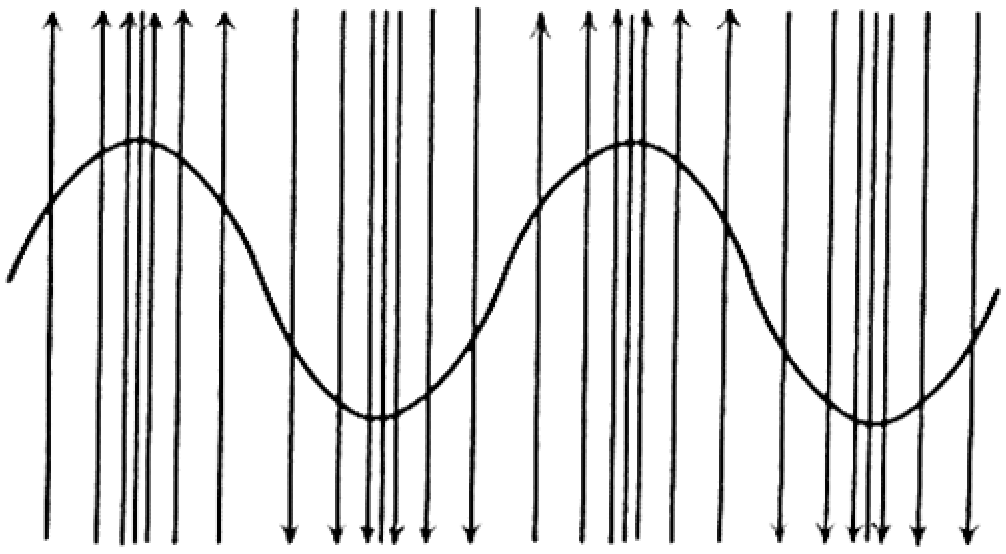


Fig. 264

Meanwhile the other two parts of the tube have joined together and stretch across the spark gap, now charged in the opposite direction. The process is repeated and, since the discharge oscillates backwards and forwards, a succession of oppositely directed loops will be radiated as in Fig. 263. The dotted lines represent the lines of magnetic force (in the horizontal plane only).

Only a single tube has been shown in each loop, but actually there should be a number of tubes. The distribution of tubes and magnetic lines of force is shown in Figs. 264 and 265 which represent the fields along a straight line from the middle of the spark gap. The fields vary sinusoidally and hence the tubes and magnetic lines of force are spaced as shown. The distance of any point on the sine curves from the axis represents the strength of the field.

In the wave the electric and magnetic fields are in phase; they reach their maximum and zero values simultaneously. At the oscillator, however, there is a phase difference of 90° between the electric and magnetic fields. The magnetic field at the oscillator is a maximum when the current across the spark gap is a maximum and thus when the electric field is zero. The change of phase may be considered to occur as a result of the breaking away of a loop.

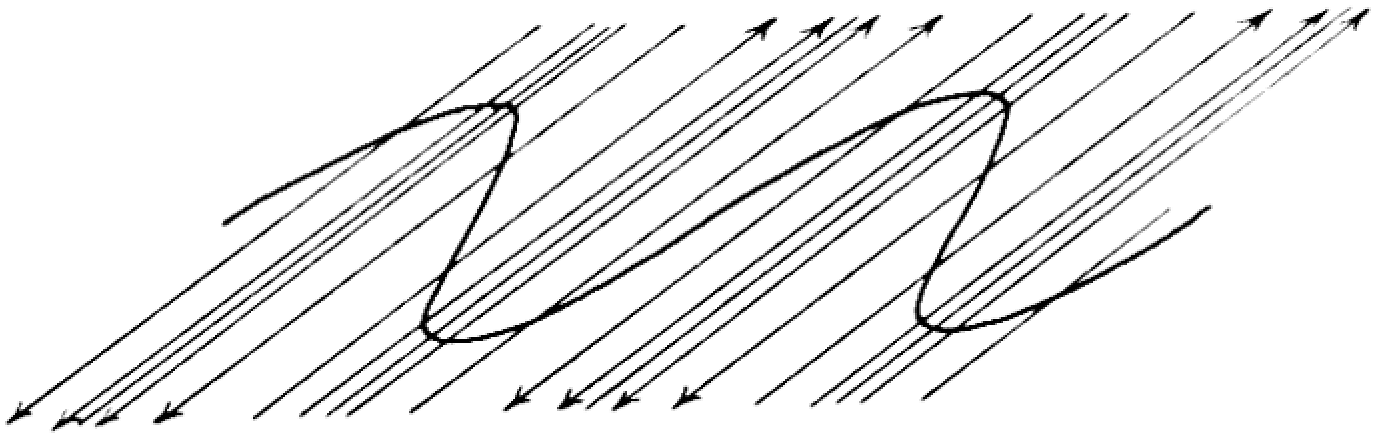


Fig. 265

On arrival at the receiver a loop must break and its free ends must become attached to positive and negative charges. The movement of the negative charge as the tube collapses constitutes an electric current in the receiver.

Wave-length and frequency

The distance between similarly directed loops of electric force, or lines of magnetic force in a wave is called the wave-length, λ (Fig. 263). If f waves are radiated per second and v is the velocity of the waves, it is clear that $v = f\lambda$.

Now $v = 3 \times 10^{10}$ cm. per sec., and in Hertz's experiment $f = 5{\cdot}5 \times 10^7$ cycles per second; hence $\lambda = \frac{3 \times 10^{10}}{5{\cdot}5 \times 10^7} = 540$ cm. $= 5{\cdot}4$ metres.

The frequency used in radio varies from about 100,000 cycles per second, i.e. 100 kilocycles, to 3×10^{10} cycles per second, i.e. 30,000 megacycles; the corresponding wave-lengths are 3000 metres and 1 cm.

To radiate appreciable energy it is essential that the frequency should be high. The alternating current in the Grid, for example, with a frequency of 50 cycles per second radiates negligible energy. If a discharge across a spark gap takes place leisurely, as it does for low frequencies, the tubes of electric force have time to collapse into the gap and no loop is formed.

Marconi

The commercial application of electromagnetic waves received its chief and primary impetus from Guglielmo Marconi (1874–1937). Although he received a Nobel prize and more honours than any other man of his time, Marconi's gifts were as much those of a good business man as of a physicist and inventor.

Marconi was the son of an Irish mother and an Italian father and hence he spoke English as well as Italian. He made his first experiments in the gardens of his home, the Villa Griffone, near Bologna, with his father's gardeners as assistants. 'The idea of transmitting messages through space by means of etheric waves', he said, 'came to me suddenly as a result of having read in an Italian electrical journal about the work and experiments of Hertz.... My chief trouble was that the idea was so elementary, so simple in logic, that it seemed difficult to me to believe no one else had thought of putting it into practice. Surely, I argued, there must be much more mature scientists than myself, who had followed the same line of thought and arrived at an almost similar conclusion.'

In 1896, after he had succeeded in transmitting messages a distance of 1¾ miles, be came to England where his mother had relatives and friends. The Engineer-in-Chief of the G.P.O. became interested in his experiments and a test transmission took place from the roof of the post office at St Martins-le-Grand to a receiving station on the Thames Embankment. Then followed a series of transmissions over increasing distances, 4, 6½ and 8 miles over Salisbury Plain, over the Bristol Channel from Penarth to Bream Down, across the English Channel from Wimereux to South Foreland, from St Catherine's Point, Isle of Wight to the Lizard, Cornwall, and finally the famous transmission across the Atlantic.

It was thought that it would be impossible to send signals across the Atlantic since, owing to the curvature of the earth, the waves would pass out into space. Nevertheless, Marconi determined to try.

He erected a powerful transmitting station at Poldhu, Cornwall. He was, at this time, still using a spark transmitter similar in principle to that of Hertz, but he had discovered the value of a high aerial. His receiver was much more sensitive than that of Hertz, its main component being a 'coherer', consisting of filings of nickel and silver between two silver cylinders; these filings cohered when electrical oscillations from the receiving aerial passed through them, and the resulting decrease in resistance enabled a battery to supply a current and cause a click in telephones or a mark on a moving tape.

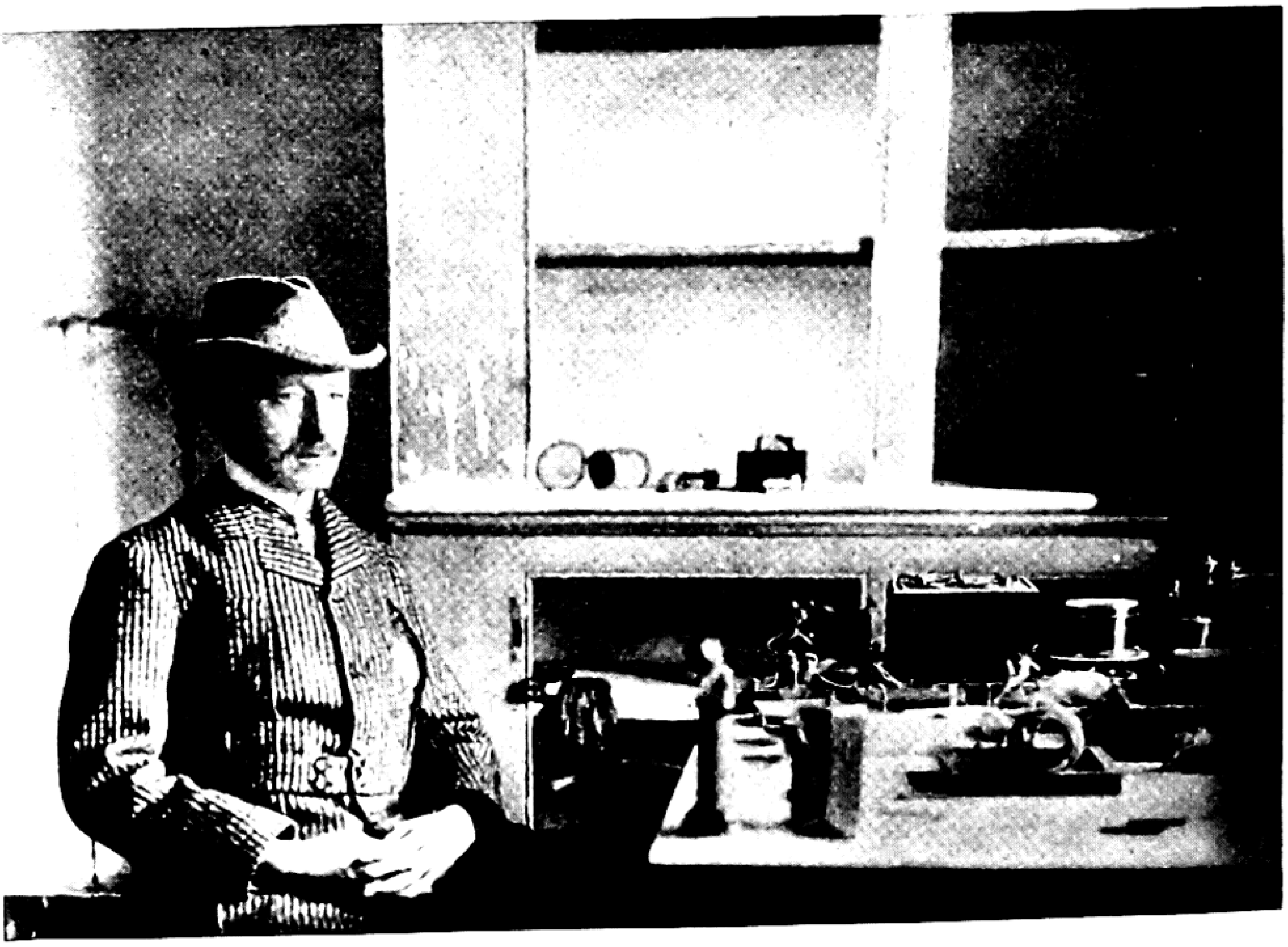


Fig. 266. Marconi in the barracks on Signal Hill, St John's, Newfoundland, with the apparatus which received the first wireless signal transmitted across the Atlantic.

Marconi sailed for St John's, Newfoundland, and set up his receiving station in the old military barracks on Signal Hill. For nearly a fortnight the winds were tremendous. Balloons sent up to raise the receiving aerial broke away and disappeared in heavy mist. On 12 December 1901 he managed to fly a kite despite the gale and drenching rain. About 12.30 p.m. three faint clicks were heard in his receiver, and the letter S, the prearranged signal from Poldhu, 1800 miles away, appeared on his tape machine (Fig. 266).

The suggestion that Marconi might have picked up stray signals proved untrue. In a short time a station was set up at Glace Bay on Cape Breton Coast in Canada, and transmission across the Atlantic was in regular operation.

The ionosphere

In 1902 Kennelly and Heaviside independently put forward an explanation of Marconi's success in transmitting waves across the Atlantic, despite the curvature of the earth. They suggested that there was an electrical reflector in the upper atmosphere, now called the Kennelly-Heaviside layer. In 1924–5 Dr (now Sir Edward) Appleton proved its existence by experiment and found its height to be about 60–70 miles above the earth. Later Appleton and his collaborators fired short pulses of electromagnetic waves at the layer—a method which will be described in connection with radar; they found that some of the pulses were getting through and being reflected from a height of about 150–180 miles above the earth. In this way the Appleton layer was discovered (Fig. 267).

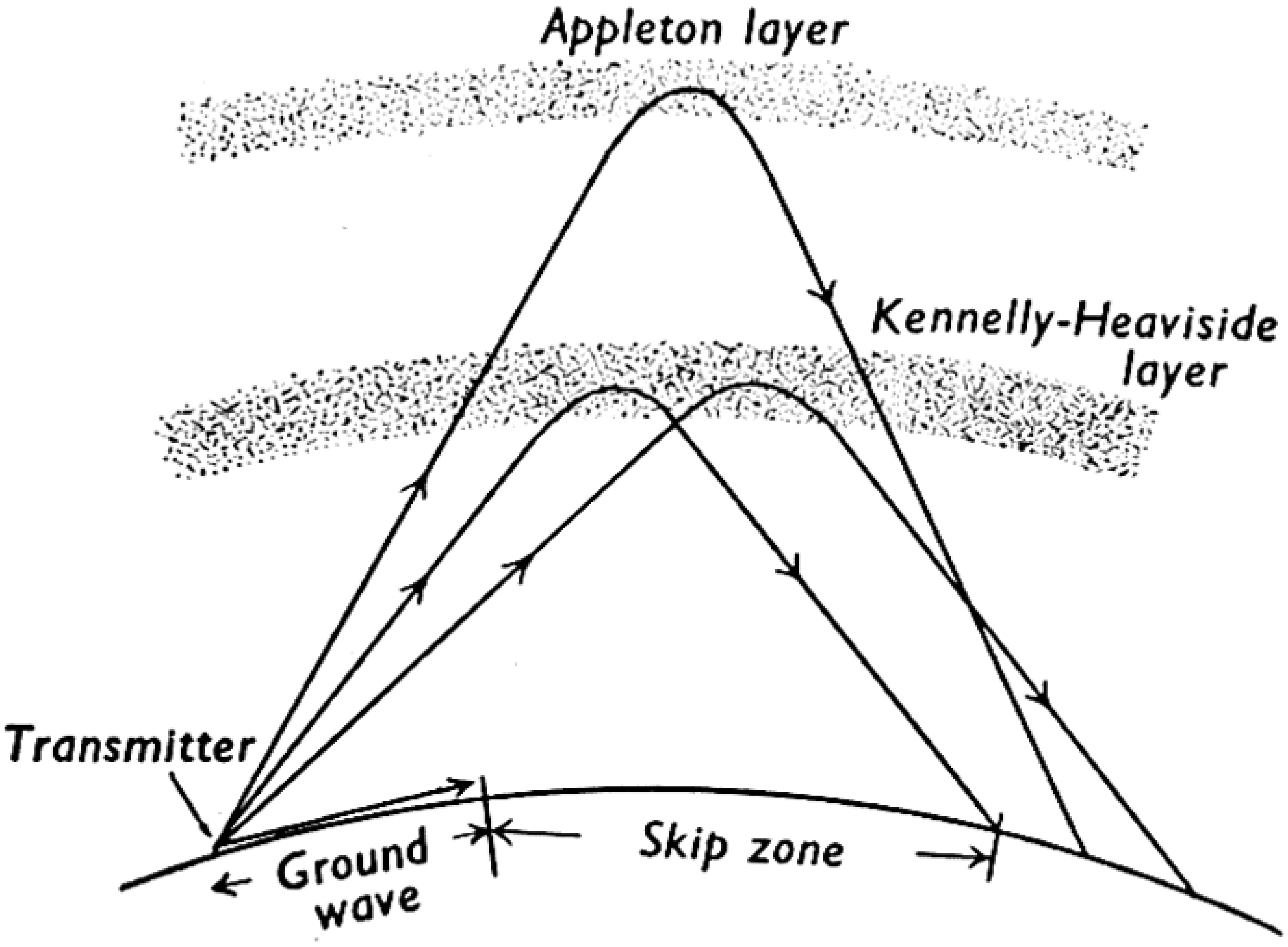


Fig. 267

The upper atmosphere is known to be ionised and hence is called the ionosphere; the rarefied air contains many charged molecules due to electrons ejected from the sun. Research on the ionosphere, now proceeding all over the world, has shown that there are several, distinct, conducting layers. The Kennelly-Heaviside layer is called the *E* region, and the Appleton layer the *F* region. The Kennelly-Heaviside layer rises during the night and drops at sunrise; its reflecting properties are more regular at night.

Electromagnetic waves penetrate a distance into the Kennelly-Heaviside layer depending on their wave-length and the obliquity of their incidence; the shorter the wave-length, the greater the penetration and the greater the obliquity the less the penetration. Short waves, unless they are very oblique, go right through. Since the Appleton layer is about four times as strong a reflector as the Kennelly-Heaviside layer, owing to the greater concentration of ions, short waves are usually reflected there.

A transmitting aerial sends out two sets of waves, a sky wave which goes upwards and may be reflected in the ionosphere, and a ground wave which keeps along the ground. If an aerial is earthed and electrical oscillations are fed in at the bottom, an arrangement due to Marconi, waves are sent out as in Fig. 268. The lower ends of the lines of force travel along the earth and give rise to earth currents. Since the earth is not a perfect conductor the earth currents travel more slowly than the lines of force above and hence the waves are bent downwards as in Fig. 268 (*b*).

The range of the ground waves depends on the conductivity of the soil. Transmission is poor over sandy soil, and best over sea water.

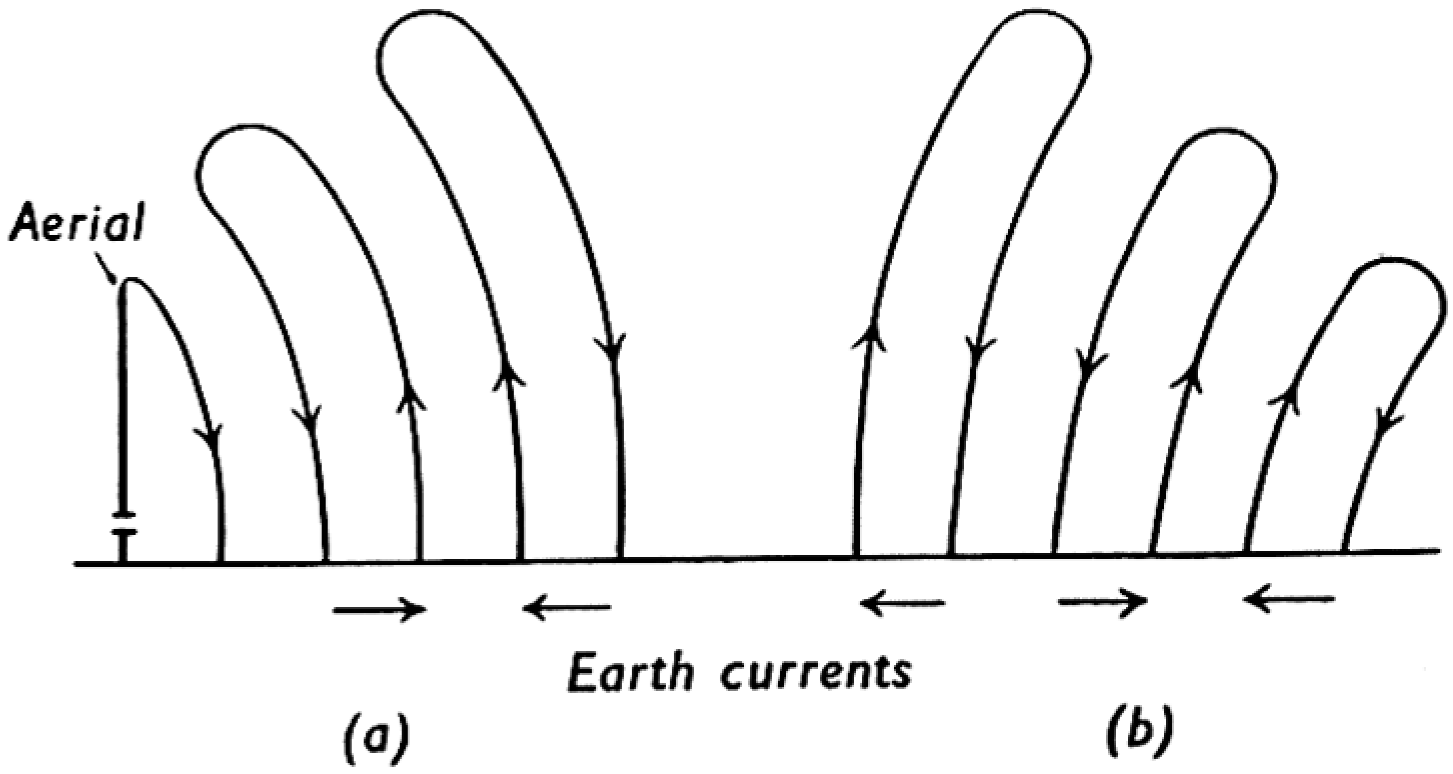


Fig. 268

There is often a region, called the skip zone (Fig. 267), where no waves are received. This region is too far away from the transmitter for the ground waves to reach it, but not sufficiently far away to receive the nearest sky waves reflected from the ionosphere.

Fading, in wireless reception, is due to interference between the ground waves and the sky waves; when the path traversed by the sky waves alters, owing to changes in the ionosphere, interference maxima and minima result.

Listeners to B.B.C. short-wave transmissions (13–20 metres) frequently receive signals direct and again after a complete journey round the earth. In these circumstances, there must obviously be successive reflections at the ionosphere and at the surface of the earth.

Radar

The word radar signifies 'radio detection and ranging', a method of locating objects by electromagnetic waves reflected from them. The first radar experiments in this country, conducted in 1935–6 under the Radio Research Board, were based on the technique developed by Appleton in his exploration of the ionosphere.

A complete radar station consists of a transmitter or sender *S* and a receiver *R* (Fig. 269 (*a*)). The transmitter emits short pulses of radiation over a broad arc. On hitting a target *T* the pulse is reflected and the time it takes to traverse the path *STR* is measured by means of a cathode-ray oscillograph. The received pulse, after being amplified and rectified, is applied to the vertical deflecting plates of the oscillograph. The horizontal deflecting plates are connected to a time-base circuit 'triggered' to start operating at the instant the pulse is sent out from *S*. The spot of light moves at uniform speed from *O* to *A* on the screen (Fig. 269 (*b*)); hence *OA* can be graduated in miles. The arrival of the pulse (or echo) causes a vertical deflection or 'blip' *E*, and the range of the target is measured by *OE* (see also Fig. 270).

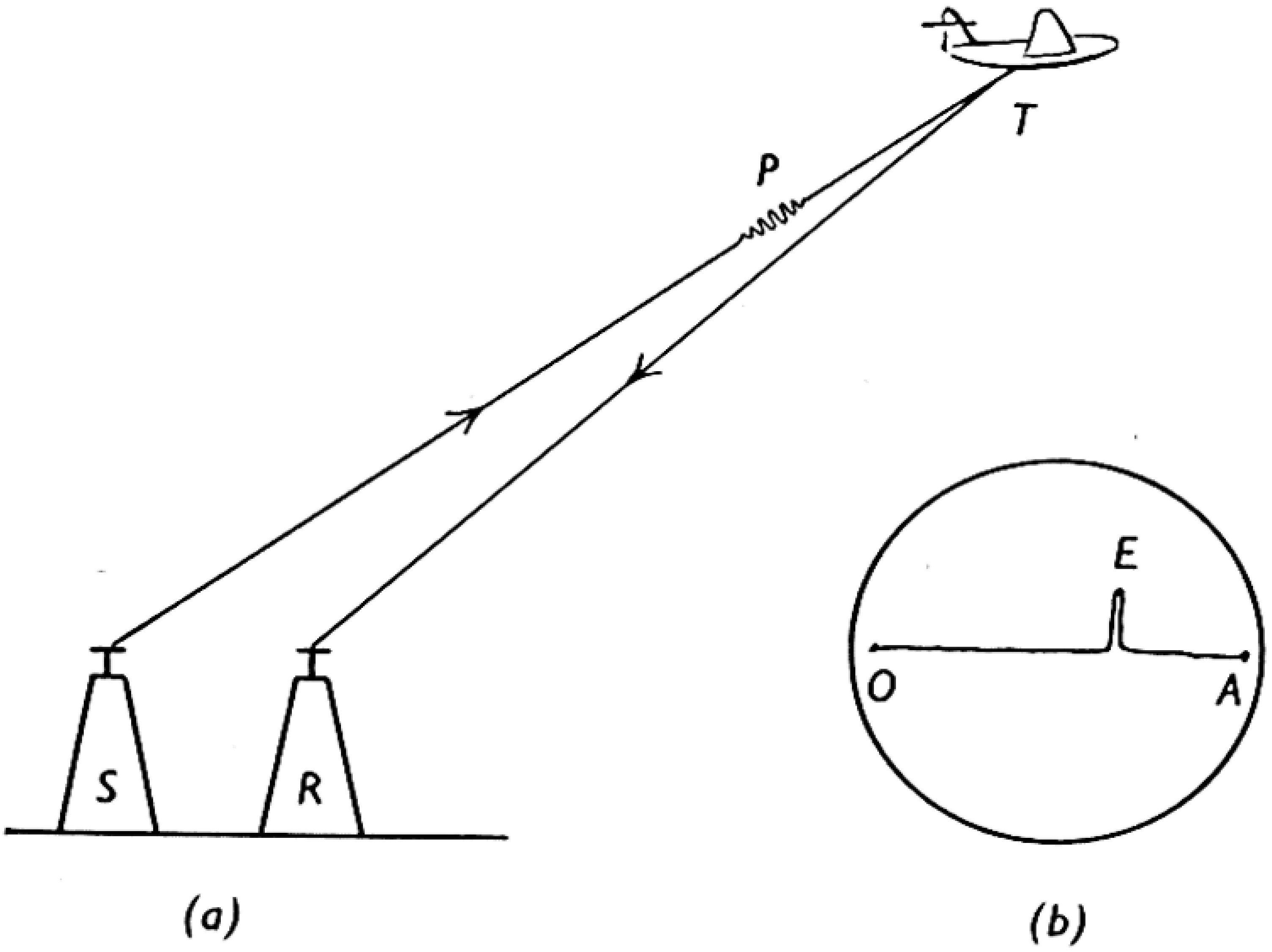


Fig. 269

It is obviously essential that the reflected pulse should arrive at the receiver before the next pulse is sent out by the transmitter. Hence the interval between the generated pulses is determined by the intended range. Suppose the range is 100 miles. The time taken by the waves (of velocity 186,000 miles per second) to travel 200 miles is

$$\frac{200}{186{,}000} \simeq \frac{1}{1000} \text{ th second.}$$

Therefore in this case the pulses must be sent out at a frequency not greater than 1000 per second.

Besides the range, the direction of the target is required; the latter can be specified by the bearing and the elevation. The bearing may be found by rotating the frame aerial, *R*, until the amplitude of the received pulse decreases to zero. In practice two fixed, frame aerials at right angles are employed and the coil of an instrument working in conjunction with them is rotated.

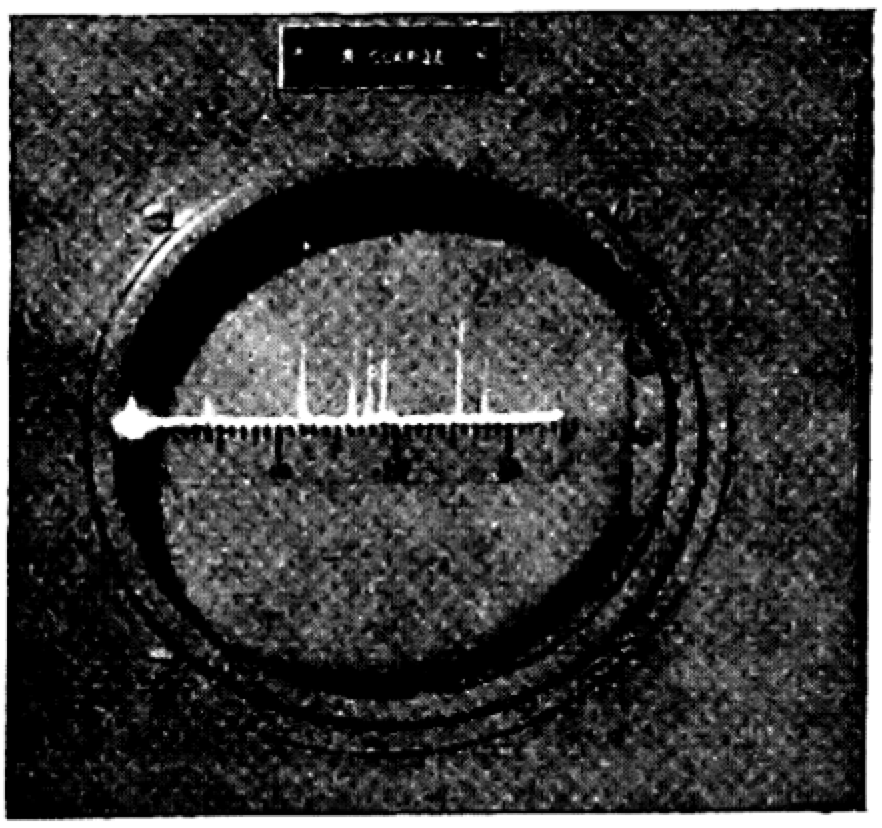


Fig. 270. Blips on cathode-ray tube.

To find the angle of elevation above the horizontal two similar aerials are mounted, one above the other, at a known distance apart. The signal in each aerial is due to two sets of waves, one being received direct from the target and the other after reflection at the ground. These waves interfere and the resultant signal depends on the elevation of the target and the height of the aerial above the ground. By comparing the strengths of the signals received in the two aerials the elevation of the target can be determined.

The Plan Position Indicator

The radiolocation method described above was used by a chain of stations on the east coast of England to direct the R.A.F. in the Battle of Britain. In later stages of the war the transmitter was equipped with a special rotating aerial which concentrated the pulses in one direction, instead of the fixed aerial which sent out radiations over a considerable area—a development which may be described as a radio searchlight in contrast to radio floodlighting, and which made it possible to use a superior type of receiver called the Plan Position Indicator (P.P.I.).

In the P.P.I. the spot of light in the cathode-ray oscillograph is made to start at the centre of the screen and not at the side. It sweeps backwards and forwards along a radius which rotates round the screen in synchronism with the rotation of the transmitting aerial—a few times per minute. The received pulse, instead of producing a 'blip' or kink, causes an intensification of the spot of light (Fig. 271), being applied to the grid of the cathode-ray oscillograph, as in a television receiver. The distance of the spot of light from the centre gives the range of the target, and its angular position the bearing. A transparent map can be placed on the screen of the oscillograph and the position of the target read at once. The screen must be coated with a material which retains its fluorescence for a comparatively long time, since the radius along which the spot of light sweeps is rotating slowly.

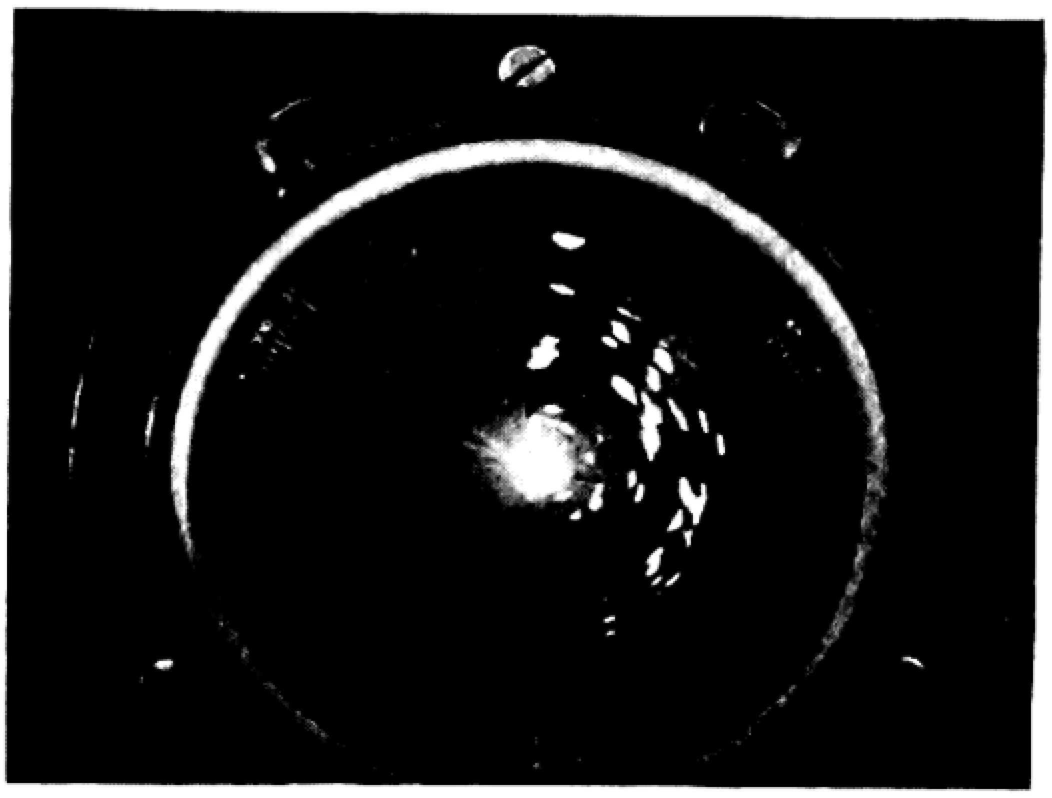


Fig. 271. Plan Position Indicator.

British night-fighters were equipped with a complete radar system, of range 2½–4 miles, and after being guided to their quarry by the ground stations, were able to aim their guns by the spot of light caused by the enemy plane on their cathode-ray tubes.

Centimetric radio

The pattern of spots in the P.P.I. is a coarse one unless very short waves are employed. Ingenious valves for the generation and reception of about 10 cm. wave-length were developed by a team of scientists working in Birmingham.

Using these very short waves, the plan of a target below them could be seen by the crew of a bomber on their 'gen box' or 'Mickey' even though the target was completely obscured by clouds. Water is a better reflector of electromagnetic waves than land, but owing to its smooth surface it reflects regularly and not diffusely. The land, particularly objects such as cliffs and houses, scatters the waves and hence the picture seen on the screen of the cathode-ray tube is similar to Fig. 272. This was a great help in night bombing.

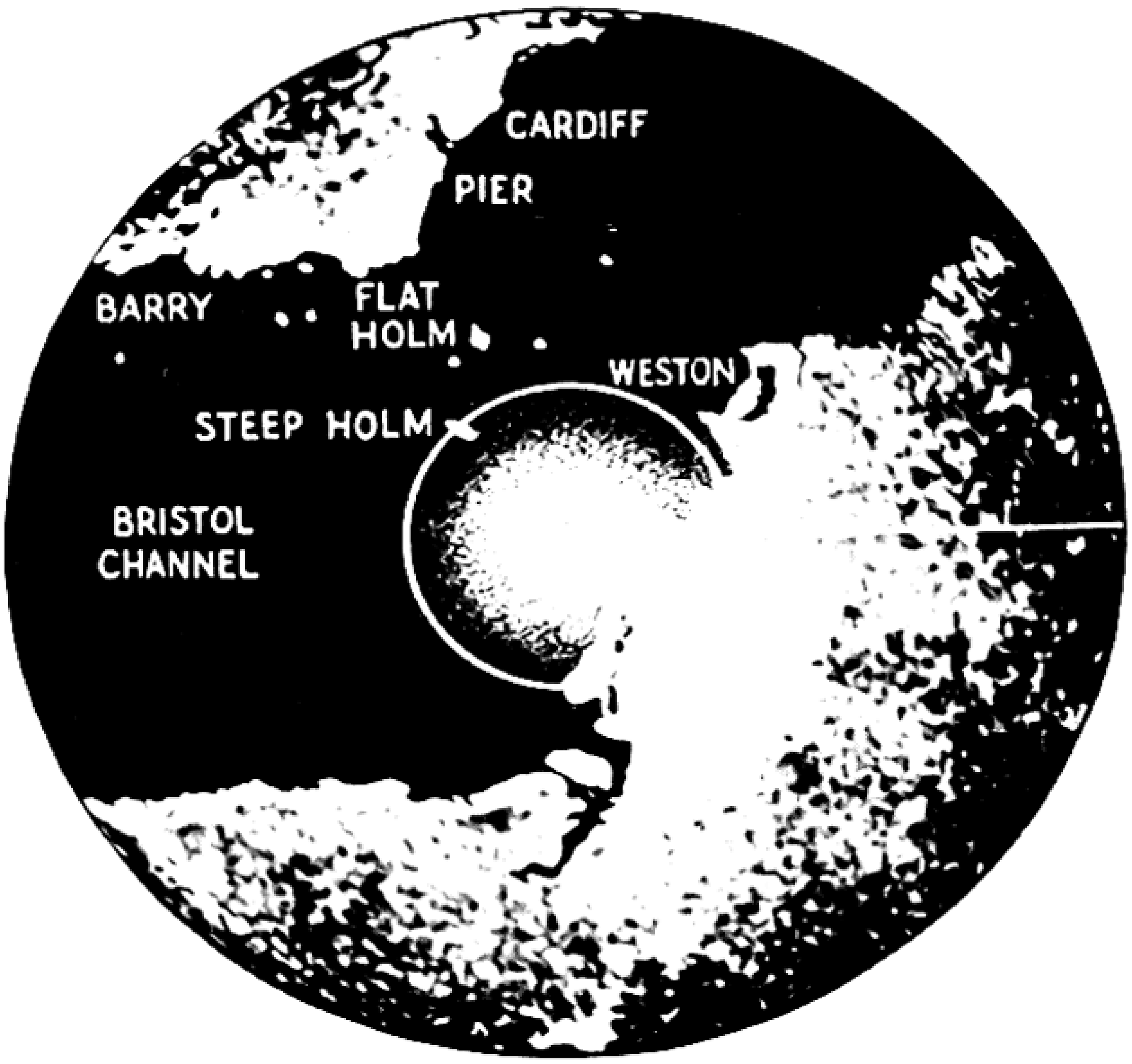


Fig. 272. Radar map of Bristol Channel.

The change to centimetric waves in the aircraft engaged in hunting U-boats in the Atlantic was the occasion of 'bitter comedy' in the long battle of wits waged by the technicians on both sides. The Germans could not understand why their U-boat commanders were failing to pick up the hunting aircraft's signals—previously used as a warning for immediate submersion. U-boats, equipped with newly modified receivers of the longer waves, chanced to operate in the area where allied aircraft were not fitted with the new centimetric radio equipment and 'came back with excited stories of signals on the new receiver that couldn't be detected on the old. Hope ran high, and worries were soothed. Yet the U-boat kills went on.' *

---

* *Radar*, H.M.S.O.

'Gee' and 'Oboe'

Radar technique has been employed in the navigational aids known as 'Gee' and 'Oboe'. 'Gee' was of great value in the assault on Normandy on D-day, and without 'Oboe' the bombing of the Ruhr would have lost much of its effectiveness.

If three transmitters on the ground send out radio pulses simultaneously an aircraft can, from the difference in the times at which they arrive, determine its position. This is the principle of 'Gee' and several areas of the world have been covered by navigational grids for the use of aircraft and ships.

'Oboe' is a refinement of 'Gee'. Two ground radar stations send out pulses which are picked up and returned automatically by the apparatus in the aircraft. One station can keep the aircraft flying towards the bomb release point while the other can tell the pilot how many minutes he is away from the target. The stations can even fly the aircraft automatically and release the bombs. The apparatus and the maps were sufficiently accurate to allow an error of no more than tens of yards at a range of 200 miles.

QUESTIONS

1. Explain qualitatively how electrical oscillations can occur in a circuit containing a condenser and an inductance. Show how such oscillations can be maintained.
Describe Hertz's experiments and explain their importance. (N.)

2. Write an account of the ionosphere and its function in radio transmission.

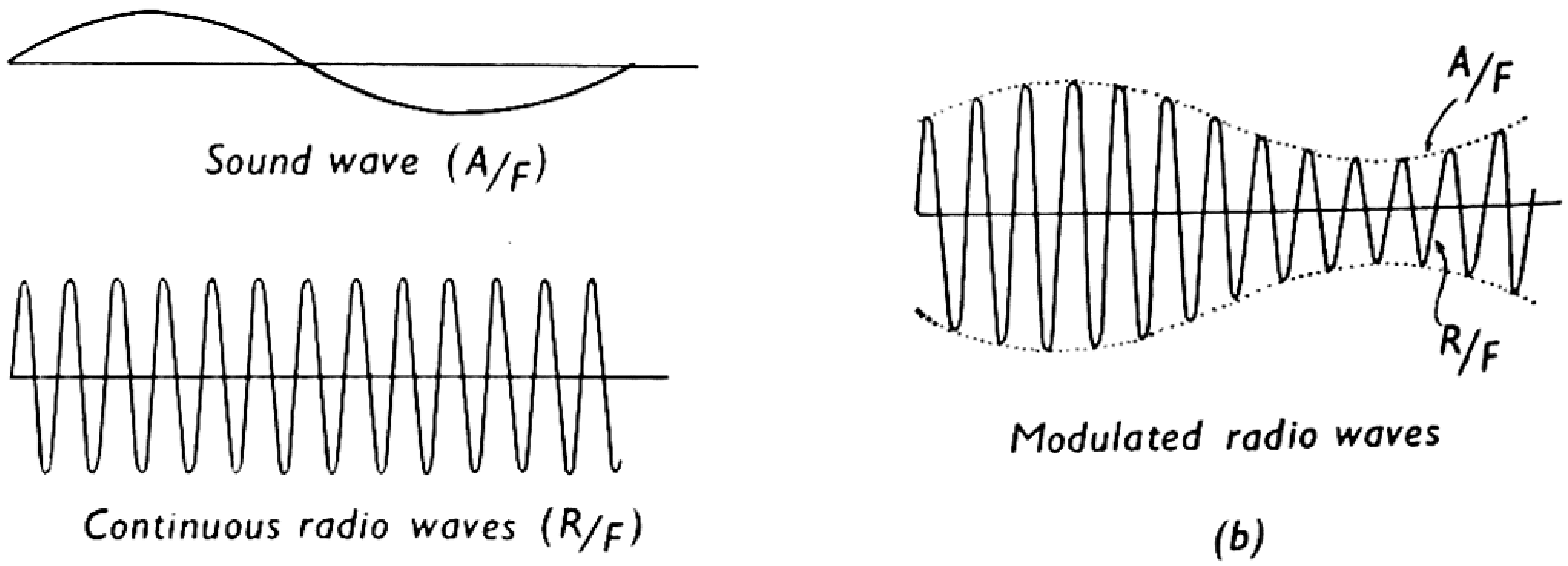
3. Write a short essay on radar.

CHAPTER 16

RADIO RECEPTION AND TRANSMISSION

HIGH-FREQUENCY alternating currents radiate electromagnetic waves. They can be generated by sparks which produce damped waves as described in the last chapter, or by thermionic valves which produce continuous waves.

For radio telephony continuous electromagnetic waves are used, and their amplitude is varied by sound waves falling on a microphone, a process called *modulation* (Figs. 273 and 274). The frequencies of the radio waves in the broadcast medium wave-band are of the order of 10^6 cycles per second, and the frequencies of normal sound waves are of the order of 10^2 or 10^3 cycles per second; thus Fig. 273 is not drawn to scale.



(a) (b)

Fig. 273

Modulated waves are sometimes said to have a radio-frequency (*R/F*) component, and an audio-frequency (*A/F*) component. It is, of course, the *A/F* that must be reproduced by the receiver, and the waves of *R/F* are merely carrier waves.

Detection

Fig. 275 represents a very simple receiver which would function under certain circumstances. The waves from the transmitter set up *R/F* currents, whose amplitudes vary at *A/F*, between the aerial and the earth.

It may be asked why the circuit cannot be made still simpler by omitting the crystal rectifier and putting the headphones between aerial and earth. The currents between aerial and earth are of *R/F*, and the telephone diaphragm cannot vibrate at so high a frequency; like the coil of a moving-coil galvanometer connected to an A.C. supply, the diaphragm would be urged first in one direction, then in the other, so rapidly that it remained at rest.

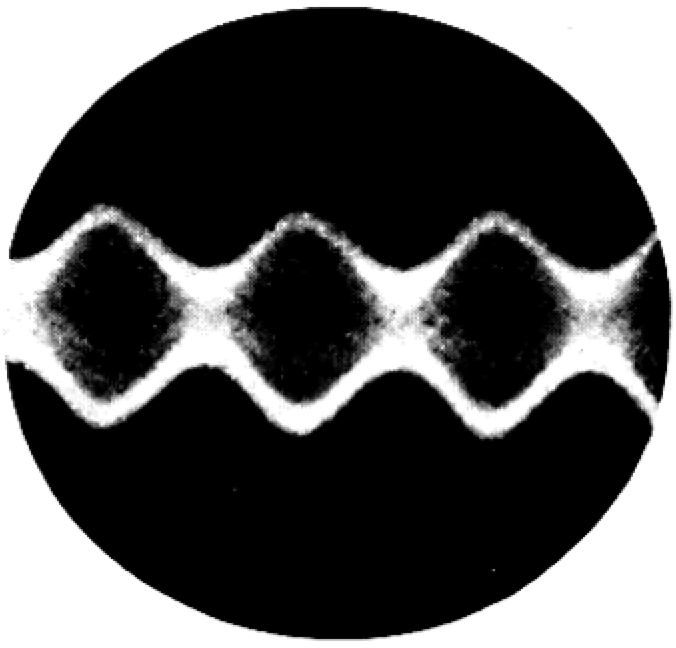


Fig. 274. Trace, taken with a cathode-ray oscillograph, showing a carrier wave of frequency 85 kilocycles per sec. modulated to a depth of about 50 % with a note of frequency 400 cycles per sec.

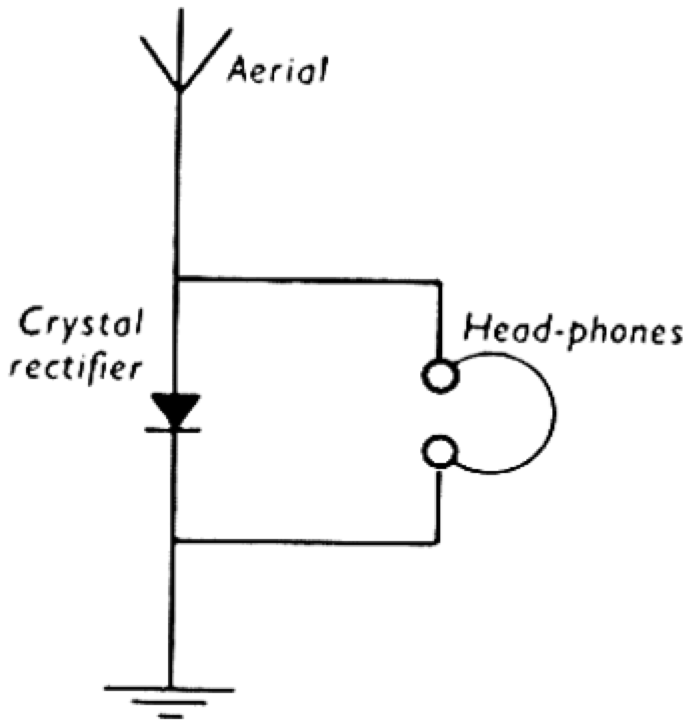


Fig. 275

Clearly some method of separating, as it were, the *A/F* from the *R/F* is necessary. A crystal of galena (i.e. sulphide of lead) touched with a phosphor-bronze wire called a 'cat's-whisker', has the property of allowing an electric current to pass through it much more readily in one direction than in the other. Thus the current through the crystal between aerial and earth is as shown in Fig. 276; the bottom halves of the carrier waves are suppressed. The half-carrier waves passing through the crystal are all in the same direction and hence reinforce each other. The curve of their average value is similar to the *A/F* frequency wave, and hence the diaphragm of the telephones vibrates in synchronism with the *A/F*. The crystal acts as a rectifier, but as this term is usually employed with reference to the conversion of A.C. to D.C. in power circuits, the crystal is more commonly called a detector or demodulator; the process is called detection or demodulation.

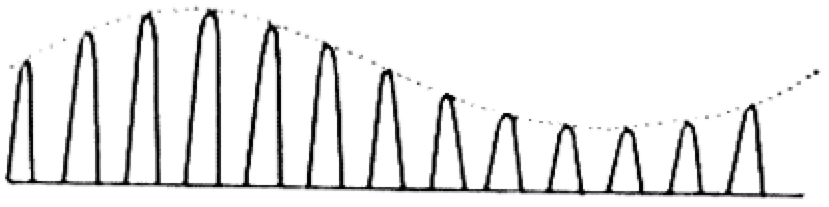


Fig. 276

Tuning

The simple receiving circuit just considered has the disadvantage that it cannot be tuned. The circuit of aerial and earth is essentially the same as the oscillatory circuit of Fig. 259 and contains inductance, L, and capacitance, C, a condenser being formed by the aerial and earth. Its resonant frequency is $\frac{1}{2\pi\sqrt{LC}}$ (see p. 246). Only if its resonant frequency corresponds to the frequency of some broadcasting station will it reproduce signals.

Tuning can be achieved by means of a variable condenser, or a variable inductance or both (Fig. 277). The condenser C_1 is in series with the condenser formed by the aerial and the earth and hence reduces the total capacitance, thus increasing the resonant frequency of the circuit. The inductance L increases the total inductance of the circuit and hence reduces the resonant frequency of the circuit.

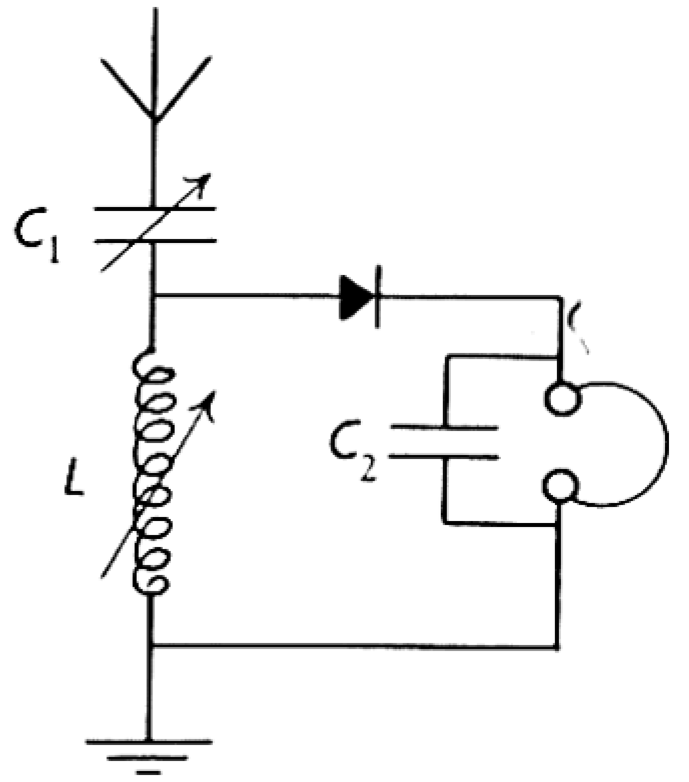


Fig. 277

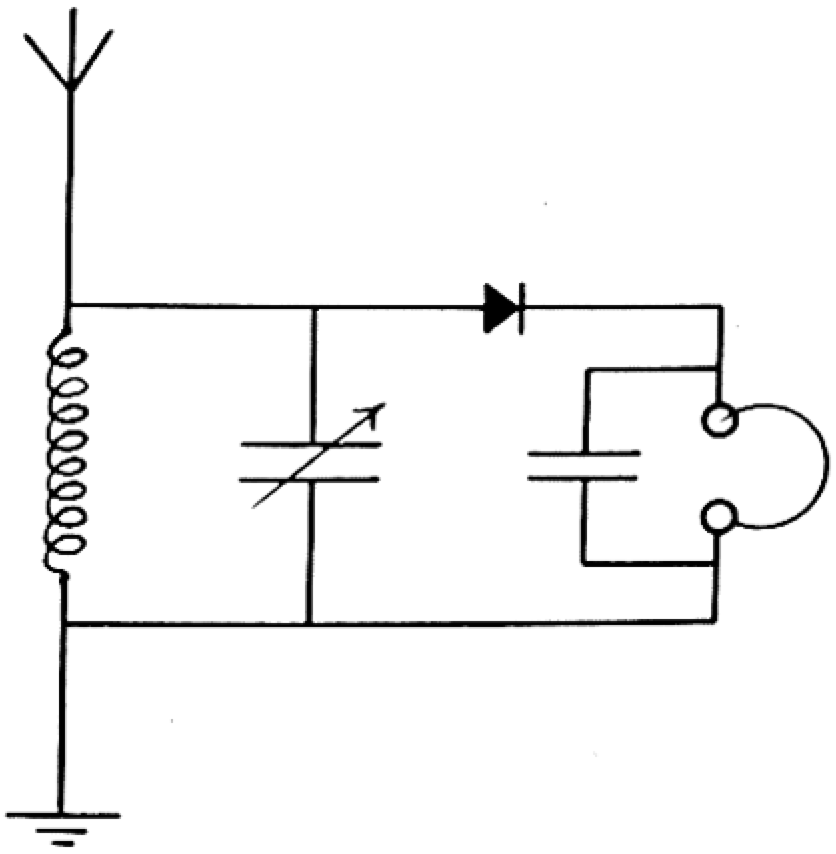


Fig. 278

The crystal has a high internal resistance and if placed between aerial and earth as in Fig. 275 it would make the tuning of the circuit flat. Its change of position in Fig. 277 is therefore an improvement. The function of the condenser C_2 will be considered later.

The tuning circuit just described is an acceptor circuit (see p. 246). When tuned it has a minimum impedance and takes a maximum current; thus there is a maximum P.D. across the inductance, to be passed on to the telephones. An alternative tuning circuit is the rejector circuit shown in Fig. 278, which has a very high impedance at the resonant frequency and also gives a high P.D. across the inductance.

The examples so far given are called single circuit tuning and are not very selective, i.e. they do not permit the separate reception of signals whose frequencies are near to one another. By coupling two circuits together and tuning both of them, greater selectivity is obtained. In Fig. 279 both circuits are acceptor circuits since the applied E.M.F. is in series with the inductance and condenser in each case. The coupling takes place through the mutual inductance of the two coils.

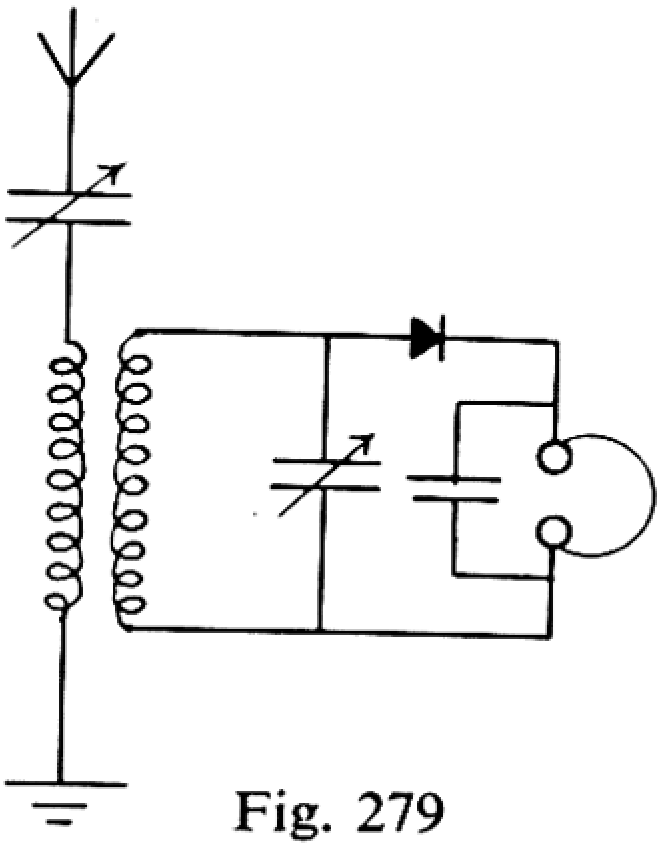


Fig. 279

The by-pass condenser

The function of condenser C_2 in Fig. 277, known as the by-pass condenser, will now be considered. Since the reactance of a condenser, $\frac{1}{2\pi fC}$, is inversely proportional to the frequency f, C_2 allows *R/F* currents to pass through it much more readily than *A/F* currents. The impedance of the telephones to *R/F* currents is high and hence C_2 acts as a by-pass for them.

Suppose an unmodulated carrier wave is received by the aerial. Each half-cycle of the *R/F* current passing through the crystal detector will charge up the plates of C_2 and, during the inactive half-cycles, C_2 will tend to discharge through the telephones. Owing to the high impedance of the telephones at *R/F* the condenser plates will remain at a steady voltage, nearly the peak value, and the diaphragm will not vibrate because the current through the telephones is steady. If, however, a modulated carrier wave is being received, the P.D. across C_2 will vary at *A/F* and hence a current varying at *A/F* will pass through the telephones. The capacitance of C_2 is kept small so as to avoid the possibility of a smoothing out of the *A/F* signal.

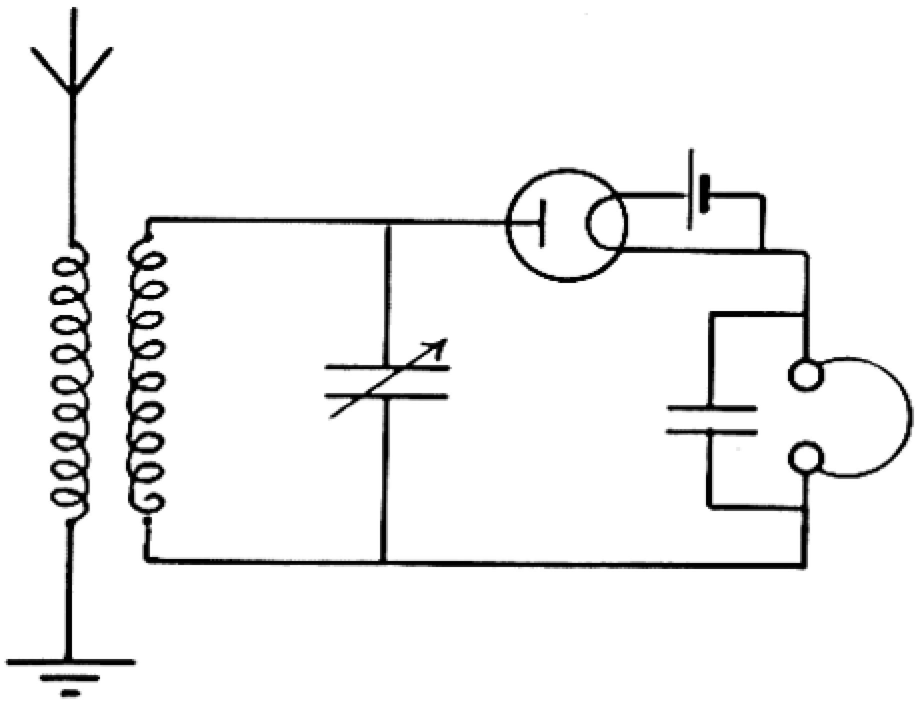


Fig. 280

The diode as a detector

A diode allows current to pass through it in one direction only, as was explained on p. 262, and hence a diode can be used instead of a crystal as a detector (Fig. 280).

After a period of disuse the diode has become the most popular detector because it introduces very little distortion.

The triode as amplifier

The meaning of the amplification factor, μ, essential to the understanding of the following account of the use of the triode as an amplifier, was described on p. 265.

The grid and filament of the triode are connected to the ends of the tuning circuit as in Fig. 281, and hence there is a P.D. between them varying at R/F. The anode is connected to an 'anode load', which may take the form of a high resistance, R ohms, across which amplified P.D.'s are produced. This is connected to the next stage in the receiver, possibly a detector.

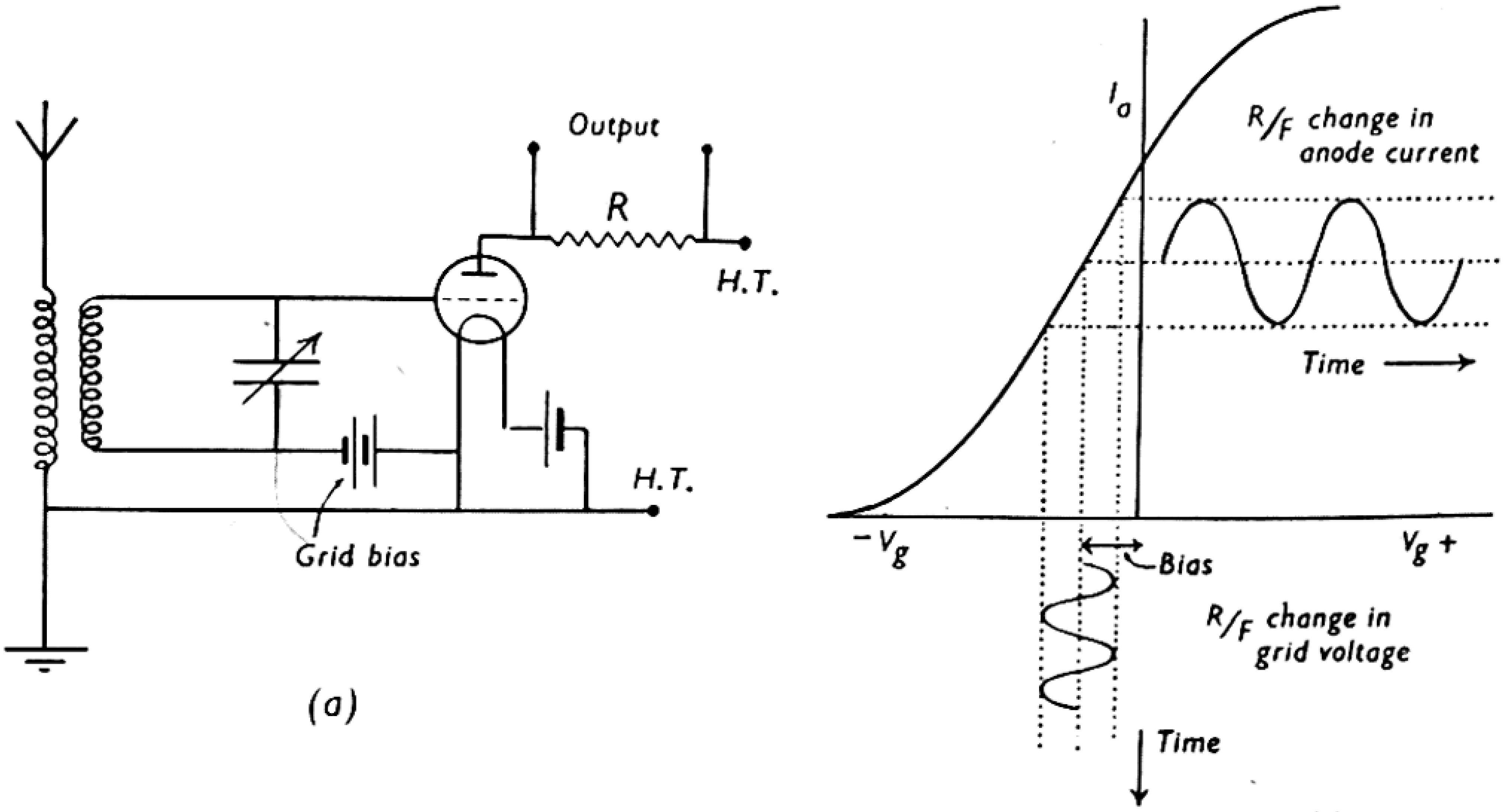


Fig. 281

(b)

It was shown on p. 265 that a change in the grid voltage of V_g has the same effect on the filament-anode current as a change of μV_g on the anode. But in Fig. 281 the current must also be changed in the anode load, R.

Suppose the impedance of the valve to A.C., defined as

$$\frac{\text{change in anode potential}}{\text{corresponding change in anode current}},$$

is R_a (typical values vary from 5000 to 30,000 ohms). The voltage change of μV_g will be across the total resistance of the anode circuit, $R+R_a$. Hence the voltage change across the anode load is $\dfrac{R}{R+R_a}\mu V_g$. The expression $\dfrac{\mu R}{R+R_a}$ is called the *voltage amplification factor*. The larger the value of R the nearer this approximates to μ. If, however, a very large value of R is used the H.T. voltage to work the valve will have to be made very large, and hence R is usually not more than 2 or 3 megohms.

To avoid distortion the valve must operate on the straight portion of its characteristic (Fig. 281 (*b*)). The grid is given a negative bias, say of -2 volts, by means of the grid-bias battery to prevent it from collecting electrons and causing a grid current in the tuning circuit which leads to a loss of selectivity due to its damping effect.

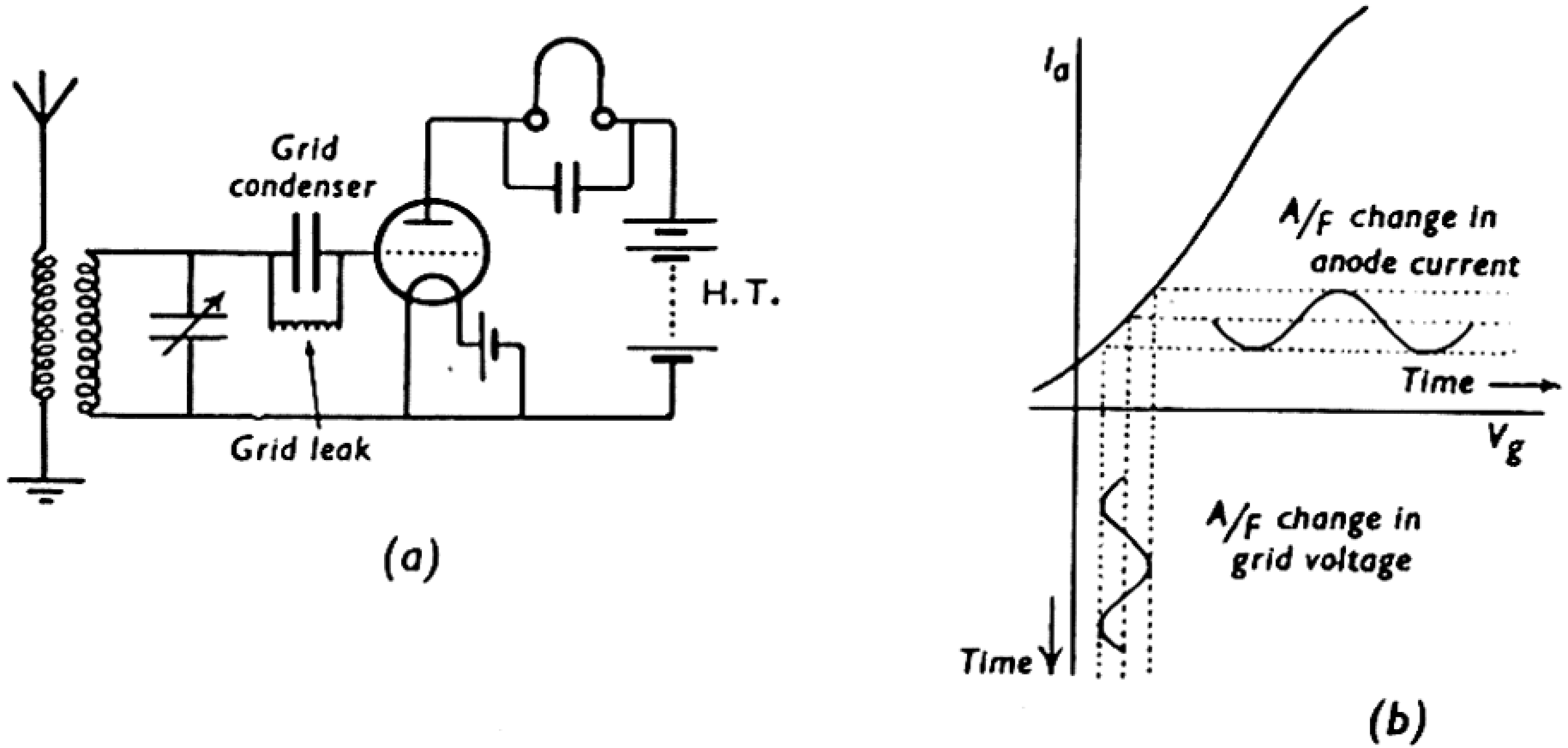


Fig. 282

The triode as a detector

The triode can be used simultaneously as a diode for detection and as a triode for amplification. Regarding the grid and the filament as a diode, the grid filament circuit of Fig. 282 (*a*) should be compared with Fig. 280. It will be seen that they are identical; the grid leak (a resistance of about 1 megohm) corresponds to the headphones and the grid condenser corresponds to the by-pass condenser.

Suppose that a constant R/F signal is applied to the circuit in Fig. 282 (*a*). The grid condenser is no barrier to R/F currents and during the half-cycles when the signal causes a positive potential to be applied to the grid, the latter collects electrons. These electrons leak away slowly through the grid leak and the coil of the tuning circuit back to the filament. After several R/F cycles an equilibrium is attained between the collection of electrons and their escape through the grid leak, the grid acquiring a negative or less positive potential and causing a decrease in the anode current.

When the strength of the R/F is varied, the grid voltage and the anode current vary in the same manner. Hence a signal modulated at audio-frequency gives rise to an A/F variation in the anode current.

Fig. 282 (*b*) shows the effect of an A/F wave on the voltage of the grid and the corresponding change in the anode current.

Clearly the grid leak could equally well have been connected directly between grid and filament.

The triode should be operated on the straight part of the characteristic with a zero or slightly positive grid bias since, in contrast to the use of a triode as an amplifier only, a grid current is required. This method of using the triode as a detector is known as *grid rectification.*

There is another method, seldom used, called *anode-bend rectification.*

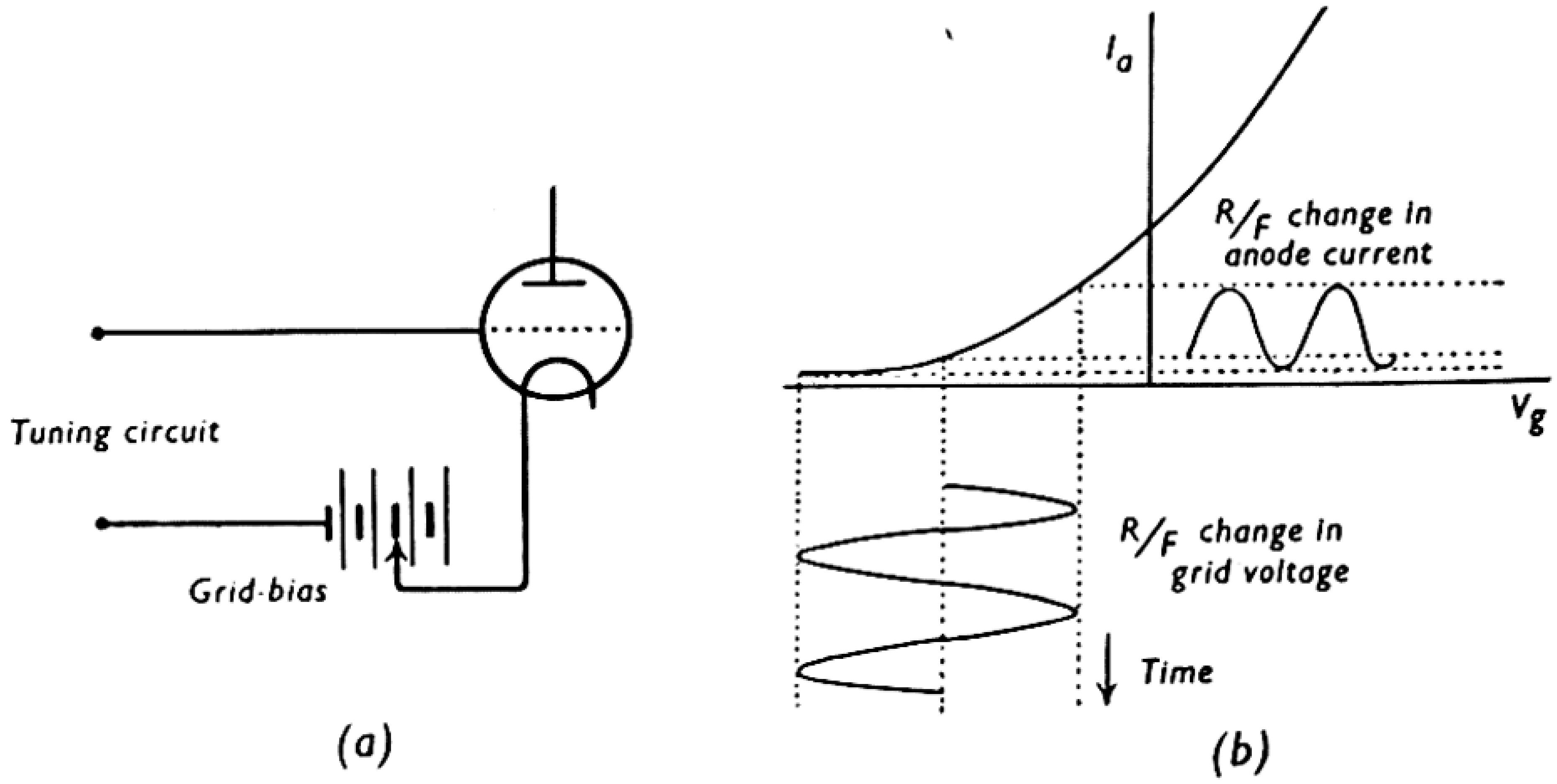


Fig. 283

The point of operation is on the bend of the characteristic, preferably the lower bend rather than the upper, because then less current is taken from the H.T. battery. It can be seen from Fig. 283 (*b*) how every alternate half-cycle of the anode current is largely suppressed owing to the curved shape of the characteristic. The full circuit has not been drawn in Fig. 283 (*a*) because what is omitted is the same as Fig. 282 (*a*). Note, however, that no grid condenser or grid leak is required and that a grid-bias battery is needed to bring the point of operation of the valve on to the bend of the characteristic. In this case a steady R/F signal causes an increase in the average value of the anode current, while a modulated one produces an A/F rise and fall of the current flowing in the anode load.

Reaction

The strength of the signal reproduced by the receiver can be increased by a method variously known as reaction, regeneration, back feed, or back coupling.

Fig. 284 represents a circuit consisting of a single triode used as a grid detector, and incorporating a 'tickler' coil L_2 to provide reaction. L_2 is connected in the anode circuit and is coupled with one of the tuning coils L_1. The alternating currents in L_1 and L_2 are in phase and hence the currents in L_2 will induce an increase in those in L_1, thereby increasing the currents through the valve and hence in L_2 itself. If L_1 and L_2 are coupled too closely the valve will behave as an oscillator and 'howling' will be heard in the telephones. The amount of back feed, as it is called, can be controlled by varying the relative positions of L_1 and L_2 or by inserting a variable condenser (not shown in Fig. 284) between the anode and filament of the valve, which acts as a by-pass and enables the current in L_2 to be varied.

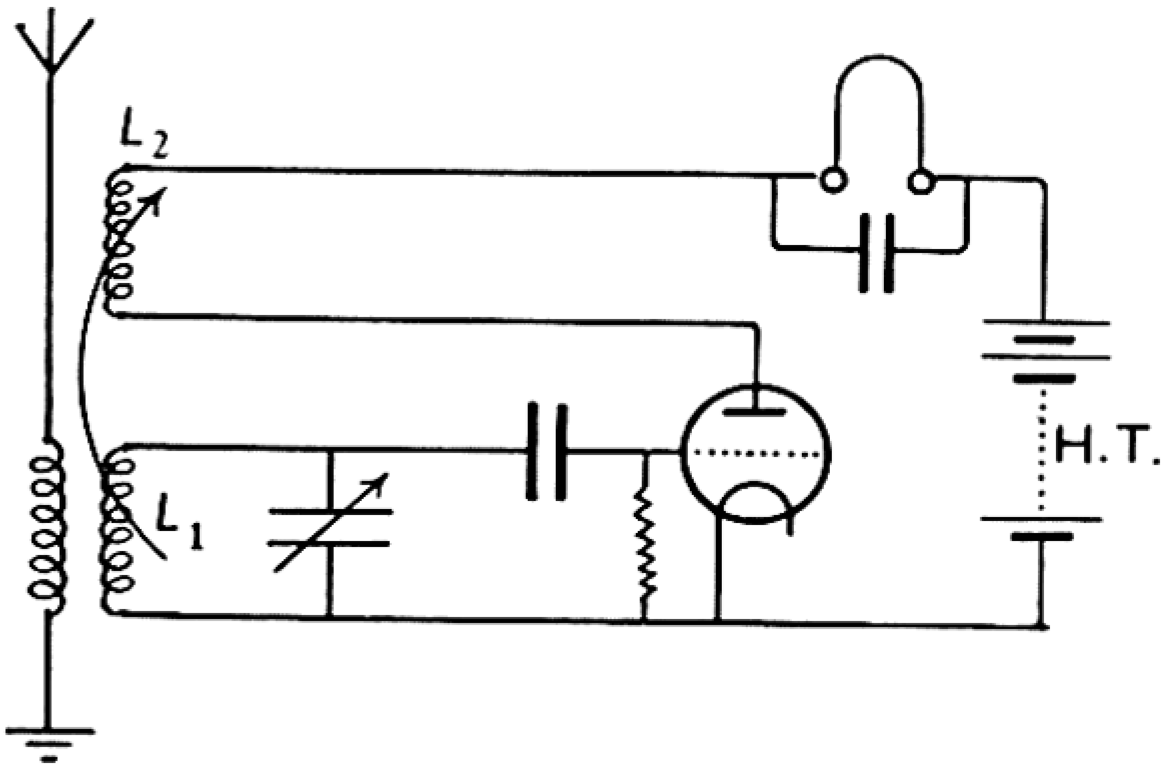


Fig. 284

The valve as an oscillator

Energy from the anode circuit is fed back into the grid circuit to produce reaction. If this energy compensates for the energy lost as heat in the grid circuit, continuous oscillations will result and the valve will behave as an oscillator (Fig. 285).

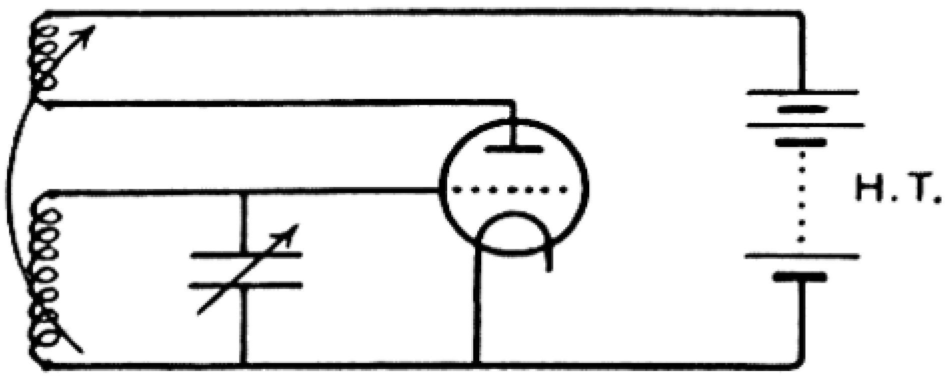


Fig. 285

Fig. 286 shows a simple circuit by means of which modulated waves can be generated. The part of the circuit generating carrier waves is similar to that of Fig. 285. The waves are modulated by the microphone M, similar in principle to a telephone transmitter. Sound waves falling on M give rise to an A/F current which is coupled through a transformer to the oscillatory circuit. The condenser C_1 acts as a by-pass for the R/F currents, and the condenser C_2 controls their frequency.

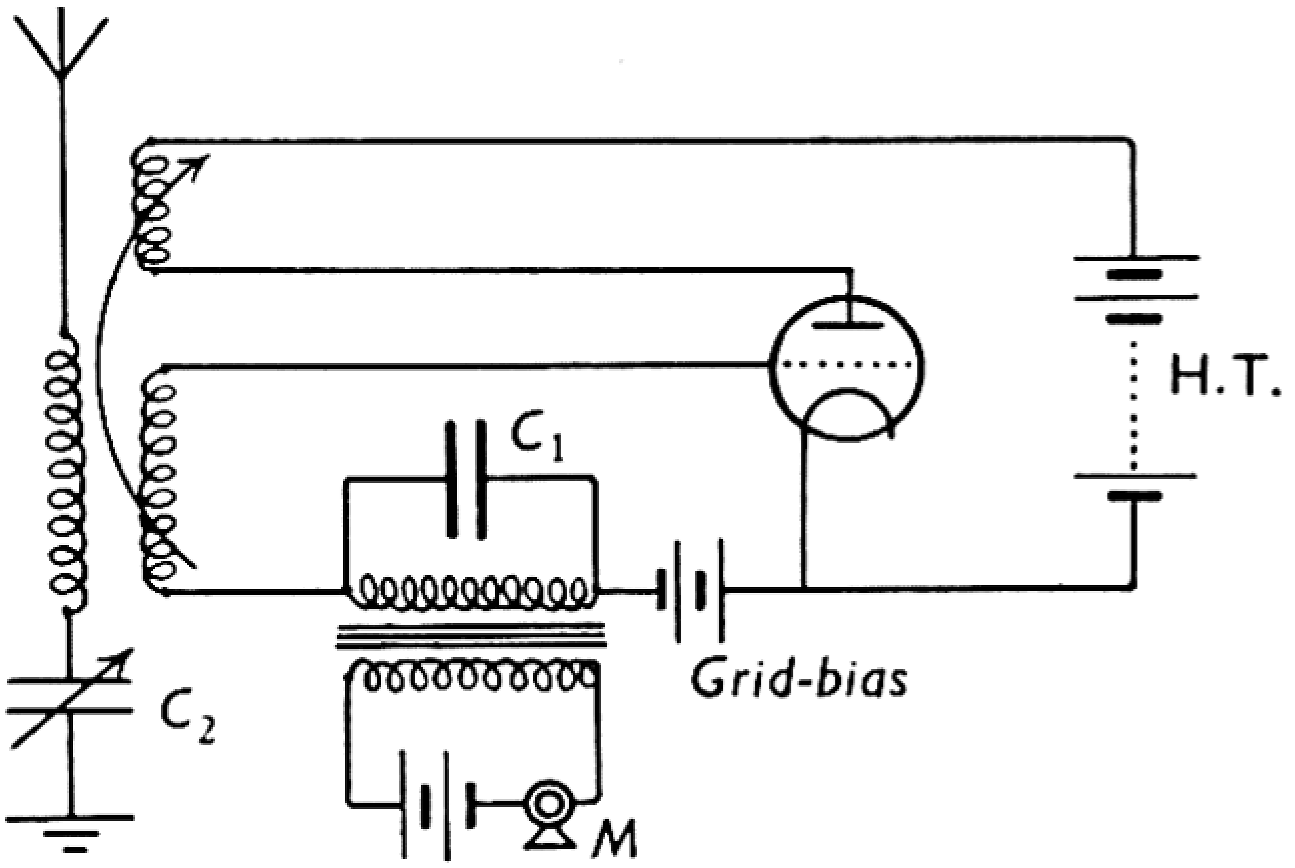


Fig. 286

Coupling of valves

In a normal receiver there are three stages whose functions are (1) R/F or high-frequency amplification, (2) detection, (3) A/F or low-frequency amplification. One or more valves may be used in each stage and hence it is necessary to consider how valves can be coupled.

Coupling can be effected by four chief methods, resistance-capacity, choke-capacity, tuned anode and transformer coupling.

Fig. 287 (*a*) illustrates resistance-capacity coupling. As has already been explained, the P.D. across R is $\frac{R}{R+R_a}\mu V_g$ and hence varies with V_g. The upper end of R is maintained at the positive potential of the H.T. battery and the potential of the lower end varies. If the lower end of R were connected direct to the grid of the second valve, the grid would receive current from the H.T. battery, and behave as an anode. Hence a condenser C, which allows alternating current to flow but not direct current, is inserted. In order that C shall not insulate the grid completely a grid leak is provided and this enables electrons collected by the grid to leak back to the filament.

The resistance R may be replaced by a choke, the rest of the circuit remaining unchanged; this is choke-capacity coupling. The advantage of a choke is that while it has a high impedance for A.C. its resistance for D.C. is low, and it permits moderate anode voltages to be used. It has the disadvantages that its impedance varies with frequency and that it has considerable self-capacity. In consequence, low notes and very high ones are not well amplified.

The resistance *R* may be replaced also by a rejector circuit, consisting of an inductance and variable capacitance, the remainder of the circuit being unchanged; this is tuned anode coupling. As explained on p. 247 a rejector circuit has a high impedance at its resonant frequency. As this is a tuned system it is applicable to one frequency only and it increases selectivity. It cannot be used in the audio-frequency stage because the amplification there must be independent of frequency.

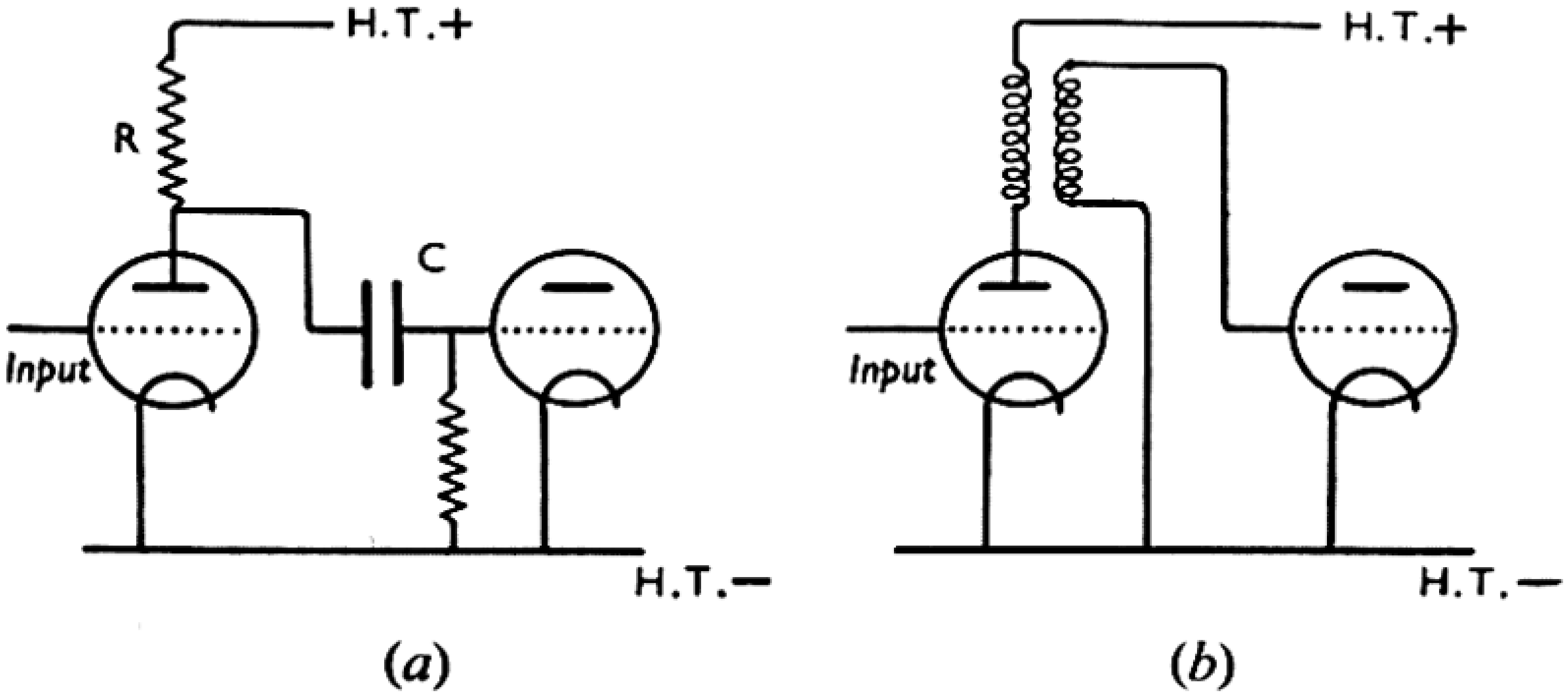


Fig. 287

Fig. 287 (*b*) illustrates transformer coupling. A condenser and a grid leak are unnecessary here because there is no direct connection between the anode of the first valve and the grid of the second. Air-cored transformers are used for *R/F* coupling and iron-cored ones for *A/F*. For *R/F* the secondary of the transformer is sometimes tuned by connecting across it a variable condenser. It might seem possible greatly to increase the amplification by using a step-up transformer of high ratio. However, the inductance of the primary must be high in order to give a large voltage amplification factor from the first valve, and a large secondary winding would have a very considerable self-capacity. Hence the step-up ratio usually employed does not exceed 1:6.

Tetrode and pentode

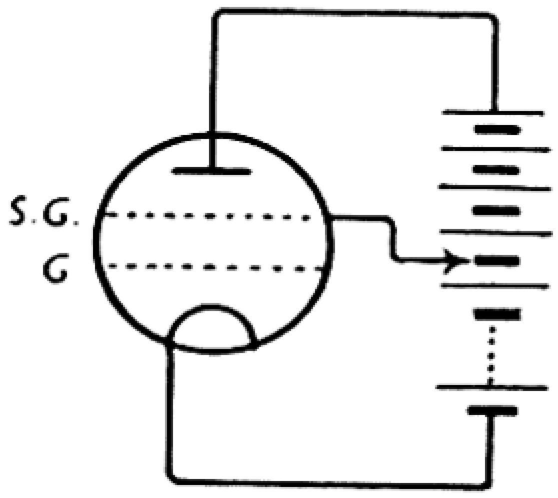


Fig. 288

The triode, particularly when used as a *R/F* amplifier, is liable to be unstable and to oscillate. The anode and the grid act as the plates of a condenser and energy is fed back from the anode circuit to the grid circuit through the capacitance. The defect can be overcome by inserting a screening grid between the anode and the control grid (Fig. 288). The resulting valve is called a tetrode or a screened-grid valve.

If the potential of the anode is 120 volts that of the screening grid is usually about 80 volts. High voltage amplification factors, from 100 to 1500, can be obtained with a tetrode.

When the electrons from the filament strike the anode with sufficient velocity they cause the emission of secondary electrons, and if the potential of the screening grid happens to be higher than that of the anode, it will collect these secondary electrons. The kink in Fig. 289 shows how, after secondary electrons begin to be emitted, the anode current of a tetrode actually decreases with increasing anode potential for a certain range of voltage. Thus distortion will result if the potential of the anode swings below that of the screening grid. The insertion between the anode and the screening grid of a suppressor grid, kept at earth potential, enables the anode to attract back the secondary electrons without losing any to the screening grid. The resulting five-electrode valve (Fig. 290), called a pentode, is often used in preference to either a tetrode or a triode for amplification purposes.

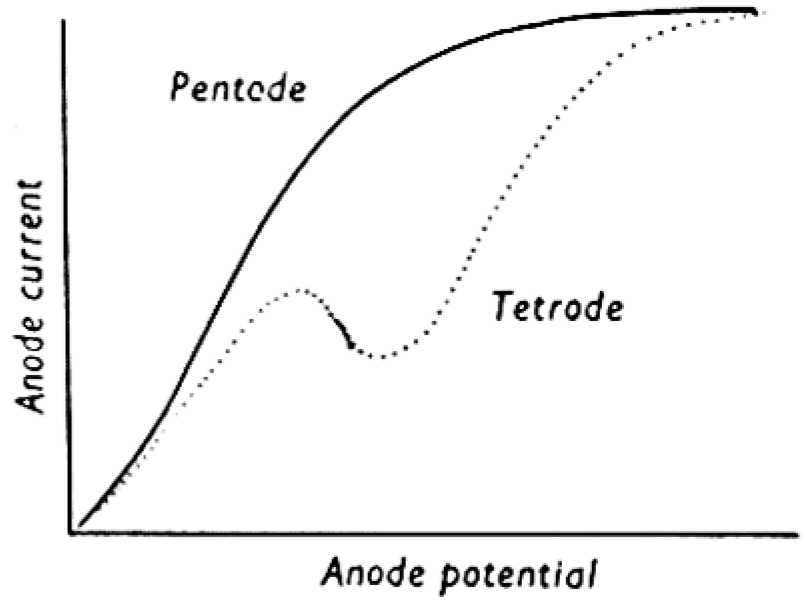


Fig. 289

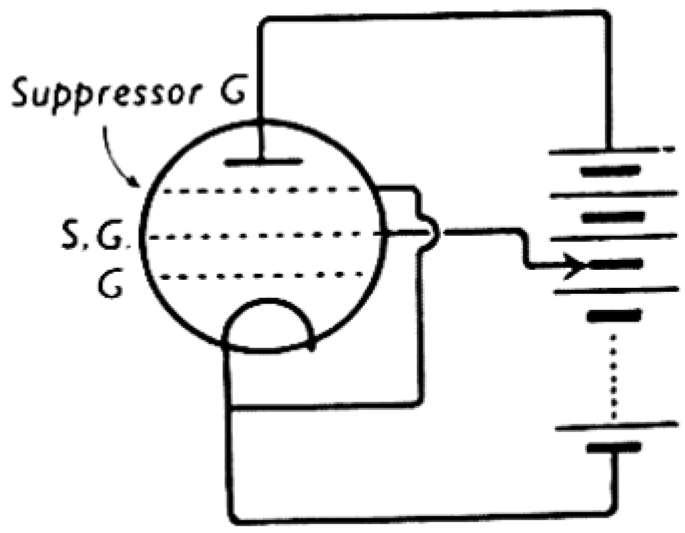


Fig. 290

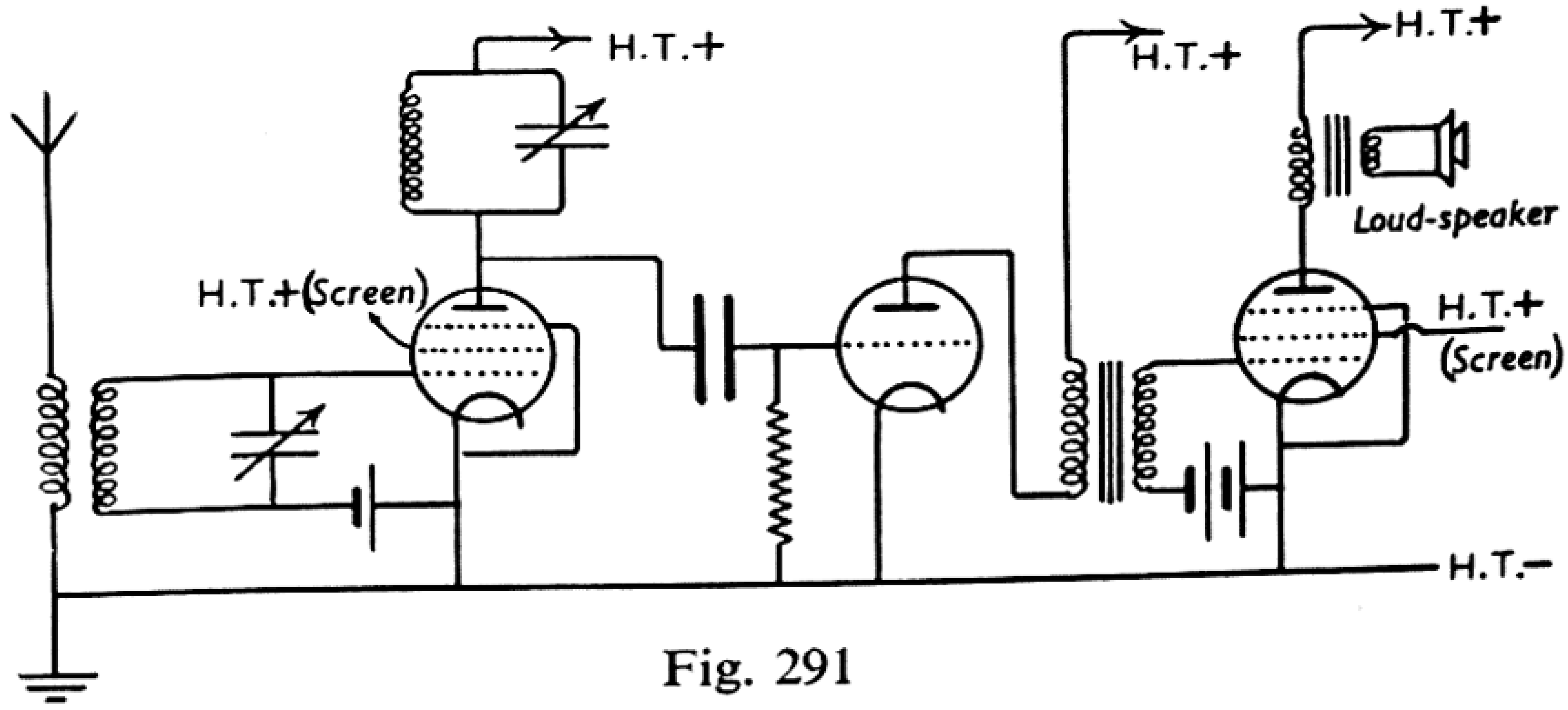


Fig. 291

Three-valve receiver

Fig. 291 represents a simple three-valve receiver. A pentode *R/F* amplifier is coupled by a tuned anode circuit to a triode acting as a grid detector. The triode is coupled through a transformer to a pentode *A/F* amplifier. The loud speaker is of the low-impedance type, and hence is connected through a step-down transformer to the anode circuit of the last valve.

QUESTIONS

1. Explain what is meant by an *oscillatory electric circuit* and distinguish between *closed* and *open* circuits. Describe how oscillatory circuits are used in wireless transmission and reception. (N.)

2. Describe the triode thermionic valve. Give an account of two applications of a thermionic valve, with a brief explanation of the action in each case. (N.)

3. Discuss the physical principles involved in the use of a triode in the reception of wireless signals as (*a*) a rectifier, (*b*) an amplifier. (N.)

4. Describe the triode valve and give a short account of its mode of action. Explain the term *voltage amplification* of the valve and describe how you would determine it experimentally. Give a labelled diagram of the circuit used. (N.)

5. Describe briefly the action of a modern radio receiver. (O. S.)

6. It has been reported in *The Times* that members of the Australian forces cut off and left behind in Timor made a wireless set and communicated with Australia. Enumerate the components which you consider they could improvise from derelict cars and machinery and explain how you would set about the task if you found yourself in their position.

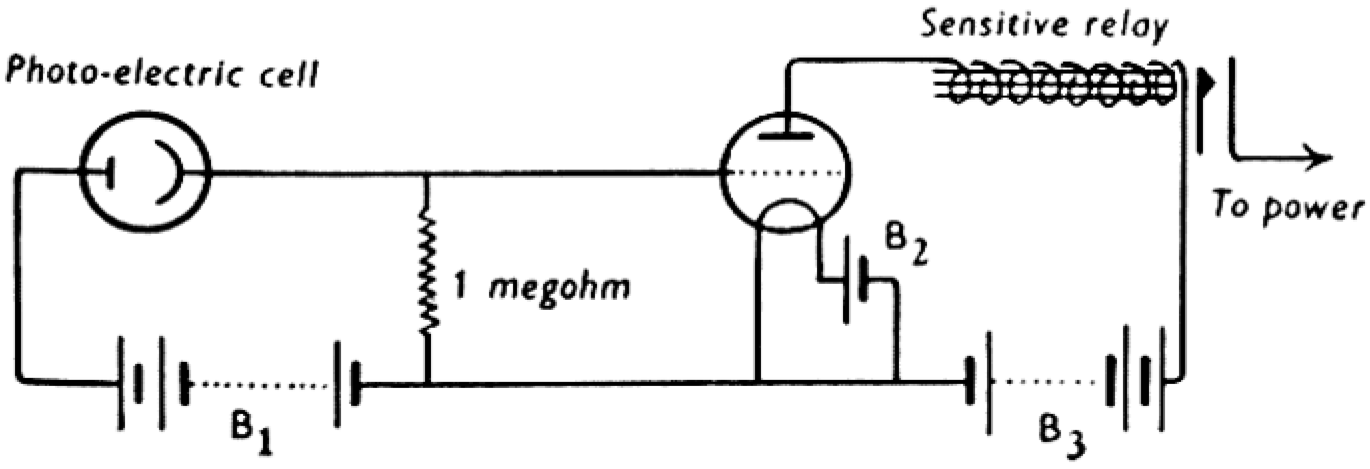


Fig. 292

7. When light falls on the photoelectric cell in Fig. 292 the resulting current is amplified and then operates a relay which switches on a power circuit not shown. Explain how the circuit works, stating clearly the purpose of the batteries B_1, B_2, B_3, and the megohm resistance. Why does the valve amplify the current?

8. Explain what is meant by reaction in a radio receiver. Fig. 293 represents 'capacitive reaction'. Explain carefully how it works.

9. The circuit in Fig. 294 represents a valve oscillator. Explain how it works and how it differs from Fig. 285.

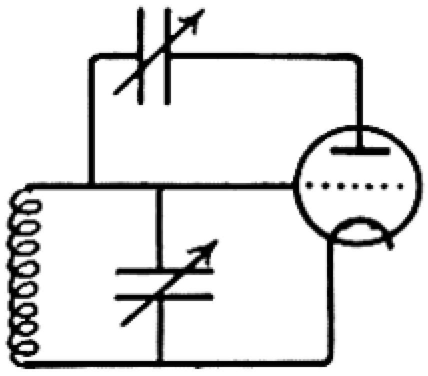


Fig. 293

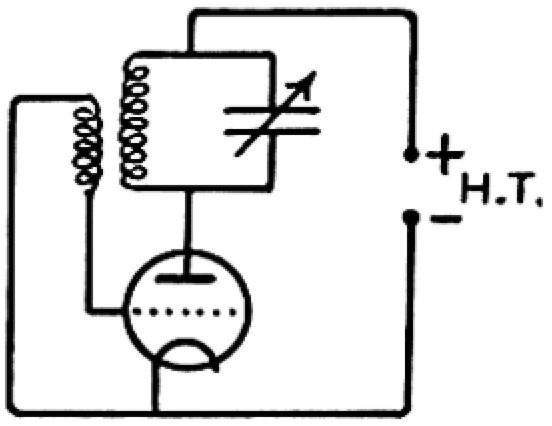


Fig. 294

10. Draw the circuit of a receiver consisting of three triodes: a H.F. amplifier, coupled by a tuned anode circuit to a grid rectifier, coupled by resistance-capacity to a L.F. amplifier.

11. Draw the circuit of a two-valve receiver consisting of a detector coupled by a transformer to a L.F. amplifier. Include reaction.

CHAPTER 17

ATOMIC PHYSICS

In the last decade of the nineteenth century three fundamental discoveries, the electron, X-rays and radioactivity, provided entry into a new and unsuspected world, that of the atom. The growth of numerous centres of scientific training and research, and latterly the stimulus of war, have enabled these discoveries to be developed and exploited with unexampled swiftness.

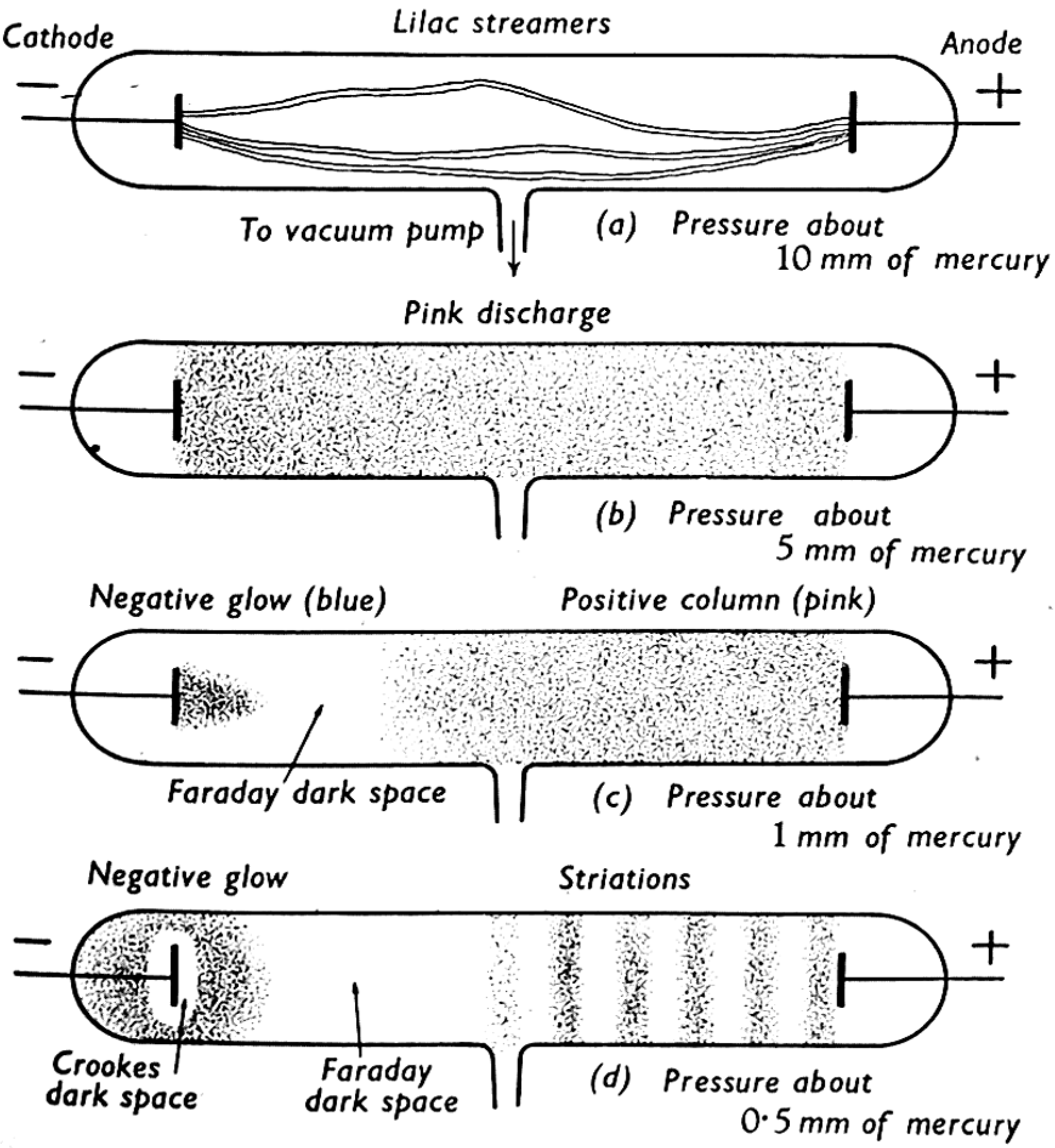


Fig. 295

The discharge of electricity through gases

The electron was discovered from the study of the discharge of electricity through rarefied gases. The preliminary stages of the discharge, as the pressure of a gas is reduced, are rather striking and beautiful. Fig. 295 represents a long glass tube having electrodes at its ends. The electrodes are connected to the terminals of an induction coil which acts as a unidirectional source of several thousand volts. As the pressure of the air is reduced by a pump, lilac streamers first pass between the electrodes. Then the tube becomes filled with a pink glow. The colour varies with the gas in the tube; neon gives a red glow, argon pale blue, mercury green and helium yellow-white. As the pressure of the air is further reduced, the pink glow, called the positive column, shrinks towards the anode, and a dark space, called the Faraday dark space, together with a blue negative glow round the cathode appear. At a still lower pressure the positive column breaks up into striations, and shrinks still further, while the Crookes dark space appears between the cathode and the negative glow. Eventually at a pressure of about 0·02 mm. the positive column, the Faraday dark space and negative glow have disappeared, and the Crookes dark space has expanded to fill the whole tube. The glass of the tube then fluoresces a green colour.

These complicated phenomena will be considered again later.

Cathode rays

The electron was discovered by the investigation of the Crookes dark space and the following experiments refer to gas pressures such that the Crookes dark space fills the tube. If an obstacle such as a mica cross is placed in front of the cathode, a shadow of the cross is thrown on the end of the tube (see Fig. 296). Certain minerals, when placed in front of the cathode, fluoresce with brilliant colours. Something appears to be emitted from the cathode and this invisible radiation was termed cathode rays. It was found that cathode rays would pass through an aluminium window. German investigators believed that they consisted of penetrating invisible light; English investigators, on the other hand, believed that they consisted of tiny particles of negative electricity. The

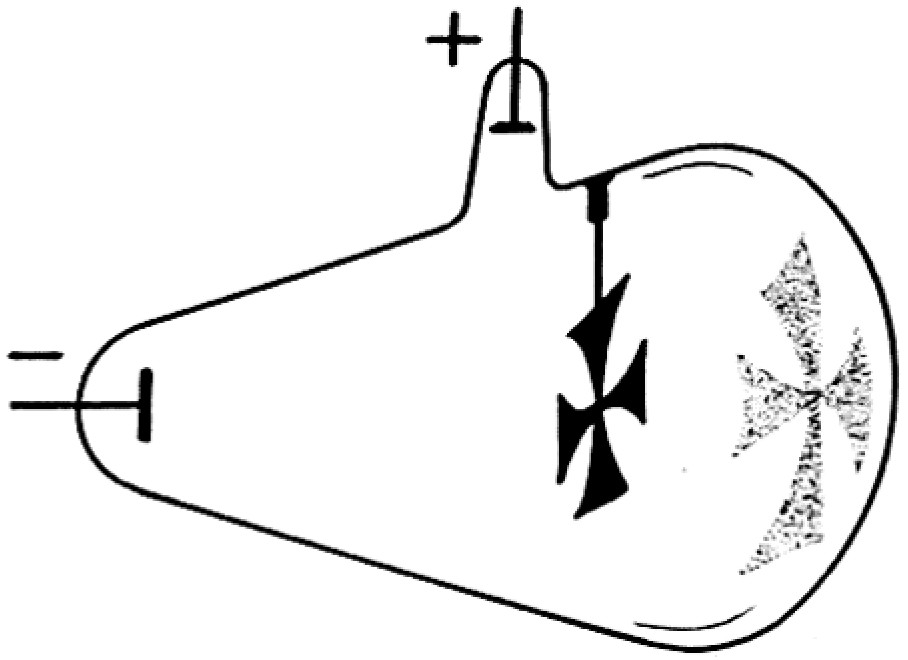


Fig. 296

latter view was substantiated by the three following experiments. A light mica paddle wheel is made to rotate on rails by the impact of the cathode rays (Fig. 297); when the polarity of the electrodes is reversed, the direction of rotation of the paddle wheel is changed. Again, the cathode rays, made visible by a fluorescent, zinc sulphite screen fitted inside the tube, are deflected by a magnet (Fig. 298); by Fleming's left-hand rule it can be seen that the cathode rays behave like a flow of negative charge. Finally, and this experiment of Jean Perrin in 1895 was crucial, the cathode rays can be collected on a metal cylinder, called a Faraday cylinder, which is connected to an electroscope, thereby proving directly that the cathode rays carry a negative charge (Fig. 299). The anode is perforated and surrounds the Faraday cylinder to shield the latter from electrostatic disturbance.

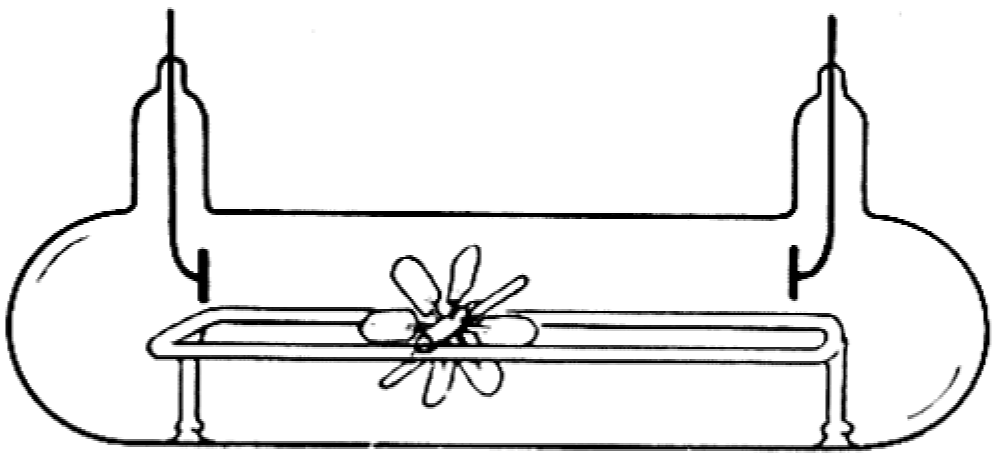


Fig. 297

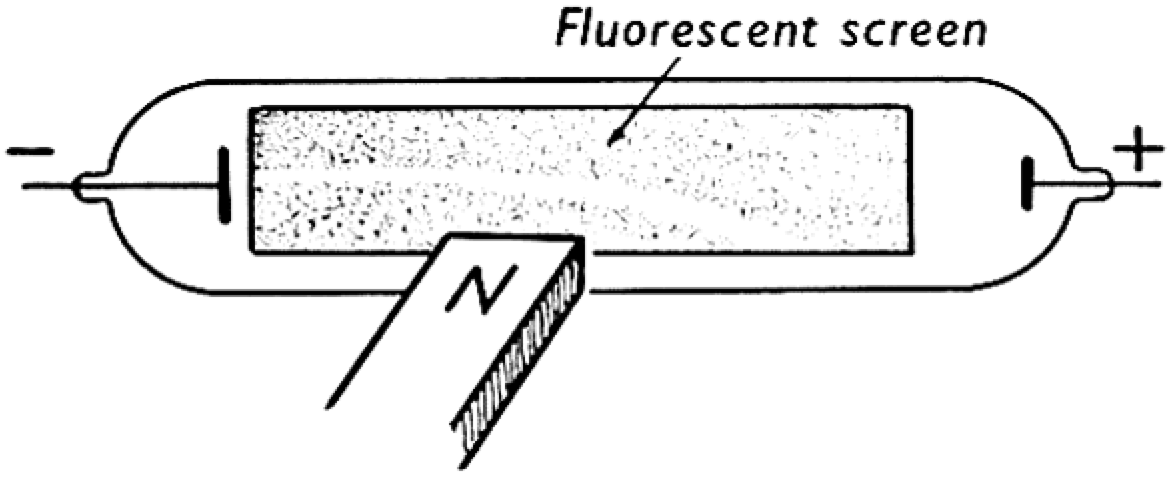


Fig. 298

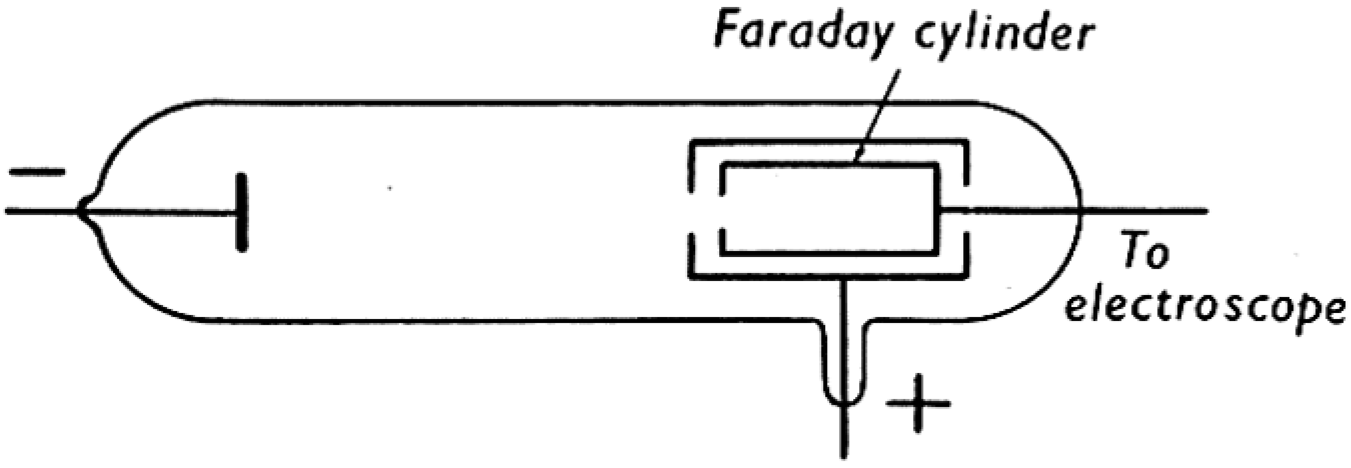


Fig. 299

Sir William Crookes, as early as 1874, suggested that the cathode rays represent 'a fourth state of matter' and remarked with extraordinary insight, 'I believe that the greatest scientific problems will find their solution in this borderland, and even beyond it; it seems to me that here we have reached the ultimate realities.'

Measurement of e/m and v for the cathode particles

Cathode rays were studied in various parts of the world, notably in the Cavendish Laboratory, Cambridge, under the direction of J. J. Thomson. In 1897 Thomson measured the ratio of the charge to the mass of the particles, and also their velocity, by deflecting them in magnetic and electric fields. His apparatus is represented in Fig. 300. A fine beam of cathode rays is produced by means of a cathode *C*, a perforated anode *A* and a slit *S*, and makes a bright spot *P* on the end of the tube. *X* and *Y* are metal plates between which an electric field can be applied and the dotted circle *M* represents two coils, one on each side of the tube, which enable a magnetic field to be applied at right angles to the paper.

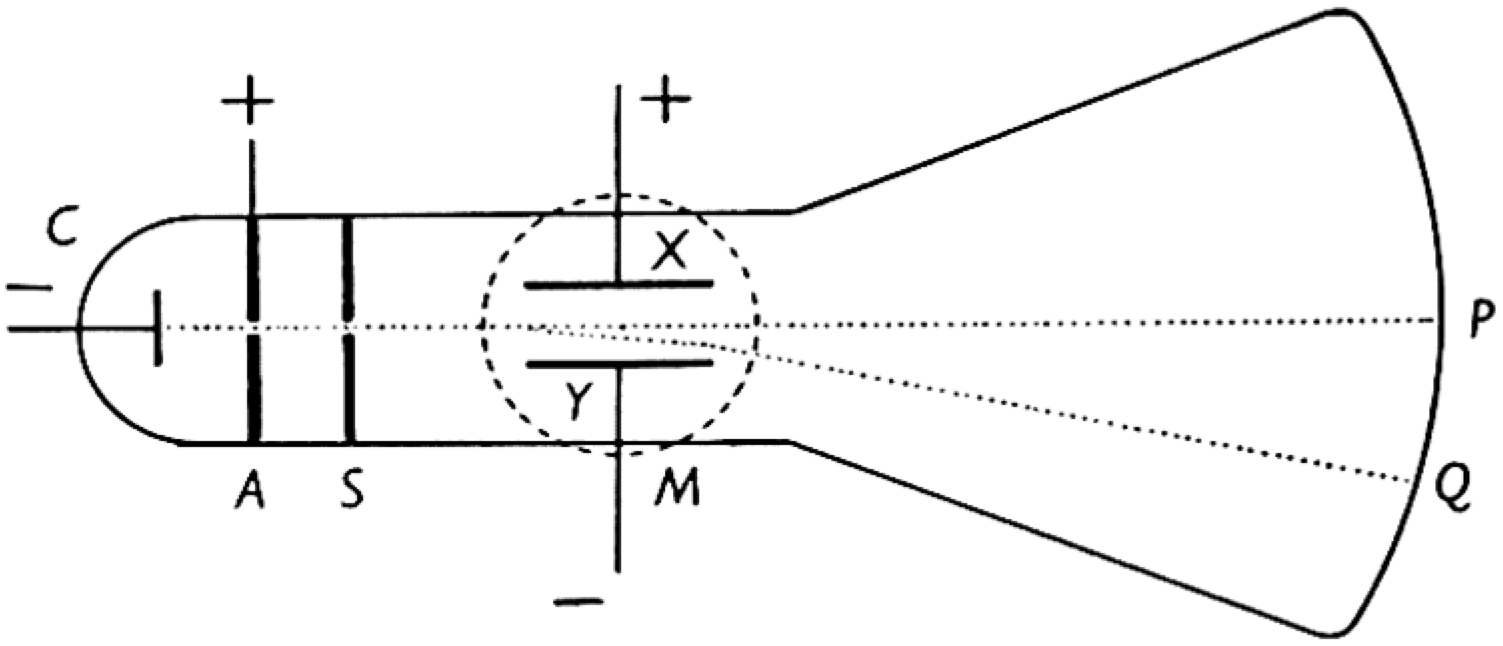


Fig. 300

Suppose a magnetic field only is applied. The cathode rays are deflected and the bright spot moves from *P* to *Q*. (If the magnetic field is reversed then the deflection will also be reversed.) The force exerted on a charged particle while passing through the uniform magnetic field is always at right angles to the magnetic field and to the direction of motion of the particle. Thus the particle will move in an arc of a circle while passing through the magnetic field. If e is the charge and v the velocity of the particle, the force exerted on it by the magnetic field, H, is Hev (see p. 88). This force will equal the centrifugal force mv^2/r, where m is the mass of the particle and r the radius of the circle in which it moves.

$$\therefore\ Hev=\frac{mv^2}{r},$$

$$\frac{e}{m}=\frac{v}{rH}.$$

The value of r is known from the distance PQ and the dimensions of the apparatus.

Now suppose that an electric field is applied between X and Y in such a direction and of such strength that it brings the spot of light back from Q to P, thus annulling the deflection due to the magnetic field. The force on each charged particle, due to the electric field E, is Ee and this must equal the force due to the magnetic field.

$$Ee = Hev,$$

$$v = \frac{E}{H}.$$

Knowing E and H, v can be calculated. Thomson found that its value was of the order of $\frac{1}{10}$th the velocity of light, and that it varied with the P.D. between the anode and the cathode.

Knowing v it is possible to obtain e/m from the above equation. Thomson found that the value of e/m is constant for all gases and electrodes.

Assuming e to be the same as the smallest charge carried by an ion in electrolysis (see p. 188) he calculated that the particles forming the cathode rays, now called electrons, had a mass only $\frac{1}{1840}$th* of that of the lightest known atom, hydrogen.

This was an astonishing result at the time, and an alternative interpretation, preferred by some physicists, was that e was 1840 times larger, rather than that m was 1840 times smaller, than the known values for the hydrogen ion. Accordingly Thomson and Townsend devised a method of finding e separately.

Determination of the charge of the electron

Townsend's method was to determine the number of drops in a charged fog formed on negative gaseous ions, and to measure the total charge. The charge on each drop could then be calculated. Gaseous ions are produced by molecules losing or gaining electrons and hence carry the electronic charge.

Water drops do not form in slightly supersaturated air unless it contains nuclei like specks of dust on which the vapour can condense. Gaseous ions act as nuclei for condensation and negative ions are more efficient in this respect than positive. Thomson, in an improved version of Townsend's experiment, ionised a gas mixed with saturated water vapour by exposing it to X-rays and caused a fog to form on the negative ions by sudden expansion. The fog was allowed to settle to the bottom of the apparatus where its total charge was measured with a quadrant electrometer, and also the total volume of the drops was found. The size of each drop could be calculated from the rate of fall, measured by a small telescope focused on the top edge of the fog as it settled. Thomson obtained an approximate value for *e* which completely confirmed his view that the electron had a mass very much less than the lightest atom.

---

* This is the modern value.

The American physicist, Millikan, modified the fog method and in 1906 made a very accurate determination of *e*. There were several serious sources of error in the earlier experiments. The fog droplets were not all of the same size and measurements were made only on the smallest, which fell most slowly. Moreover, the individual droplets did not remain the same size; the smallest tended to evaporate and the largest to grow.

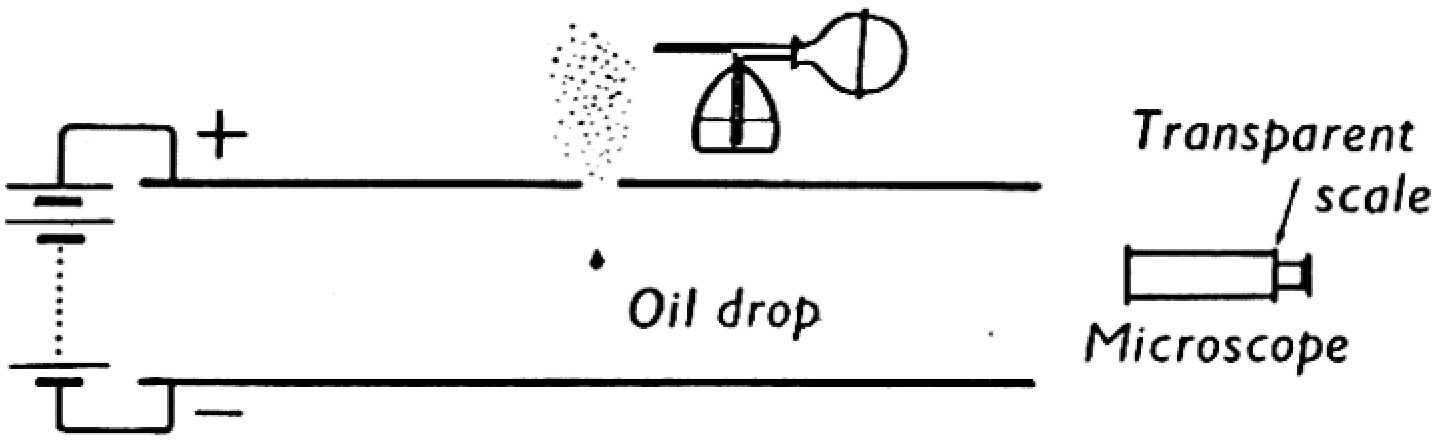


Fig. 301

Millikan formed drops of non-evaporating oil instead of water and observed the motion of a single drop instead of a whole cloud. His apparatus is indicated in Fig. 301. Oil drops formed by a spray are allowed to fall between two plates. They are brightly illuminated and observed through a low-power microscope. The plates are first earthed and the rate of fall of a drop timed by a stop-watch. From this the mass of the drop can be found. Stokes's formula, $F=6\pi\eta av$, gives the force of resistance F to the fall of the drop where η is the viscosity of the air, a is the radius of the drop and v is the velocity. When the drop has reached its terminal velocity, F is equal to the weight of the drop minus the upthrust of the air.

$$\therefore\ 6\pi\eta av=\tfrac{4}{3}\pi a^3(\rho_1-\rho_2)g,$$

where ρ_1 and ρ_2 are the densities of the oil and the air respectively. Hence the radius of the drop and its mass can be found.

Drops formed by a spray are usually found to be charged but, if necessary, they may be given a charge by directing on to them a beam of X-rays. The fall of a negatively charged drop can be arrested by giving the upper plate a positive potential. The drop can be made to

SIR J. J. THOMSON

LORD RUTHERFORD

SIR J. J. THOMSON (1856–1940) *was the discoverer of the electron and, in the words of Sir William Bragg, 'more than any other man was responsible for the fundamental change in outlook which distinguishes the physics of this century from that of the last'. At the turn of the century J. J., as he was affectionately called, attracted to the Cavendish Laboratory a brilliant team of research workers whom he directed in investigations connected with the discharge of electricity through gases. He would visit the room of each worker daily, leaving open the door, and his loud voice with its trace of a Lancashire accent would resound throughout the building; despite his own lack of manual skill he often displayed an uncanny insight into the working of their apparatus. In 1918 he became Master of Trinity and was succeeded as Cavendish Professor of Experimental Physics by Rutherford, a former pupil. He was buried in Westminster Abbey near to the tombs of Newton, Darwin, Kelvin and Rutherford.*

LORD RUTHERFORD (1871–1937), *a New Zealander, was perhaps the greatest British physicist since Newton. He was foremost in investigating radioactivity, and in 1903 put forward, with Soddy, the disintegration theory to account for it. In 1911 he propounded the nuclear theory of the atom, based on his experiments on the scattering of α-particles, and in 1919 he effected the first artificial disintegration of an atom. Sir James Chadwick has said of him: 'It is amazing that one man could so transform physics by his own effort, a man with no great mathematical equipment, good but not remarkable even in experimental technique. Nor had he an acute or subtle mind; no, his mind was like the bow of a battleship—there was so much power and weight behind it, it had no need to be as sharp as a razor. He brushed aside all irrelevancy and went straight to his mark.'*

rise and fall at will by adjusting the P.D. between the plates and a single drop can be kept under observation for hours.

Suppose the drop is held stationary by a P.D. of V volts, and the plates are d cm. apart. The electric field between the plates is V/d volts per cm., or $V/300d$ e.s.u. Thus the force on the drop of charge e is $Ve/300d$. This force must equal the weight of the drop, which is known from the rate of fall between uncharged plates. Hence the value of e can be calculated.

Millikan obtained the value $e = -4{\cdot}80 \times 10^{-10}$ e.s.u. for the smallest charge carried by a drop, although drops sometimes acquired charges exactly two or three times this value.

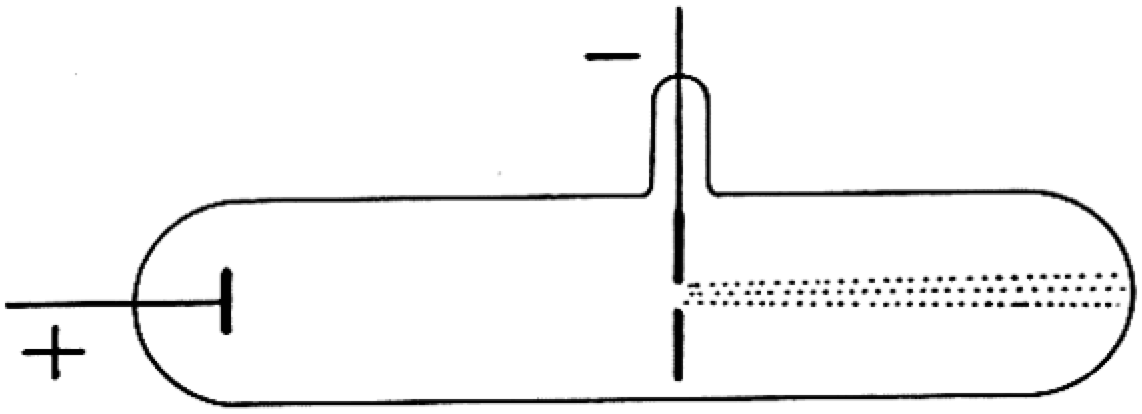


Fig. 302

Positive rays

In 1886 Goldstein discovered that if a hole is bored in the cathode of a discharge tube a reddish streamer of radiation can be seen on the further side of the cathode (Fig. 302). He called this radiation canal rays because it passed through a canal in the cathode. In 1898 Wien deflected the radiation with a powerful magnet and deduced that it consisted of positively charged particles; hence the usual name, positive rays.

In 1910–11 J. J. Thomson measured the deflection of positive rays when subjected simultaneously to a magnetic and an electric field. From the values obtained for e/m, assuming e to be equal in magnitude but opposite in sign to the charge of the electron, he found that the masses of the particles in the positive rays are those of atoms or molecules and that they vary with the nature of the gas used. The particles were therefore taken to be atoms or molecules of the gas which had lost one or more electrons, i.e. positive ions.

Explanation of the phenomena of the discharge tube

The conductivity of a gas, like that of a liquid, is due to positive and negative ions travelling in opposite directions. A discharge can start only if the field is sufficiently intense to ionise the gas, unless a separate source of ionisation, such as X-rays, is used. The phenomena of the discharge tube can be accounted for by the production of ions by collision in various parts of the tube, luminosity being a result of ionisation. The clue needed to solve the problem was the very different fall of potential in various parts of the tube, determined by inserting subsidiary electrodes.

Consider the state of the discharge in the bottom of Fig. 295. The fall of potential in the Crookes dark space is very much greater than in any other part of the tube. Here the positive ions acquire their maximum speed and, as already described, if allowed to pass through a hole in the cathode, they will ionise the gas on the other side, causing a reddish stream of illumination.

The negative ions or cathode rays, caused probably by the bombardment of the positive ions on the cathode, move with increasing speed in the opposite direction and collide with molecules of the gas, causing the ionisation in the negative glow. They lose energy by collision, becoming incapable of further ionisation, and the Faraday dark space results. On being speeded up again by the field they acquire sufficient energy to cause ionisation in the positive column. The striations are accounted for by alternate ionisation and subsequent increase of speed of the negative ions to produce fresh ionisation.

Isotopes

An interesting discovery was made by Thomson during his investigation of positive rays which recalled a discarded hypothesis put forward by Prout in 1815, that the atomic weights of all the elements should be exact integers. The gas neon, of atomic weight 20·2, was found to consist of two kinds of atoms, of masses 20 and 22. The existence of atoms, having identical chemical properties but different masses, was earlier postulated by Soddy from his experiments on radioactivity, and he called them *isotopes*, the Greek derivation of the word indicating that they have the same place in the periodic table of the elements.

Aston refined Thomson's method and undertook the task of examining all the elements. When his apparatus was perfected in 1919, scarcely a week passed without the announcement of new isotopes. There are more than 250 isotopes of the 92 elements. The most complex element is tin with no fewer than ten isotopes.

One of the most important isotopes is heavy hydrogen of mass 2 discovered by Urey in America in 1932 and now called deuterium. Its compound, heavy water or deuterium oxide, has been isolated by repeated electrolysis of water; the light hydrogen comes off first leaving behind a progressively richer mixture of deuterium oxide which, when pure, has specific gravity of 1·1 and freezing- and boiling-points of 3·8° and 101·4° C. respectively.

Aston's mass spectrograph

Aston's apparatus, called a mass spectrograph, is shown in Fig. 303. A is the anode and C the cathode. The positive rays emerging from the fine hole in the cathode pass through a narrow slit S; they are deflected downwards by an electric field between the plates XY and upwards by the magnetic field between two poles M of a powerful electromagnet. Aston's chief improvement on Thomson's method was the focusing of the rays; all particles of the same mass are caused to converge to a focus. The slower particles are deflected more than the faster ones by the electric field since they are under its influence for a longer time. The magnetic field gives a similar effect but in the opposite direction, so that, by adjustment of the field strength, the dispersion can be annulled. Particles having different masses are focused at different points and thus images P_1, P_2, etc. are produced on a suitably placed photographic plate for each of the isotopes which are present.

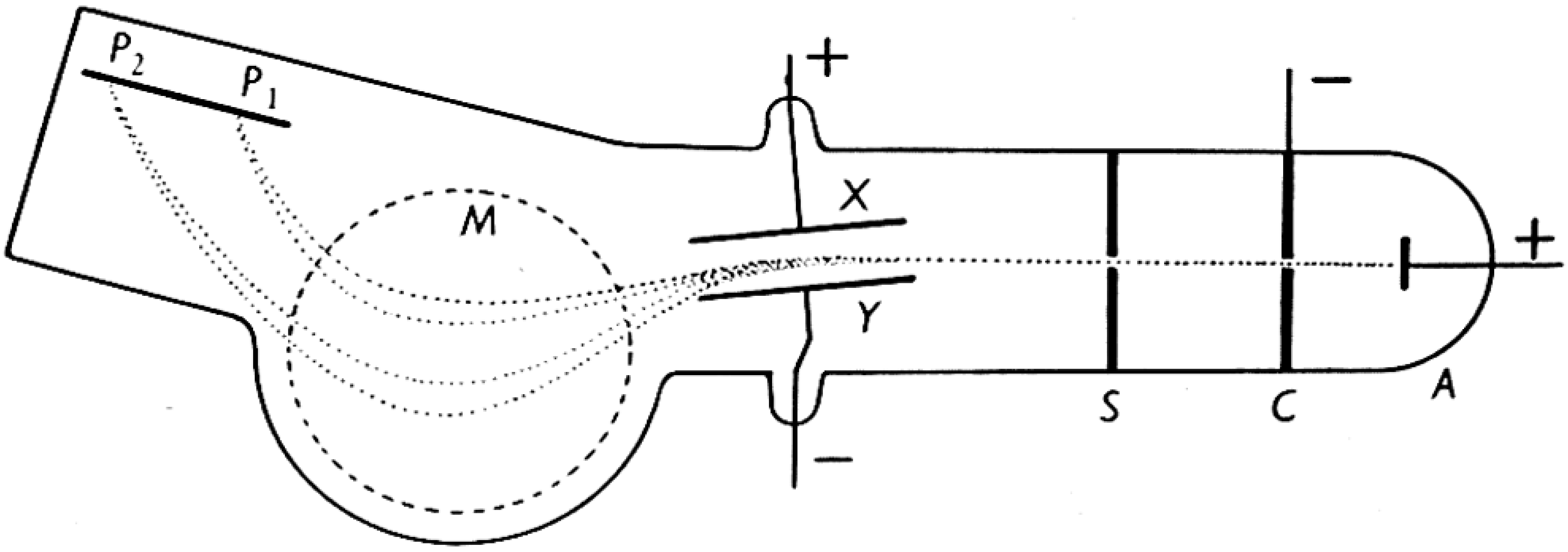


Fig. 303

The type of photograph obtained is shown in Fig. 304. From the dimensions of the apparatus and the strengths of the fields, the masses of the particles can be calculated.

To obtain positive ions of elements which are not normally gaseous Aston used an anode having a cavity packed with a paste containing one of the compounds of the element.

Aston's original apparatus had an accuracy of 1 in 1000 which was later improved to 1 in 10,000. The masses of all isotopes are not exact whole numbers and the slight discrepancies have proved to be of great theoretical importance. Einstein obtained the following simple relation between a mass m and its equivalent energy E: $E=mc^2$, where c is the velocity of light. Thus if 1 g. of matter could be annihilated the energy released would be $1 \times (3 \times 10^{10})^2 = 9 \times 10^{20}$ ergs, which is the energy obtained from burning about 4000 tons of coal.

The helium nucleus, of mass 4·00280, is now believed to be made up of two protons, each of charge 4e and mass 1·0081, and two neutrons, each of zero charge and mass 1·0089. Thus the separate components of the helium nucleus have a mass $2 \times 1{\cdot}0081 + 2 \times 1{\cdot}0089 = 4{\cdot}0340$, which is considerably more than the mass of the helium nucleus. Astronomers now account for the radiation of the sun and stars by the annihilation of mass during the building up, from protons and neutrons, of helium and more complex elements. It is calculated that the sun is losing 4,000,000 tons of mass per second, but would take 15×10^{12} years to radiate its whole mass. The atomic bomb and the atomic pile also derive their energy from the annihilation of mass.

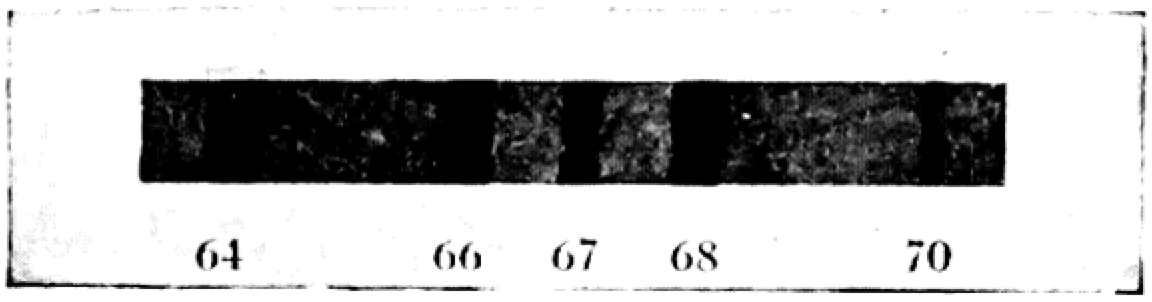


Fig. 304. Mass spectrogram of zinc's five isotopes, 64, 66 and 68 in greatest quantity, 67 in less and 70 in very small quantity.

X-rays

In 1895 Röntgen, Professor of Physics at Würtzburg, while experimenting with a discharge tube, discovered that photographic plates wrapped in black paper and left near the tube, were fogged. His laboratory assistant noted that a silhouette of a key resting on the box of plates appeared on the plates after development. The tube was found to be emitting radiation of a penetrating type which Röntgen called X-rays. Sir William Crookes, who was also bothered by the fogging of his plates when photographing discharges at low pressures, moved them to another room and so, while saving his plates, possibly missed discovering X-rays.

X-rays are produced whenever fast moving electrons are stopped suddenly. The earliest type of X-ray tube known as a gas tube, is shown in Fig. 305. The gas in the tube is at such a pressure that the Crookes dark space fills the tube. The cathode C is concave in order to focus the cathode rays on to a target or anti-cathode A.C. The anti-cathode may also be used as the anode, but it is found that a separate anode A gives a steadier discharge. The pressure of the gas in the tube gradually decreases with use owing to occlusion of gas by the electrodes; this causes the X-rays produced to become less intense but 'harder' or more penetrating. 'Softening' devices for releasing a little more gas into the tube when necessary are usually provided.

A greatly improved type of tube was invented by Coolidge in 1913 (Fig. 306). The tube is exhausted as highly as possible so that no discharge would pass through it if it were used as a gas tube. The electrons are emitted by a tungsten filament C and the rate of their emission, which determines the intensity of the X-rays, can be controlled by the heating current through the filament.

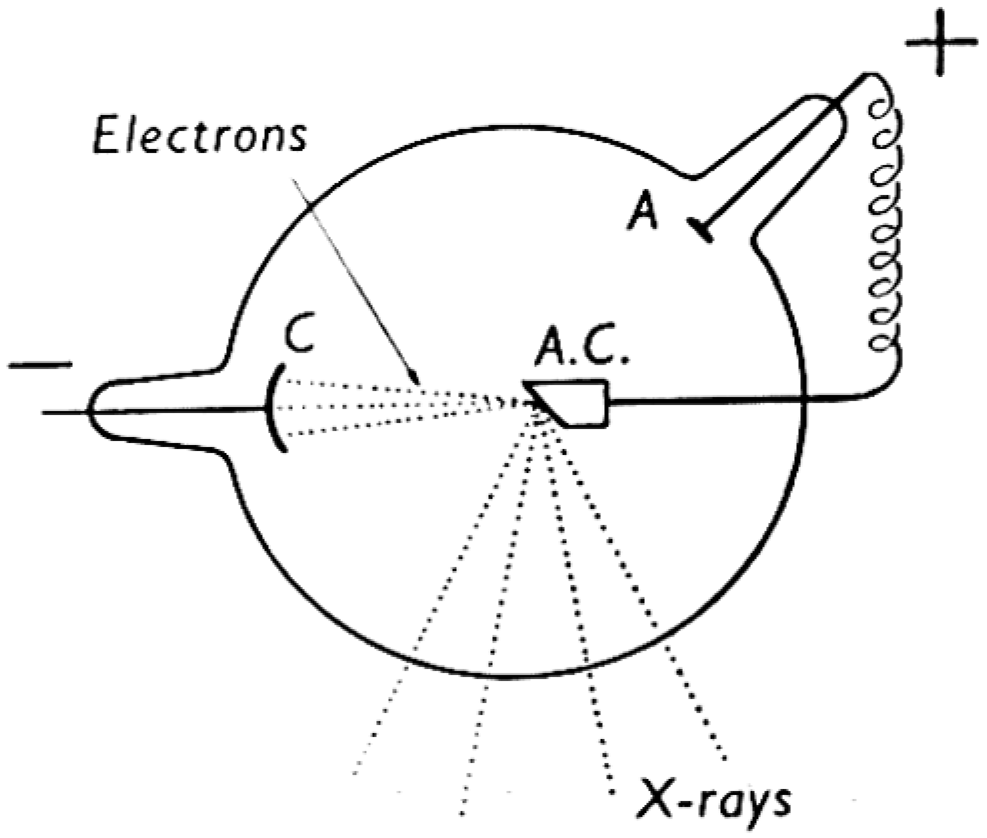


Fig. 305

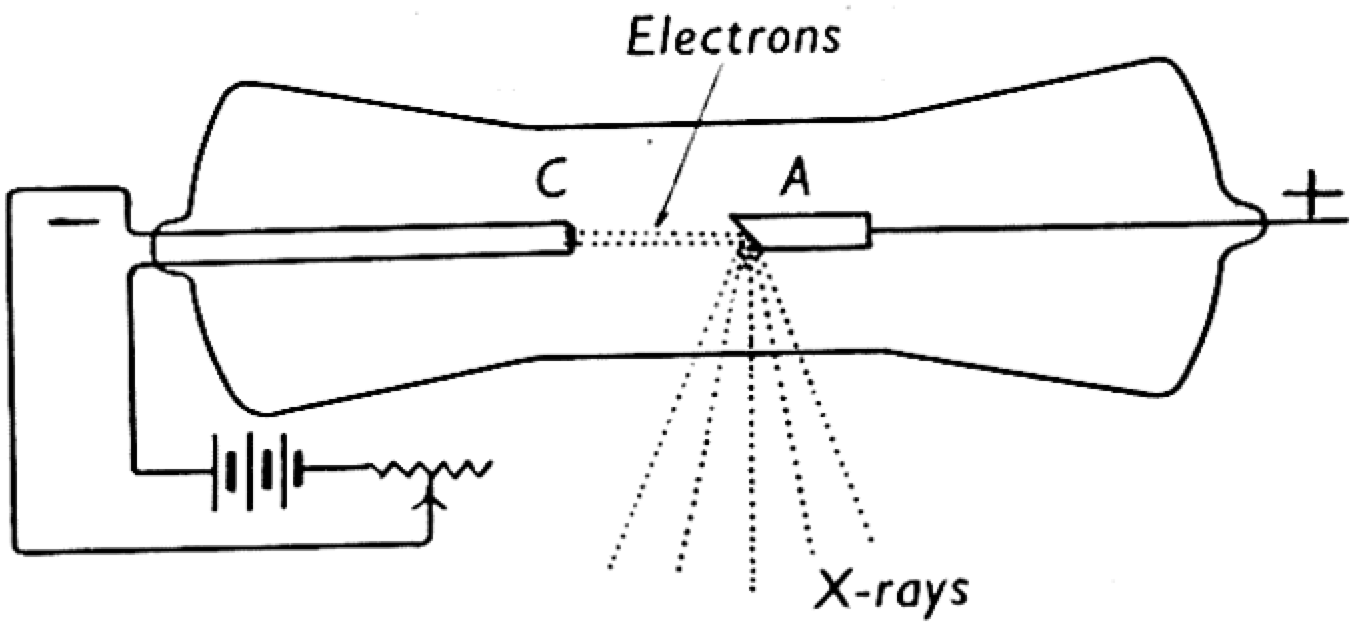


Fig. 306

The hardness of the X-rays is controlled separately by the P.D. between A and C. X-rays are electromagnetic waves like light but of much shorter wave-length. Their hardness depends on their frequency and this is greater the faster the speed of the electrons hitting the target. The phenomenon is a kind of reverse photoelectric effect and a similar relation holds:

$$Ve = h\nu,$$

where V = P.D. through which the electrons fall, e = charge of the electron, h = Planck's constant, ν = maximum frequency of the X-rays emitted.

Only about 1% of the energy of the electrons is converted into X-rays. The rest of the energy appears as heat, and hence the target is made of a metal of high melting-point, such as tungsten or molybdenum, embedded in a solid block of copper which is a good thermal conductor.

Until 1912 the true nature of X-rays was unknown. Some believed that they were waves like light and others, among whom was Sir William Bragg, who was later to measure their wave-length, believed that they were particles. Their known properties were as follows:

(1) They travel in straight lines and are undeviated by magnetic and electric fields.

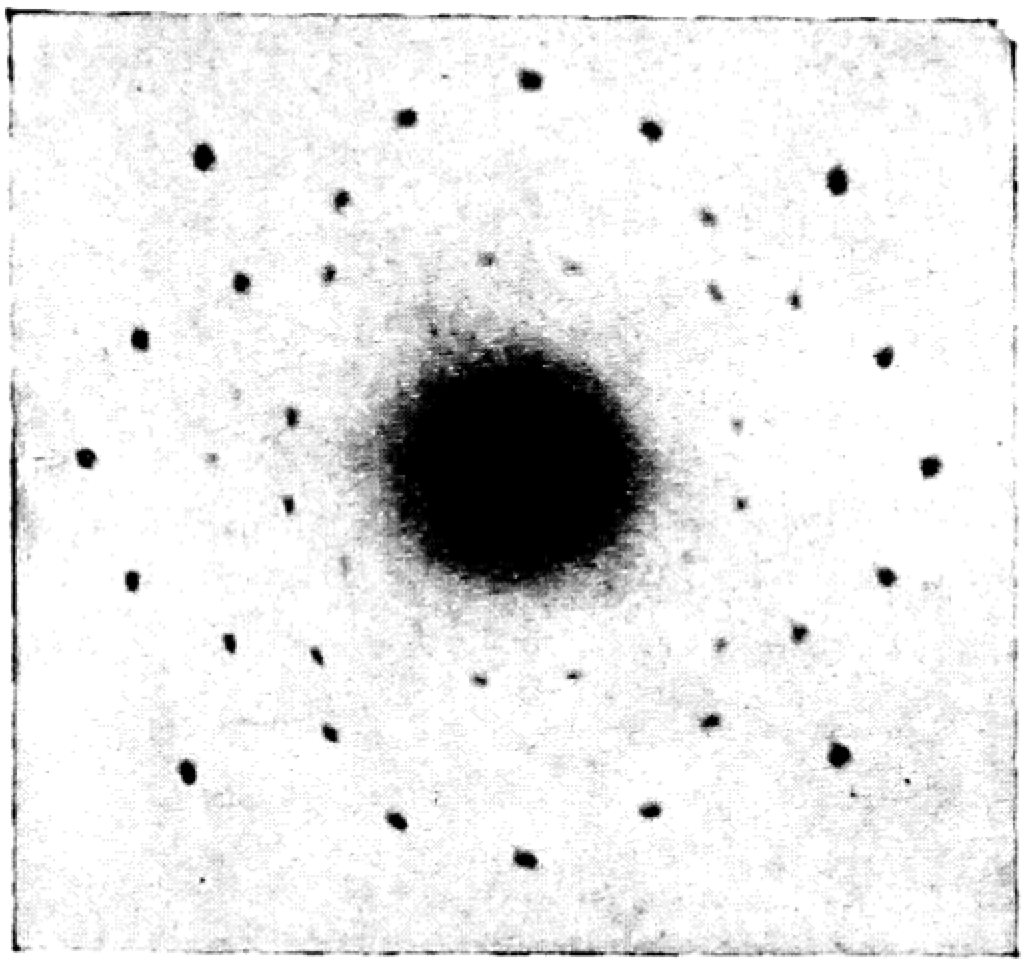


Fig. 307. Laue photograph of rock salt.

(2) They affect a photographic plate and cause luminescence on a screen of zinc sulphide or barium platinocyanide.

(3) They are very penetrating. Their absorption depends on the density and thickness of the matter through which they pass. Thus bones absorb them more readily than flesh; a thickness of 1 mm. of lead is opaque to X-rays of moderate hardness.

(4) They liberate electrons from air (ionising it) and from certain metals. Thus they cause a charged electroscope to become discharged.

Early attempts to diffract X-rays with the most closely ruled gratings were unsuccessful, and this was the fact which weighed most heavily with the supporters of the particle theory. In 1912, however, von Laue, of the University of Munich, conceived the idea that crystals, with their regularly spaced atoms, might behave as gratings and produce diffraction patterns with X-rays. The experiment was tried and was immediately successful (Fig. 307).

Determination of the wave-length of X-rays

In 1913 Sir William Bragg and his son, now Sir Lawrence Bragg, devised a comparatively simple method of measuring the wave-lengths of X-rays by reflecting them from the surface of a crystal. In the method of von Laue the rays are transmitted through the crystal.

Huyghens explained the reflection of light waves by assuming that each point on the reflector struck by the incident waves acts as a source of secondary waves and he showed that these secondary waves reinforce each other when the angle of incidence is equal to the angle of reflection. The atoms (or more strictly the ions) of a crystal act as sources of secondary waves when struck by X-rays. Fig. 308 shows reflection of X-rays at two successive layers of ions in a crystal. The lines with arrows represent the direction of travel of the waves and hence are rays. The complete reflected wave will be strong only if the reflections from successive layers, probably from many millions of layers, reinforce each other. This will occur if the difference in path, $AB+BC$, is an integral number of wave-lengths, i.e. if

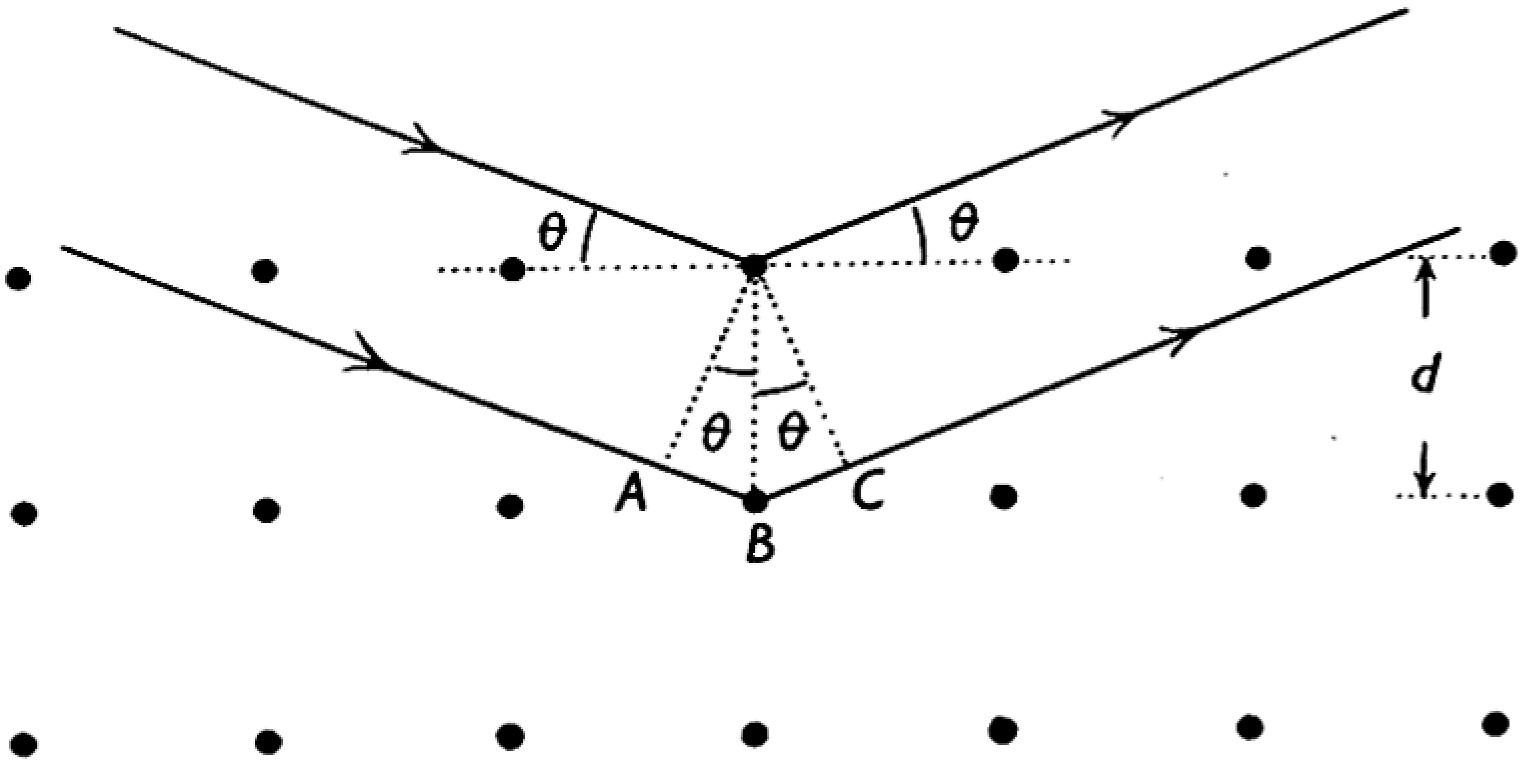


Fig. 308

$$2d\sin\theta = n\lambda,$$

where d = distance apart of the adjacent layers of ions, n = an integer, λ = wave-length of X-rays.

The Bragg X-ray spectrometer is shown in Fig. 309. The intensity of the 'reflected' waves is measured by means of an ionisation chamber, consisting of a lead cylinder containing air in which is fitted an insulated electrode. When a beam of X-rays enters the chamber it ionises the air, enabling a current to flow between the walls of the chamber and the electrode, and causes a deflection of the gold-leaf electroscope. The crystal and the ionisation chamber are rotated together at suitable rates and measurements made of the glancing angle θ and the corresponding ionisation current.

It can be seen from the equation $2d \sin \theta = n\lambda$ that for X-rays of a particular wave-length λ there will be a series of angles θ corresponding to $n = 1, 2, 3$, etc., at which a strong beam is reflected. Thus spectra of several orders must be distinguished.

The value of d for rock salt (NaCl) can be calculated quite simply since the crystal form of the latter is known to be that of a cubic lattice. Each cube in the lattice in Fig. 310 has an ion at each corner, eight in all, and in a crystal containing a large number of cubes each ion, except those on the surface, will be shared by eight cubes (like the centre ion in Fig. 310). Thus there will be, on an average, one ion per cube. Now a gram-molecular weight of rock salt, i.e. $23 + 35{\cdot}5 = 58{\cdot}5$ g., contains $6{\cdot}06 \times 10^{23}$ atoms, or $2 \times 6{\cdot}06 \times 10^{23}$ ions. The density of rock salt is $2{\cdot}18$ g. per c.c. and hence 1 c.c. contains $\frac{2 \times 6{\cdot}06 \times 10^{23} \times 2{\cdot}18}{58{\cdot}5}$ ions. But for each ion there is 1 cube of the lattice of volume d^3, where d is the distance apart of the ions.

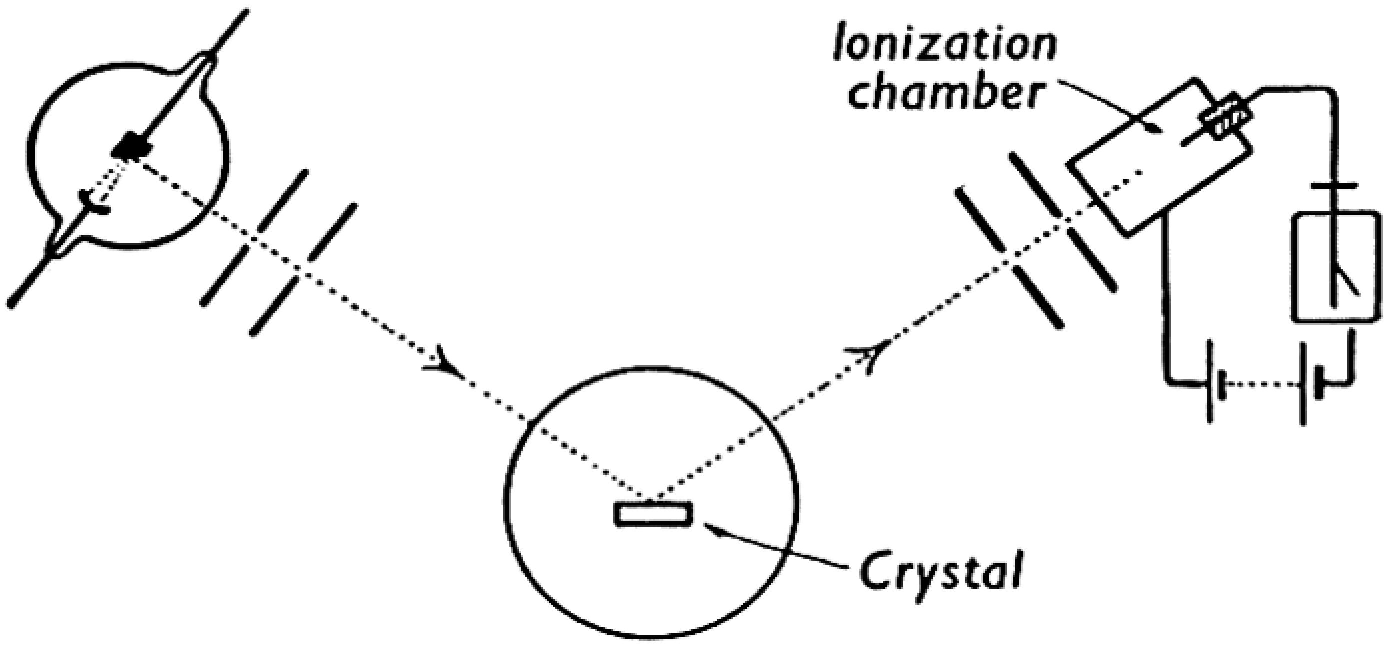


Fig. 309

$$\therefore \quad \frac{2 \times 6{\cdot}06 \times 10^{23} \times 2{\cdot}18}{58{\cdot}5} \times d^3 = 1,$$

$$d = 2{\cdot}8 \times 10^{-8} \text{ cm.}$$

Typical values obtained by the Braggs for strong radiation from palladium, using a crystal of rock salt, were

n	θ
1	5° 59′
2	12° 3′
3	18° 14′

Substituting for the first value,

$$\begin{aligned}\lambda &= 2d \sin\theta \\ &= 2 \times 2{\cdot}8 \times 10^{-8} \sin 5^\circ\ 59' \\ &= 0{\cdot}586 \times 10^{-8}\text{ cm.}\end{aligned}$$

Once the wave-length of X-rays is known the method can be used to determine the spacing of the ions in crystals.

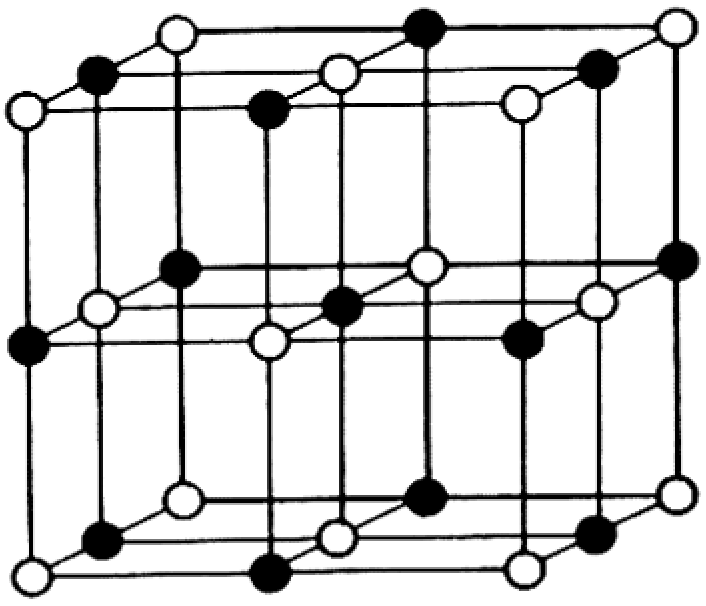


Fig. 310

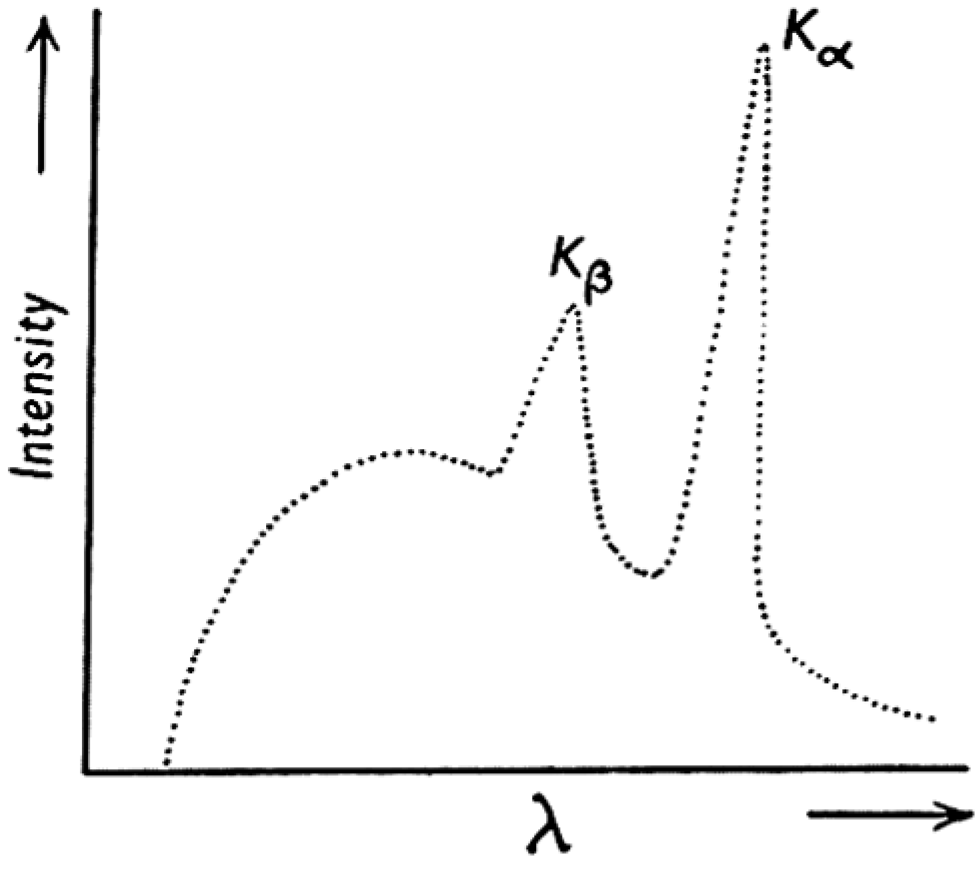


Fig. 311. X-ray spectrum of molybdenum.

Moseley's experiments

An X-ray tube emits a background of radiation, like the continuous spectrum of white light, on which is superimposed certain stronger radiations, like luminous line spectra (Fig. 311). The X-ray line spectra vary in wave-length with the element of which the anti-cathode is composed, and have been classified into a hard K radiation, and a softer L radiation. There is, in addition, a still softer M radiation in the case of the heavy elements.

In 1913 Moseley undertook the investigation of the X-ray spectra of the elements. He mounted a series of elements on a trolley in an X-ray tube and moved them so that they were struck in turn by a stream of electrons. The resulting X-rays were examined by a crystal. Moseley discovered that there is a regular shift of the spectral lines from element to element (Fig. 312). Each element can be given a number, beginning with hydrogen 1, helium 2, lithium 3 and so on to uranium 92 which, if plotted against the square root of the frequency of, say, the K_α line, gives a straight line graph. This number, now called the *atomic number*, was found to be equal to the positive charge on the nucleus of the atom and also to the number of planetary electrons.

Moseley's discovery enabled the elements to be classified in a new and more satisfactory manner. If the elements are arranged in the order of their atomic weights certain anomalies occur, since elements with similar chemical properties do not fit into the periodic classification. Thus potassium precedes argon instead of following it and the same is true of nickel and cobalt. On arranging the elements in the order of their atomic numbers these anomalies are removed. Moseley found gaps in the table corresponding to numbers 21, 43, 61, 72 and 75. A search was made for the missing elements and the gaps were filled by scandium, masurium, illinium, hafnium and rhenium respectively.

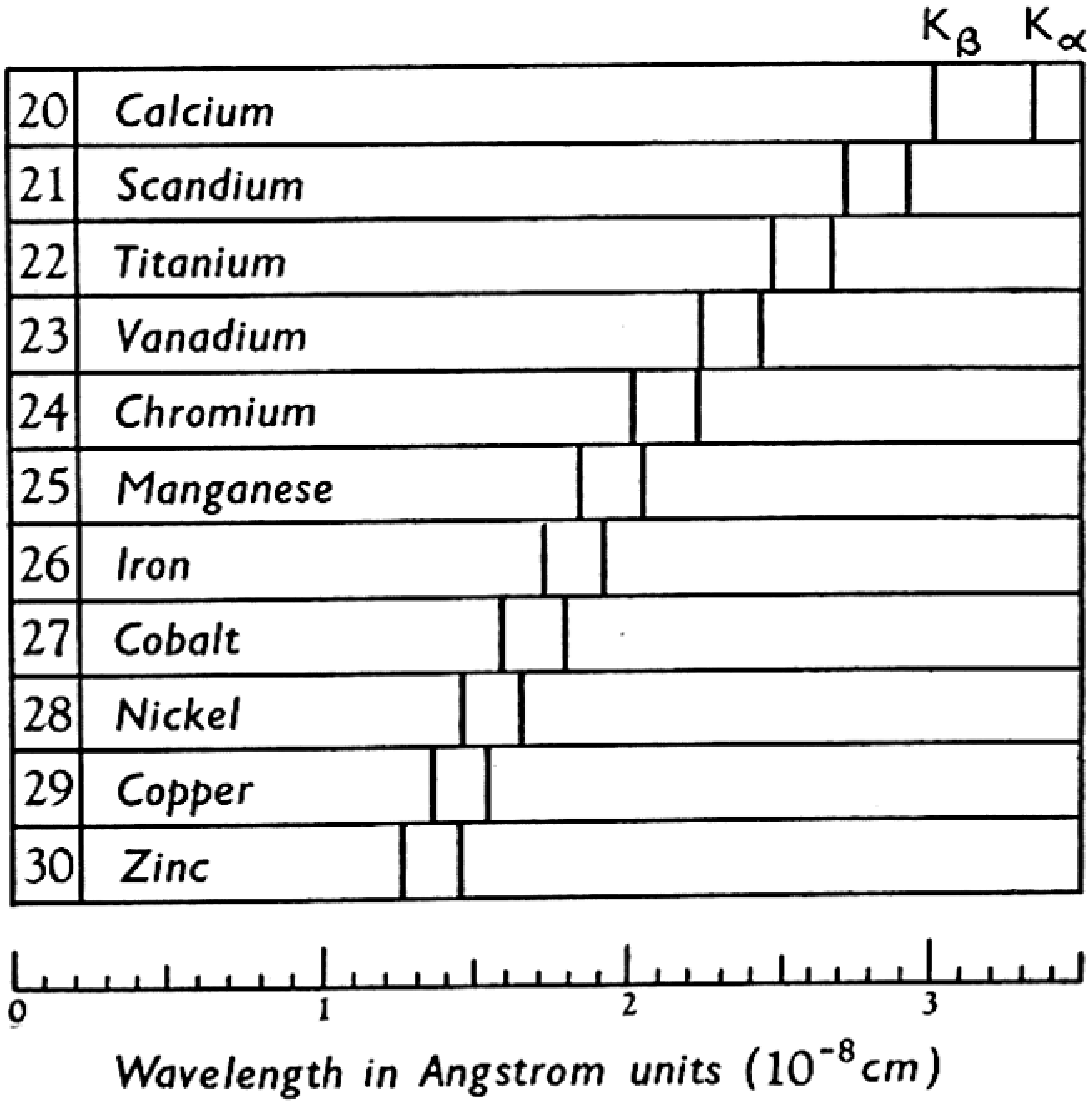


Fig. 312. X-ray spectra of the elements showing K_α and K_β lines.

Moseley's early death at the Gallipoli landing in 1915 was a tragic loss to science.

Radioactivity

X-rays cause certain substances, such as zinc sulphide or barium platinocyanide, to fluoresce. Henri Becquerel examined fluorescent substances to see whether they gave off radiation and in 1896 he found that uranium and its compounds, like X-rays, affect a photographic plate through a wrapping of paper. That the effect was not due to vapours was proved by inserting a sheet of glass between the uranium salt and the plate.

The phenomenon was investigated by Pierre and Marie Curie. The Austrian Government provided them with a ton of pitchblende, the ore of uranium, from Bohemia. In a leaky, abandoned shed near the School of Physics in Paris they began the laborious task of separating the radioactive components from the ore. First they isolated a new, active element which was associated with the bismuth in the ore. This was named polonium in honour of Madame Curie's native country. They realised, however, that there was a far more active element associated with the barium constituent and eventually they succeeded in extracting $\frac{1}{5}$ g. of it from 1 ton of ore. The new element, radium, had an activity about two million times that of uranium. Its supply of energy was prodigious and apparently inexhaustible; it provided enough heat every hour to raise its own weight of water from freezing- to boiling-point.

α, β and γ rays

The most acute and successful investigator of radioactivity was Rutherford, who left Cambridge in 1898 to take up a professorship at McGill University, Montreal. At this time practically all the radium in the world was in the possession of the Curies. Dr Giesel, however, put radium bromide on the market at £1 per mg. and Rutherford bought 30 mg.; later the price rose to £12 per mg.

The radiation from radioactive substances is complex as can be shown by deflecting it in a magnetic field (Fig. 313). Part, called by Rutherford α-rays, is deflected in such a direction as to indicate that it carries a positive charge; another part, the β-rays which proved to be electrons, is deflected in the opposite direction; and a third part, the γ-rays which are X-rays of very short wave-length, is undeviated. The α-rays in Fig. 313 are less bent than the β-rays because they are heavier, and they are less dispersed because they are homogeneous as regards velocity, whereas β-rays are not.

In 1903 Rutherford measured the deflection of α-particles in a magnetic field and calculated the ratio of charge to mass. He deduced that the α-particles are charged atoms of helium, of mass 4 (taking the hydrogen atom as of mass 1) and of charge $+2e$ (where $-e$ is the charge of the electron). That the α-particles might be helium was suggested by the fact that radioactive minerals contain small quantities of helium. Rutherford obtained confirmation by collecting α-particles in an evacuated tube and showing that they gave the spectrum of helium when rendered luminous by an electrical discharge.

The velocity with which the α- and β-particles are emitted is enormous, that of the α-particles varying between $\frac{1}{100}$ and $\frac{1}{10}$ the velocity of light, while that of the β-rays is sometimes as high as 99 % of the velocity of light. The γ-rays are given off whenever β-rays are ejected, which is understandable since X-rays are produced by a sudden change in the velocity of electrons.

The penetrating powers of the different rays in different media were closely studied; α-rays can penetrate only a small fraction of a millimetre of aluminium foil, β-rays penetrate several millimetres of aluminium, and γ-rays will penetrate several centimetres of lead. The range of α-particles in air at atmospheric pressure varies between 3 and 8 cm. and can be used to identify different radioactive substances.

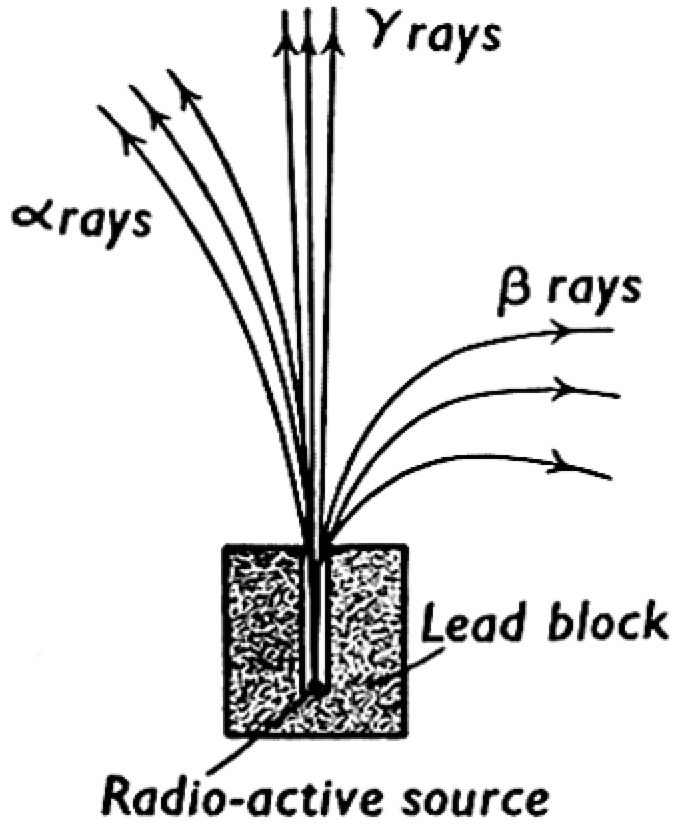


Fig. 313. Deflection of α- and β-rays by a magnetic field whose direction is into the plane of the paper.

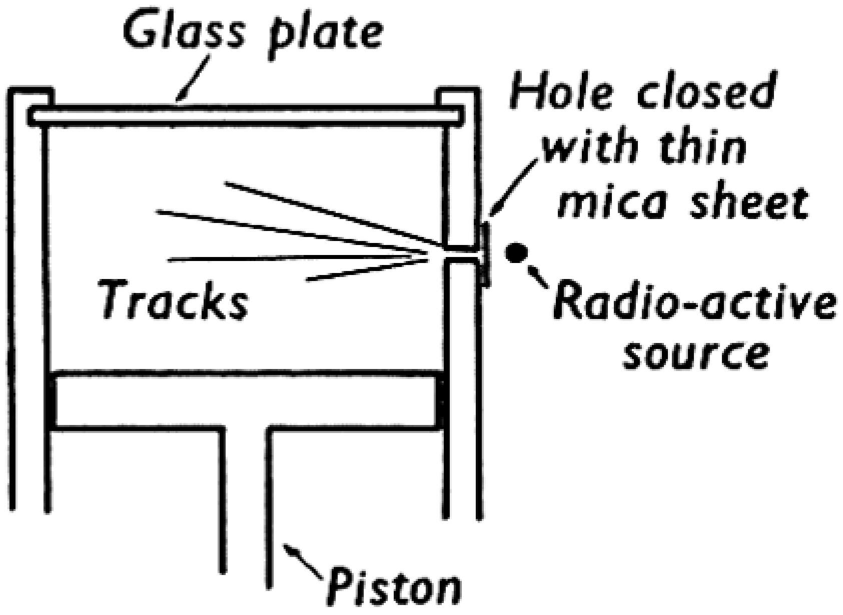


Fig. 314

The Wilson cloud chamber

In 1912 C. T. R. Wilson devised an apparatus known as the cloud chamber for making visible the tracks of charged particles (Fig. 314). The space inside a vessel containing dustless air and saturated water vapour is made to expand suddenly by the rapid movement of a piston. Cooling and supersaturation result, and if the expansion is not too great (about $\frac{1}{3}$ of the original volume), condensation will take place on charged ions without a general fog being formed.

An α-particle, as it ploughs its way through the air in a cloud chamber, knocks out electrons from thousands of molecules and leaves a trail of

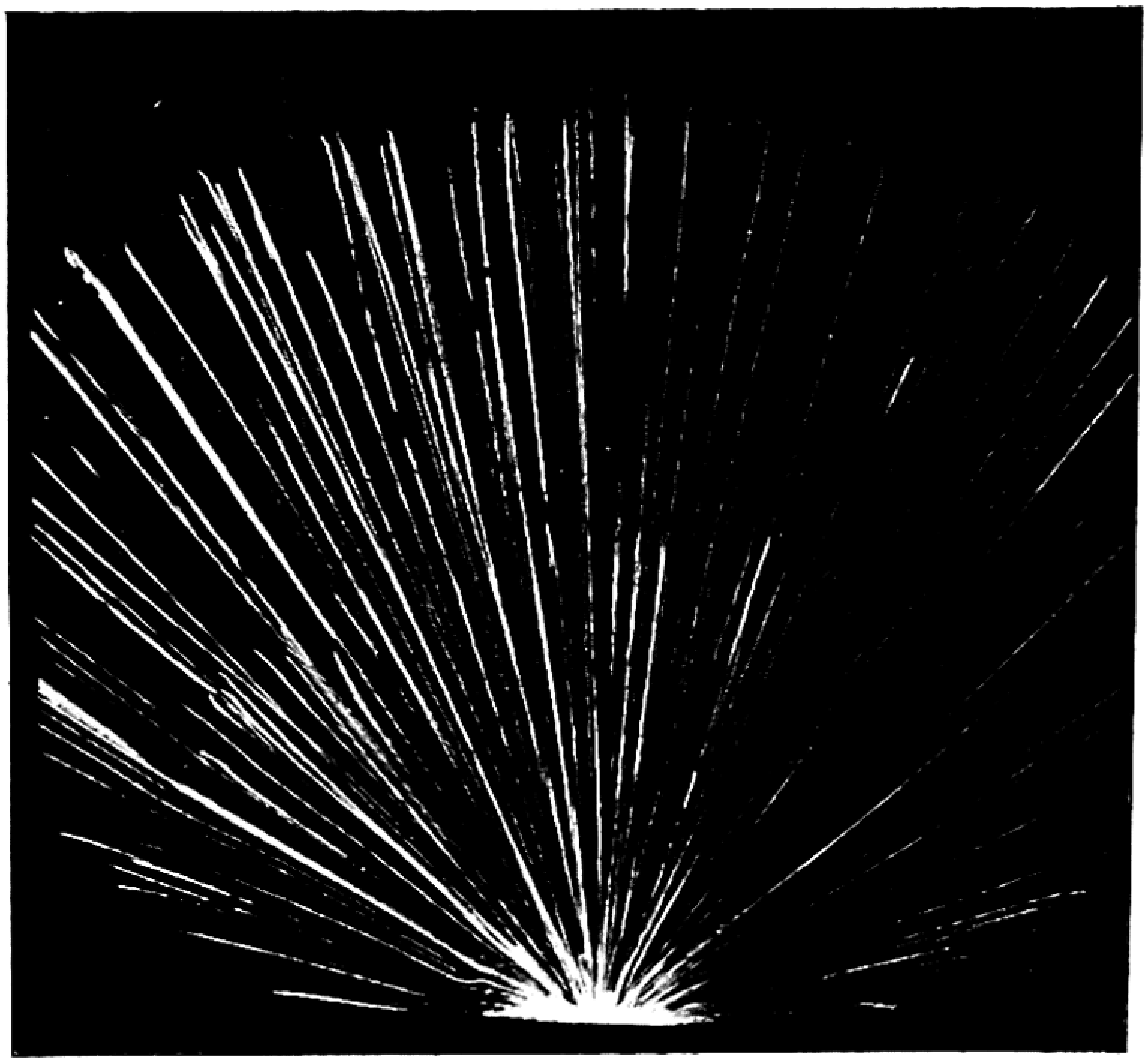


Fig. 315. Tracks of α-particles from thorium (C and C') showing the two groups of ranges 8·6 and 4·8 cm. in air.

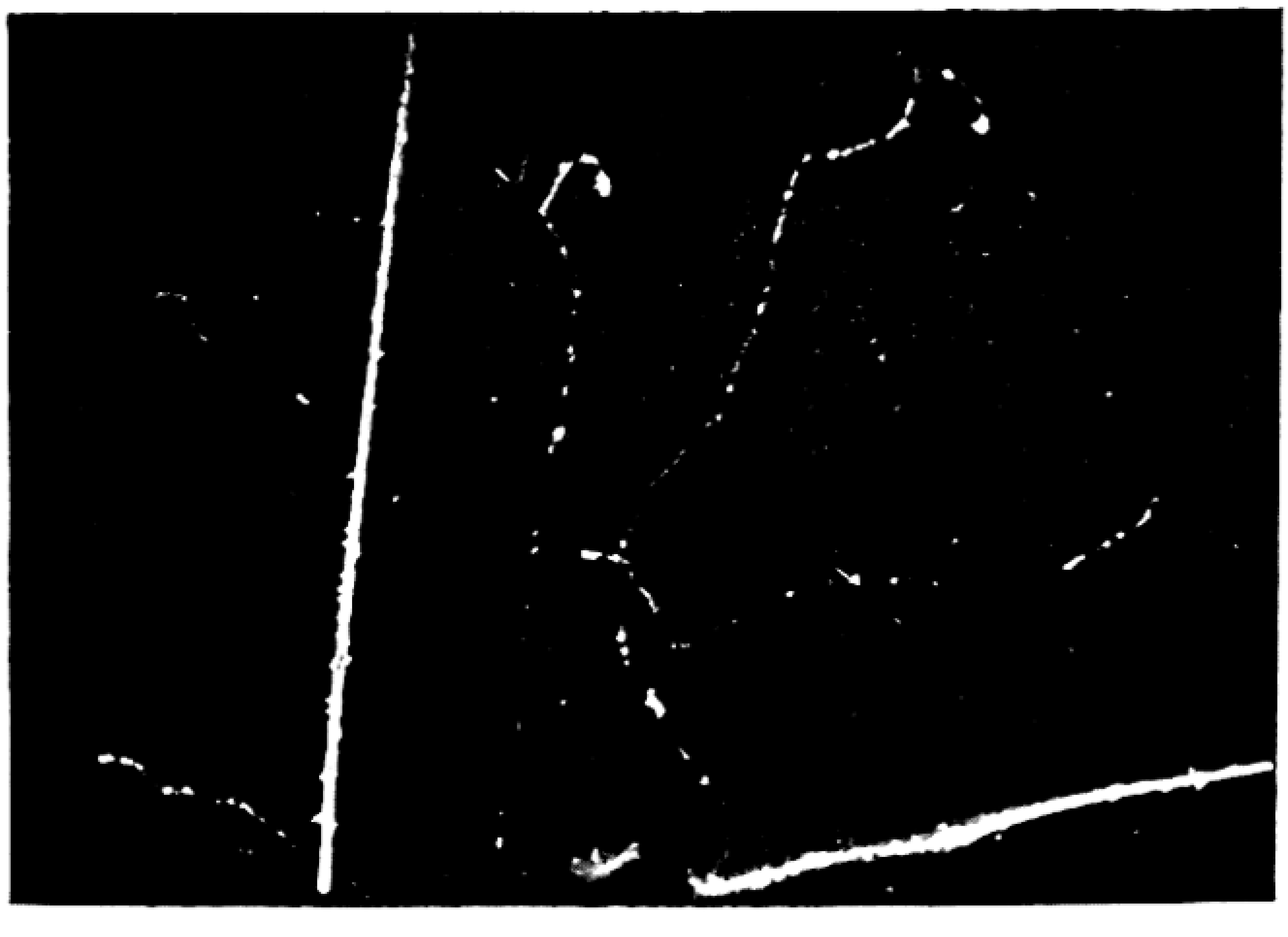


Fig. 316. Tracks of α- and β-particles in hydrogen. The thin curved tracks are those of β-particles.

condensation on the ions produced (Fig. 315). A β-particle, being far lighter, is deflected by its collisions with the air molecules and produces far fewer ions than an α-particle (Fig. 316). X-rays cause electrons to be ejected from air molecules and produce tracks as in Fig. 317.

The cloud chamber was described by Rutherford as 'the most original and wonderful instrument in scientific history'; it is still of great value in atomic research.

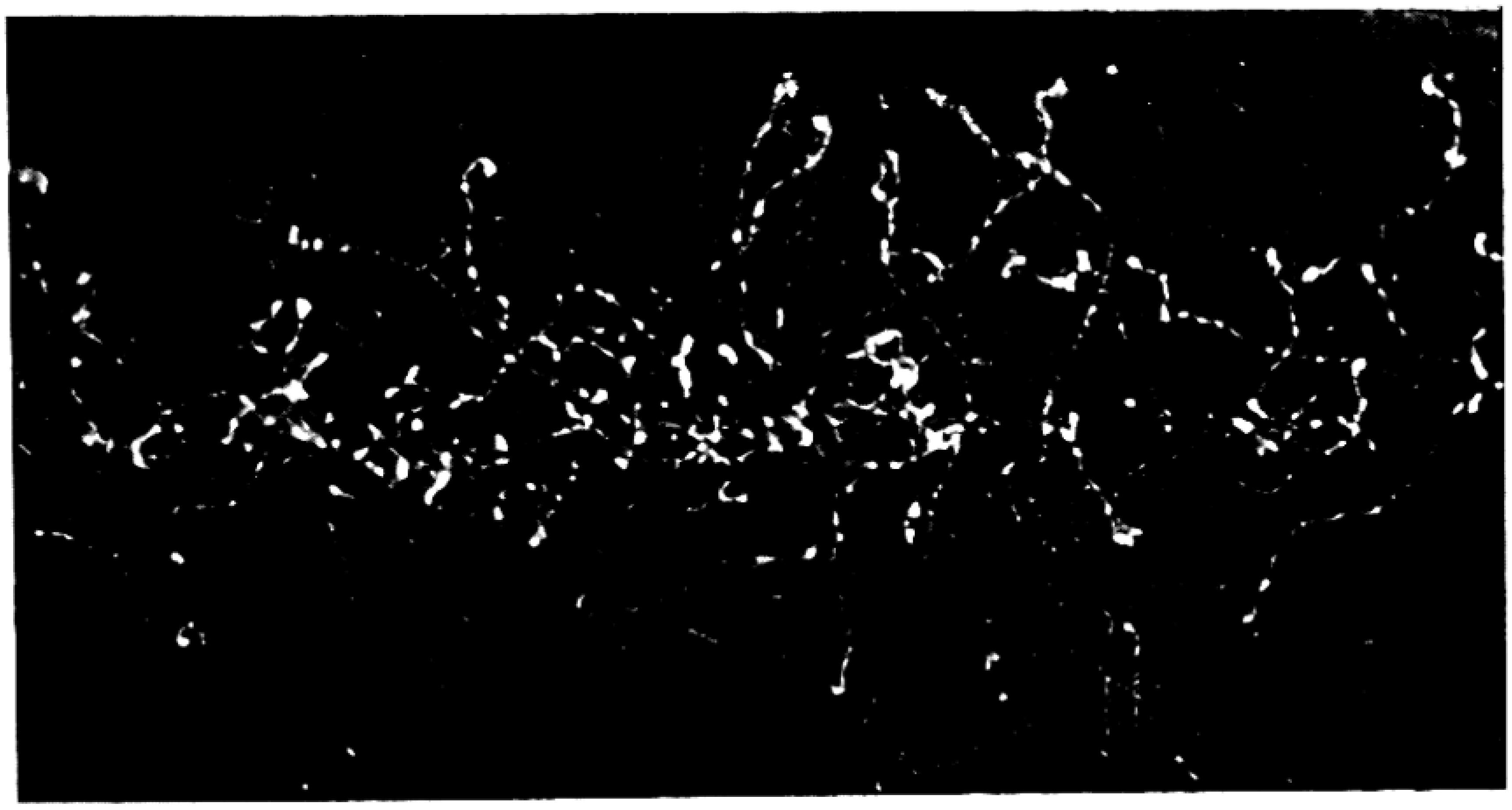


Fig. 317. Electron tracks produced by a beam of X-rays in a gas.

The disintegration theory

In 1903 Rutherford and Soddy presented to the British Association their disintegration theory, that the atoms of a radioactive element explode with great violence, hurling out α- and β-particles and being changed in the process to a different element. The theory was so novel and startling that it aroused much opposition, including that of Lord Kelvin. Some other source of the immense energy of radioactive atoms was suggested, such as the absorption of hitherto undetected universal radiation.

The subsequent experimental unravelling of the intricate radioactive changes has, however, fully borne out the theory. There is some inherent instability in atoms of atomic number greater than 82, causing disintegration. The process cannot be hastened or delayed by physical or chemical means. Its speed may be represented by the 'half-life period' which is the time required for half the element to break up and be transformed. The half-life period of radium is 1590 years; thus 1 g. of radium is reduced to $\frac{1}{2}$ g. in 1590 years, to $\frac{1}{4}$ g. in a further 1590 years, and so on. The law is statistical. It is impossible to tell when an individual atom will disintegrate, but in the case of a very large number of atoms it is known that a certain fraction of them will break up every second.

The table below shows the uranium-radium family, each element being produced by the disintegration of the element immediately above it in the table. The emission of an α-particle causes a loss of mass of 4 and a loss of charge of $+2e$. Since the atomic number of an element is the charge on the nucleus it is clear that the loss of an α-particle causes the element to be displaced two places down the atomic number table. When radium disintegrates with the ejection of an α-particle it gives rise to the radioactive gas radon, a member of the family of inert gases. The disintegration may be represented as follows:

$$_{88}\mathrm{Ra}^{226} = {}_{86}\mathrm{Rn}^{222} + {}_{2}\mathrm{He}^{4}.$$

The subscript represents the charge or atomic number and the superscript the atomic mass.

The emission of a β-particle during disintegration has a negligible effect on the mass of an atom, since the mass of an electron is only $\frac{1}{1840}$ of the mass of the hydrogen nucleus or proton, but it results in a loss of charge of $-1e$ which is equivalent to a gain of charge of $+1e$. Thus the element moves up one place in the atomic number table. For example,

$$_{82}\mathrm{Ra\ B}^{214} = {}_{83}\mathrm{Ra\ C}^{214} + {}_{-1}e^{0}.$$

Uranium-radium series of elements

Element	Abbreviation	Atomic number	Atomic mass	Particle ejected	Half-life period
Uranium I	U I	92	238	α	$4{\cdot}6 \times 10^{9}$ years
Uranium X_1	U X_1	90	234	β	245 days
Uranium X_2	U X_2	91	234	β	1·14 min.
Uranium II	U II	92	234	α	3×10^{5} years
Ionium	Io	90	230	α	$8{\cdot}5 \times 10^{4}$ years
Radium	Ra	88	226	α	1590 years
Radon	Rn	86	222	α	3·82 days
Radium A	Ra A	84	218	α	3·05 min.
Radium B	Ra B	82	214	β	26·8 min.
Radium C	Ra C	83	214	β, α*	19·7 min.
Radium C′	Ra C′	84	214	α	10^{-6} sec.
Radium C″	Ra C″	81	210	β	1·32 min.
Radium D	Ra D	82	210	β	22·3 years
Radium E	Ra E	83	210	β	5·0 days
Polonium	Po	84	210	α	139 days
Lead	Pb	82	206	—	Stable

* Ra C may pass to Ra D either by way of Ra C′ or of Ra C″. When Ra C ejects a β-particle Ra C is formed; when Ra C ejects an α-particle Ra C″ is formed.

There are two other families of radioactive elements, those of thorium and of actinium, making in all about 40 natural radioactive elements. The stable end-product of each family is an isotope of lead; Pb^{206} for the uranium family, Pb^{208} for the thorium family, and Pb^{207} for the actinium family.

The nuclear atom

The earliest hypothesis of the structure of an atom was the 'positive jelly model' of J. J. Thomson, in which electrons were regarded as embedded in a sphere of positive electricity.

In 1911 a 'planetary model', in which the electrons revolve like planets round a small, massive, positively charged nucleus resulted from Rutherford's experiments on the scattering of α-particles by gold or platinum foil. The apparatus is represented in Fig. 318; α-particles from a radio-active source, *R*, hit a thin metal foil, *F*, and are scattered. The number scattered in different directions can be counted by the scintillations which α-particles produce when they hit a fluorescent screen, *S*, at the focus of a low-powered microscope *M*.

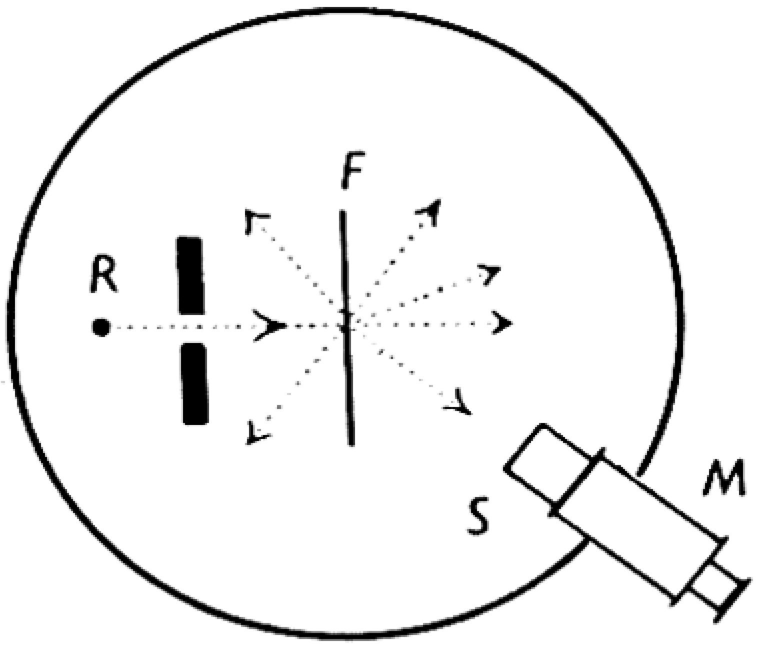


Fig. 318

Rutherford by this time had left Canada and was Professor of Physics in Manchester. He relates that one day Geiger came to see him and asked, 'Don't you think that young Marsden, whom I am training in radioactive methods, should begin a small research?' Rutherford replied, 'Why not let him see whether any α-particles can be scattered through a large angle?', feeling, at the same time, that there was little chance of this because of the immense speed and energy of the α-particles. Shortly afterwards Geiger reported that some α-particles had been observed which were scattered through angles greater than 90°. 'It was almost as incredible as if you had fired a fifteen inch shell at a sheet of tissue paper and it came back and hit you', said Rutherford. No diffuse positive jelly could account for such a result; the atom must contain something extremely hard and massive. By careful measurements of the scattering Rutherford was able to estimate that the diameter of the nucleus of an atom must be of the order of 10^{-13} cm. It was known that the diameter of an atom is about 10^{-8} cm. Thus the nuclear atom must consist largely of empty space.

The nucleus is now believed to consist of protons of mass 1 and charge $+1e$, and neutrons of mass 1 and charge 0. Surrounding the nucleus are planetary electrons of mass $\frac{1}{1840}$ and charge $-1e$. The constitutions of the three lightest elements are as follows:

	Nucleus	Planetary electrons
Hydrogen	Charge 1 mass 1 (1 proton)	1
Helium	Charge 2 mass 4 (2 protons and 2 neutrons)	2
Lithium	Charge 3 mass 7 (3 protons and 4 neutrons)	3

The atoms of the remainder of the elements are built up in a similar way. The atomic number gives the charge on the nucleus and also the number of planetary electrons; the atomic mass minus the atomic number gives the number of neutrons in the nucleus.

The Bohr atom

The difficulty with Rutherford's model atom is that, according to the laws of classical physics, it cannot exist. The electrons rotating round the nucleus must have an acceleration towards the nucleus and consequently should radiate energy continuously, spiralling towards the nucleus to provide the energy. Moreover, although our solar system would be completely disrupted if a large star came sufficiently close to it, the atoms of a gas are colliding with each other millions of times a second and have a remarkable stability.

In 1913 the Dane, Niels Bohr, was working with Rutherford in Manchester just after the nuclear atom theory had been put forward. He applied to the nuclear atom the quantum theory, originally propounded by Planck to account for the radiation from a black body and subsequently used by Einstein to explain the photoelectric effect. The quantum theory postulates that energy is radiated as small packets of energy, $h\nu$, called quanta, where h is a universal constant and ν is the frequency of the radiation.

Bohr assumed that the planetary electrons in an atom can exist only in a limited number of stable orbits or stationary states having definite amounts of energy but not emitting radiation, and that radiation occurs only when an electron jumps from one stable orbit to another. He assumed

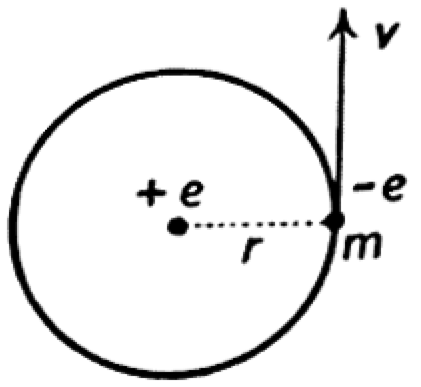


Fig. 319

$$h\nu = E_2 - E_1,$$

where ν is the frequency of the energy radiated when an electron jumps from an orbit of energy E_2 to one of energy E_1.

The hydrogen atom consists of a single electron of charge $-e$ revolving round a nucleus of charge $+e$. Suppose the electron has a mass m, that it revolves in a circle, and that when the radius of its orbit is r its velocity is v (Fig. 319). The electrostatic attraction between the nucleus and the electron must equal the centrifugal force; thus

$$\frac{e^2}{r^2}=\frac{mv^2}{r}. \qquad (1)$$

At this point Bohr made a further assumption which seems purely arbitrary; on the new wave mechanics, however, its meaning becomes more understandable. He assumed that the angular momentum of the electron, mvr, is always an exact multiple of $h/2\pi$; thus

$$mvr=\frac{nh}{2\pi} \qquad (2)$$

when n is an integer called the quantum number. The angular momentum is then said to be quantised.

The next step is to find the energy of the electron in its orbit. The potential energy of the electron is the work done in bringing it from infinity to its orbit and this is $-e^2/r$ (see p. 6); the negative sign indicates that work is done by the electron as it approaches the oppositely charged nucleus. The kinetic energy of the electron is $\frac{1}{2}mv^2$ and from equation (1) this is equal to $\frac{1}{2}e^2/r$.

$$\therefore \text{ Total energy of electron}=\text{K.E.}+\text{P.E.}$$

$$=\frac{1}{2}\frac{e^2}{r}-\frac{e^2}{r}=-\frac{1}{2}\frac{e^2}{r}.$$

From equation (1) $\quad mv^2r=e^2.$

Squaring equation (2) $\quad m^2v^2r^2=\dfrac{n^2h^2}{4\pi^2}.$

Dividing $\quad r=\dfrac{n^2h^2}{4\pi^2e^2m},$

$$\therefore \text{ Energy of electron}=-\frac{1}{2}\frac{e^2}{r}$$

$$=-\frac{2\pi^2e^4m}{n^2h^2}.$$

$$\therefore\ h\nu=E_2-E_1=\frac{2\pi^2e^4m}{h^2}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right),$$

$$\nu=\frac{2\pi^2e^4m}{h^3}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right).$$

In 1884 Balmer, a Swiss schoolmaster, had discovered a formula representing the series of visible spectral lines of hydrogen. The formula can be written in the form

$$\nu = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right),$$

where m is an integer greater than 2. The value of R, called the Rydberg constant, was known from the measured frequencies of the spectral lines. Bohr was able to calculate the value of the constant corresponding to R in his formula, i.e. $\dfrac{2\pi^2 e^4 m}{h^3}$, from the known values of e, m and h. The agreement was perfect within the limits of experimental error.

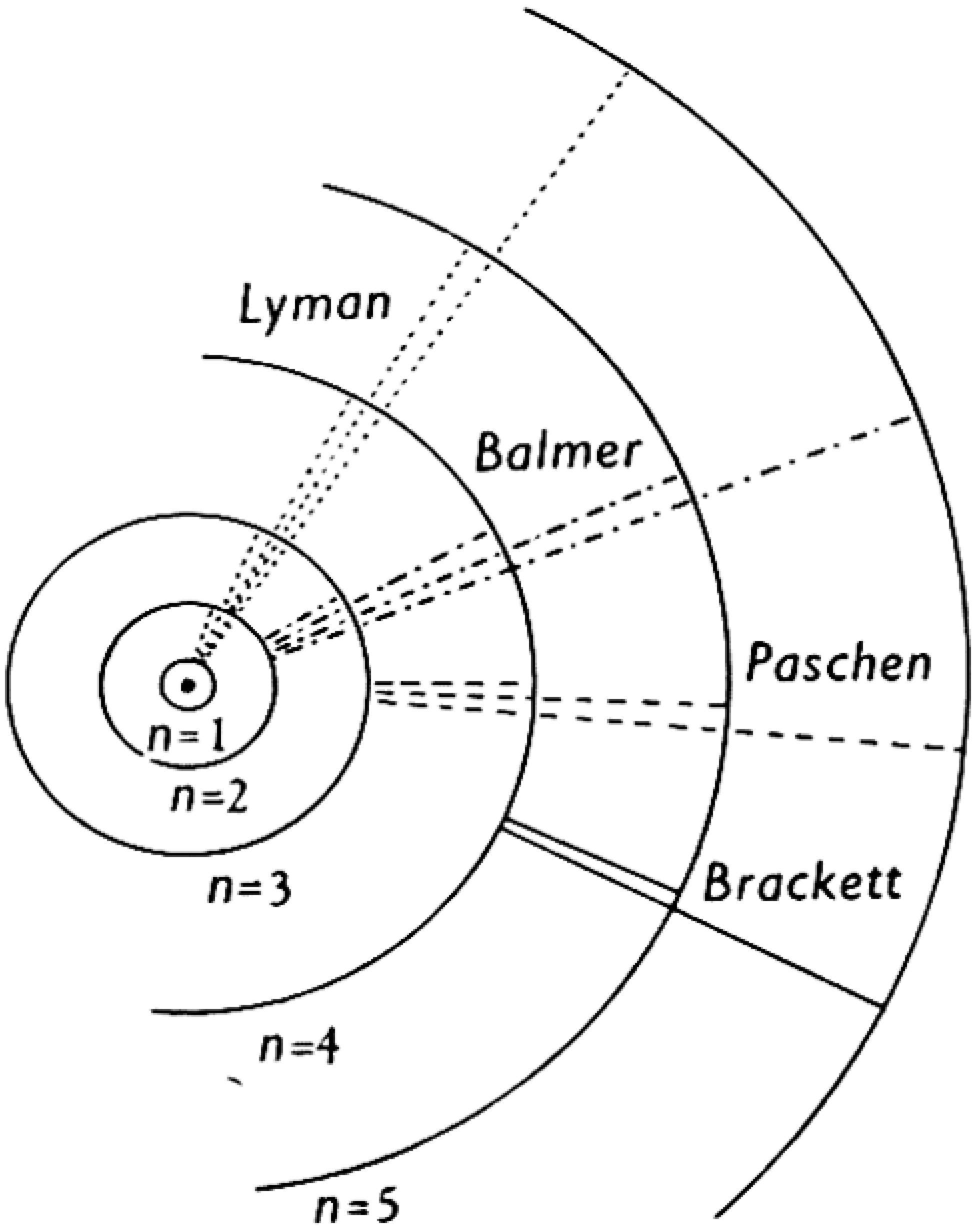


Fig. 320

Bohr explained the emission of the visible spectral lines of hydrogen as due to electrons jumping from outer orbits to the second orbit (Fig. 320). Other spectral series for hydrogen were also known, the Lyman series in the ultra-violet, the Paschen and the Brackett series in the infra-red. These are due to electrons jumping into the first, third and fourth orbits respectively.

The radius of the orbits can be calculated from the above theory. Putting $n=1$, the radius of the first orbit is 5×10^{-9} cm., the other orbits having radii 4, 9, etc. times as large. Again the velocities of the electron can be calculated; in the first orbit the velocity is $\frac{1}{137}$ that of light and the frequency of revolution is 7×10^{15} revs. per second.

The agreement of Bohr's theory with experiment was so convincing that its revolutionary break with classical physics had to be accepted. Such a break required boldness and genius of the highest order. Rutherford himself said that he regarded Bohr's theory as one of the greatest achievements of the human mind.

Excitation potentials

In the normal condition of the hydrogen atom the electron is in its innermost orbit, for which $n=1$. As a result of collision, say in a discharge tube, the electron may be knocked into other orbits for which $n=2$ or 3, etc.; the atom is then said to be excited. The electron will jump back to the innermost orbit, possibly in one jump or in stages, giving out the appropriate radiation. If the electron is completely removed the atom is said to be ionised.

A most satisfying confirmation of Bohr's theory of energy levels may be made by projecting a stream of electrons into rarefied hydrogen. Electrons of small energy collide elastically with the hydrogen atoms and lose no energy, but if the energy of the electrons is increased to $1{\cdot}6\times10^{-11}$ ergs, which is the difference in the energies of the first and second orbits, then the first spectral line of the Lyman series flashes out. The electrons projected into the gas have just sufficient energy to knock electrons from the first to the second orbits in the atoms and the first Lyman line is emitted during the jump back from the second to the first orbit. Similarly with electrons of greater energies, other lines can be produced.

Electron shells in the atoms

Assuming that the electron structure of an atom is responsible for its chemical properties it is possible to work out a scheme for the arrangement of the electrons in the atoms of all elements. The most inactive elements are the rare gases, helium (atomic number 2), neon (10), argon (18), krypton (36), xenon (54), and it is reasonable to assume that these have a very stable electron structure. Atoms having one electron more than these, lithium (3), sodium (11), potassium (19), rubidium (37), caesium (55), are all very active chemically and have similar properties to each other. The same is true of atoms having one electron less, fluorine (9), chlorine (17), bromine (35), iodine (53).

Atoms are therefore believed to be built up of shells of electrons. When these shells are completely filled the element is inert and when the outermost shell has one electron too many or too few the element is particularly active chemically. Successive shells are filled when they contain $2n^2$ electrons where $n=1, 2, 3$, etc.: the K shell contains 2 electrons, the L shell 8, the M shell 18, the N shell 32, and the O shell 50. Thus helium has the K shell filled, and neon the K and L shells filled. To explain the case of argon it is necessary to assume that the M shell is divided into sub-shells and a similar assumption is necessary in the case of the N and O shells.

The alkali metals, for example sodium, give very simple line spectra. Sodium, atomic number 11, has complete K and L shells and an additional single electron in the M shell. Bohr assumed that this outermost electron is responsible for the emission of the spectral lines in a manner similar to the hydrogen atom. It is called the valence electron, and is primarily responsible for the chemical properties of sodium.

The origin of X-ray spectra

X-rays are produced when atoms are excited by collision with very fast-moving electrons. Deep lying electrons in the K and L shells are knocked out of their orbits and X-rays are emitted when they jump back. The K series of X-ray lines are caused by electrons jumping back from other shells into the K shell—usually from the L shell, and the L series by a jump into the L shell from an outer shell—usually from the M shell. The regular shift in the lines from element to element, observed by Moseley, is due to the steady increase in charge on the nucleus as we pass from element to element in the table.

Wave mechanics

After a decade of triumph, evidence began to accumulate that something was wrong with the Rutherford-Bohr atom. The mathematical difficulties in dealing rigidly with two electrons rotating round a nucleus are insuperable and an atom like mercury has 80. As J. J. Thomson remarked, the problem is like that of a bucketful of worms. Bohr was able to account for the spectra of the heavier elements in a general way by assuming that only one electron is responsible for optical spectra, as already indicated in the case of sodium. But he was obliged to invent arbitrary rules called Selection Rules to avoid the prediction of many lines not present in spectra. A weakness of his theory was that it involved quantities not susceptible of experimental verification, such as the momentum of an electron and the radius of its orbit.

In 1925 Heisenberg, working under the inspiration of Bohr, developed his Matrix Mechanics. A matrix is a tabular set of values; the methods of multiplying and manipulating such matrices had been worked out previously by the mathematician Cayley. Heisenberg, confining himself strictly to observable quantities, such as spectral frequencies and intensities, and refusing to attempt the construction of a model, obtained results agreeing more closely with experiment than could be obtained from the Rutherford-Bohr model atom.

A different approach had already been made by de Broglie in 1922. He assumed that electrons, and indeed all material particles, can behave like waves. The energy of a photon of frequency ν is $h\nu$ or hc/λ, where λ is the wave-length and c is the velocity of light. Also the energy equivalent of a particle of mass m, according to Einstein's theory of relativity, is mc^2.

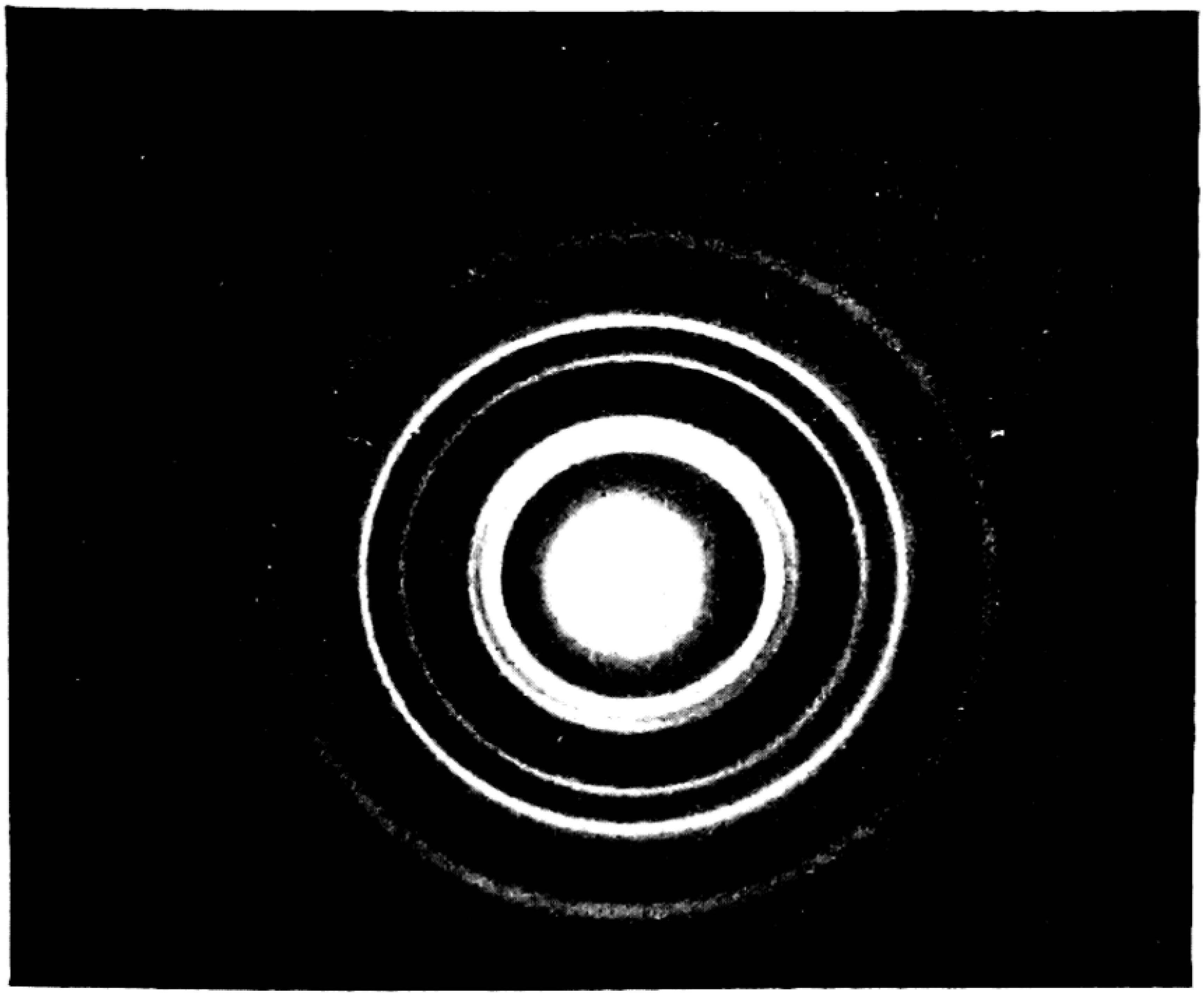


Fig. 321. Diffraction of electrons by a thin metal film.

$$\therefore \frac{hc}{\lambda}=mc^2 \quad \text{taking } m \text{ as the mass of a photon.}$$

$$\lambda=\frac{h}{mc}.$$

By analogy de Broglie suggested that small particles like electrons, of mass m and velocity v, may be considered to behave like waves of wave-length λ, where

$$\lambda=\frac{h}{mv}.$$

Thus electrons accelerated through a P.D. of 100 volts should behave like waves of $\lambda=1{\cdot}22\times10^{-8}$ cm., about the same as for X-rays.

In 1927 Davisson and Germer diffracted a beam of electrons from the surface of a nickel crystal, in a manner similar to the Bragg experiment for X-rays, and obtained a value for the wave-length agreeing with de Broglie's theory. The following year G. P. Thomson, the son of J. J. Thomson, passed electrons through very thin gold or silver foil, an experiment analogous to the von Laue method for X-rays, and obtained diffraction patterns similar to Fig. 321.

In 1926 Schrödinger developed equations going beyond those of de Broglie by including the amplitude of the electron waves. It was suggested by Born that the square of the amplitude of the electron waves is proportional to the probability of finding the electron particle in unit volume at that place. Schrödinger assumed that the motion of an electron, regarded as a system of waves, in a stable Bohr orbit round the nucleus, is comparable with stationary waves. The length of the path of the electron round the nucleus, $2\pi r$, must be an integral number of wave-lengths, $n\lambda$. Since

$$\lambda=\frac{h}{mv},$$

$$n\lambda=n\,\frac{h}{mv}=2\pi r.$$

This is Bohr's postulate that the angular momentum of an electron mvr, must be $nh/2\pi$.

Schrödinger's theory enables us to picture the atom rather as a ball of mist, the probability of the electron's presence at any point being proportional to the density of the mist. This represents, not a single atom, but the statistical behaviour of a large number of atoms.

Dirac proved that the theories of Heisenberg and Schrödinger were equivalent, and further generalised them.

Artificial disintegration

In 1919 Rutherford found that when nitrogen was bombarded with α-particles from radium C, scintillations could be obtained on a fluorescent screen far beyond the range of the α-particles. The particles causing the scintillations were deflected by a magnetic field and shown to be protons, that is, nuclei of hydrogen. Rutherford concluded that, when the nucleus of a nitrogen atom receives a direct hit from an α-particle travelling at high speed, it disintegrates to form an isotope of oxygen. This can be represented as follows:

$$\underset{\text{nitrogen}}{{}_{7}\mathrm{N}^{14}} + \underset{\alpha\text{-particle}}{{}_{2}\mathrm{He}^{4}} = \underset{\text{oxygen}}{{}_{8}\mathrm{O}^{17}} + \underset{\text{proton}}{{}_{1}\mathrm{H}^{1}}$$

Blackett, using a Wilson cloud chamber, was able to photograph the effect (Fig. 322).

Rutherford's experiment was the first artificial transmutation of an element—the old dream of the alchemists. He and Chadwick showed that about a dozen of the lighter elements can be transmuted, like nitrogen, by bombardment with α-particles.

Now 1 mg. of radium gives off 37,000,000 α-particles per second, but, owing to the very large spaces inside the atoms of the substances bombarded, only one α-particle in a million causes transmutation. Accordingly, more copious sources of fast-moving particles were devised. In 1932, Cockcroft and Walton, working with Rutherford in the Cavendish Laboratory, Cambridge, accelerated protons in a discharge tube through a P.D. of 400,000 volts (Fig. 323). They were able in this way to transmute the light elements lithium and boron. Lithium breaks up into two α-particles of very high speed, indicating a large release of energy derived from a slight annihilation of mass.

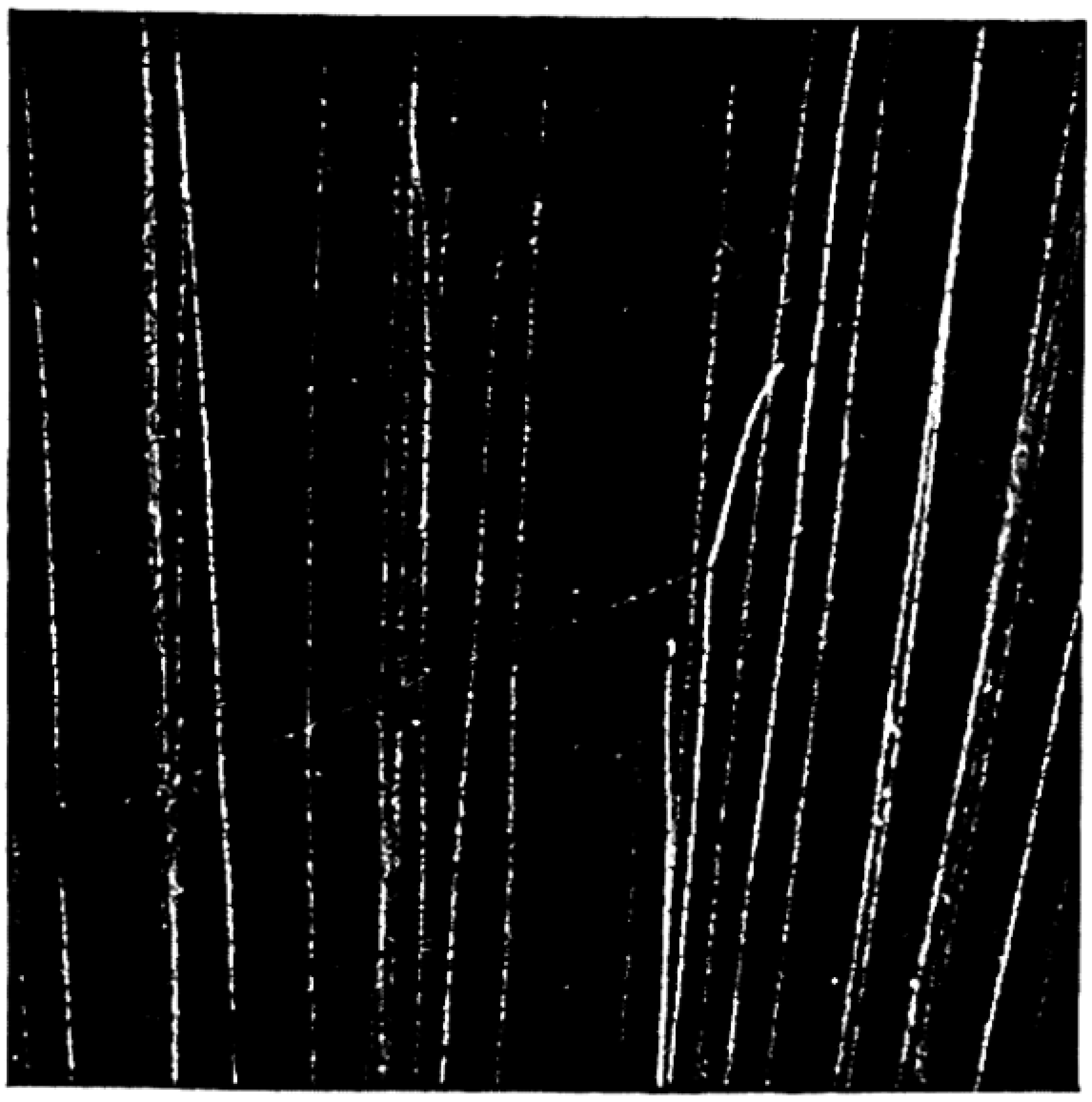


Fig. 322. A photograph showing the artificial disintegration of nitrogen. The almost parallel tracks are those of α-particles in nitrogen. One α-particle has entered a nitrogen atom which has ejected a proton (the long track going to the left) and become oxygen (the short track veering to the right).

$$_3\mathrm{Li}^7 + {}_1\mathrm{H}^1 = 2\,{}_2\mathrm{He}^4.$$

Only light elements can be transmuted by the comparatively small P.D. used by Cockcroft and Walton: the large positive charge on the

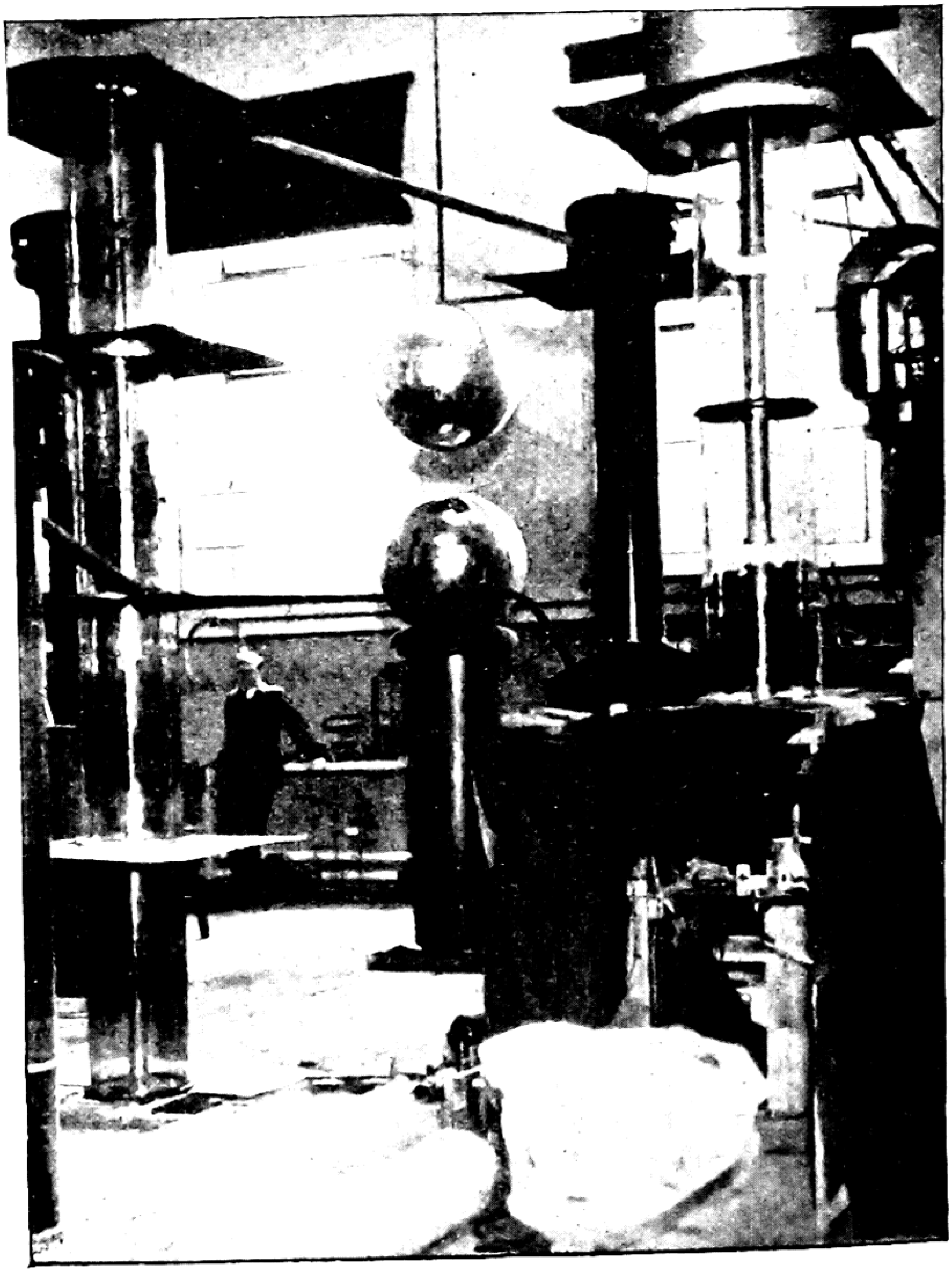


Fig. 323. The apparatus of Cockcroft and Walton. In the lower right-hand corner of the picture is the observing cabin, covered with lead to provide protection from stray X-rays and above the cabin is the accelerating tube. The dark pillar immediately to the left of the accelerating tube contains a system of condensers. To its left are two large metal spheres to provide a spark gap for measuring the high voltage obtained and on the extreme left is a tower of rectifiers.

nucleus of heavier elements causes a large repulsion of the bombarding particles and reduces the chance of a direct hit. Very much faster protons, deuterons ($_1H^2$), or α-particles can be produced by a machine called a cyclotron, invented by Professor Lawrence of the University of California. The largest cyclotron at present in action is the

184 inch cyclotron at Berkeley, California, which produces 190 million-electron-volt deuterons or 380 million-electron-volt α-particles and contains as much steel and copper as a couple of destroyers. Cyclotrons have been erected in this country in Cambridge and Liverpool, and one is being constructed in Birmingham.

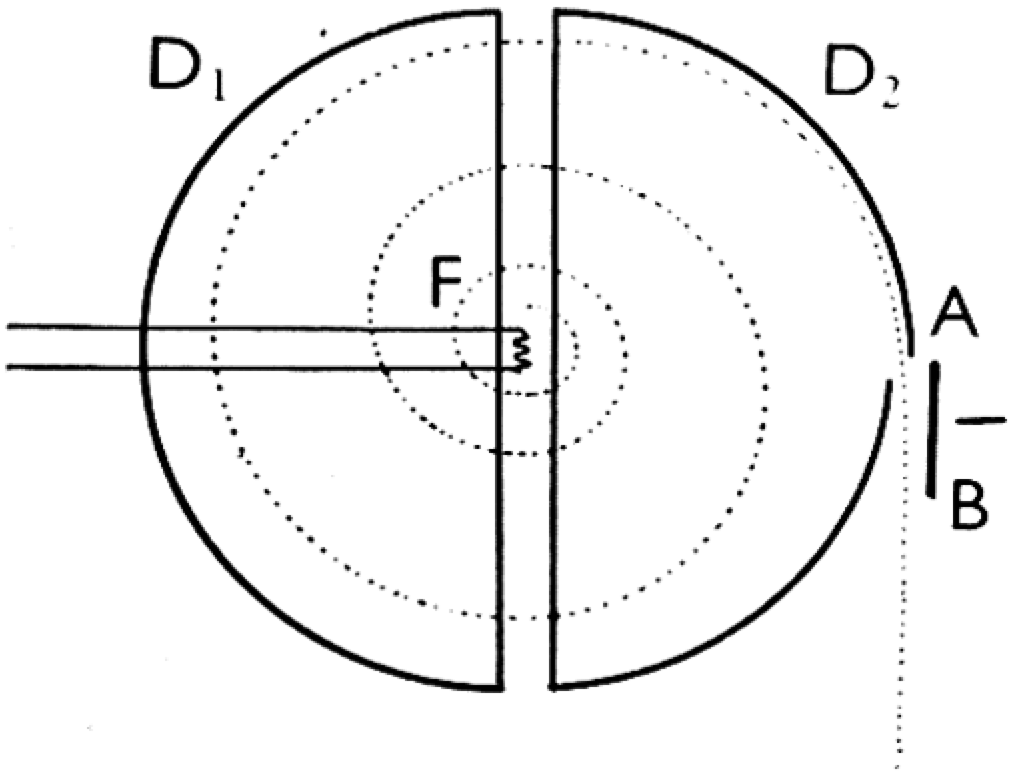


Fig. 324

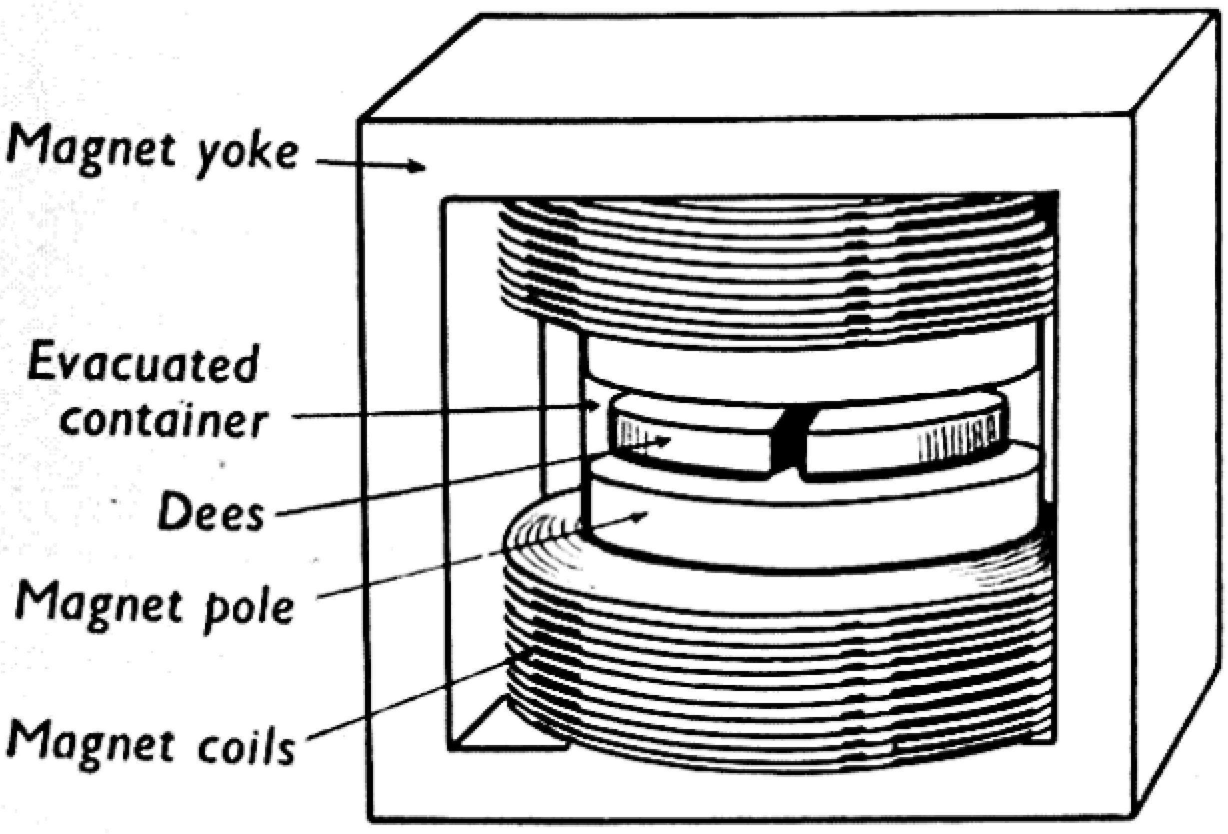


Fig. 325

The principle of the cyclotron is shown in Figs. 324 and 325. Lawrence is said to have invented it because he found difficulty in following a proof in a German periodical that it was unworkable. Two hollow, metal, half-cylinders D_1 and D_2, called dees, like a pill-box sliced in two, are contained in a larger evacuated vessel and insulated from each other. If a beam of protons is required, a little hydrogen is admitted and some of the gas is ionised by the hot tungsten filament, *F.* An alternating P.D., of say 100,000 volts, is set up between the dees. A proton formed near the centre will be accelerated across the gap between the dees from whichever of them is positive at the moment to the negative one. A magnetic field at right angles to the paper, set up by a very large and powerful electromagnet, causes the proton, now travelling at a uniform speed inside a dee, to traverse a circular path. If, by the time it is about to cross the gap again, the P.D. between the dees is reversed, it will be accelerated through another 100,000 volts. Since its velocity is now greater it will travel in a circle of a larger radius, but the time it takes to complete a half-circle is the same as before; its increased speed is exactly compensated by the increased length of its path. Thus the proton can be made to traverse a spiral and if it does 200 turns it acquires a velocity equivalent to that produced by a P.D. of 200 × 100,000 = 20,000,000 volts.

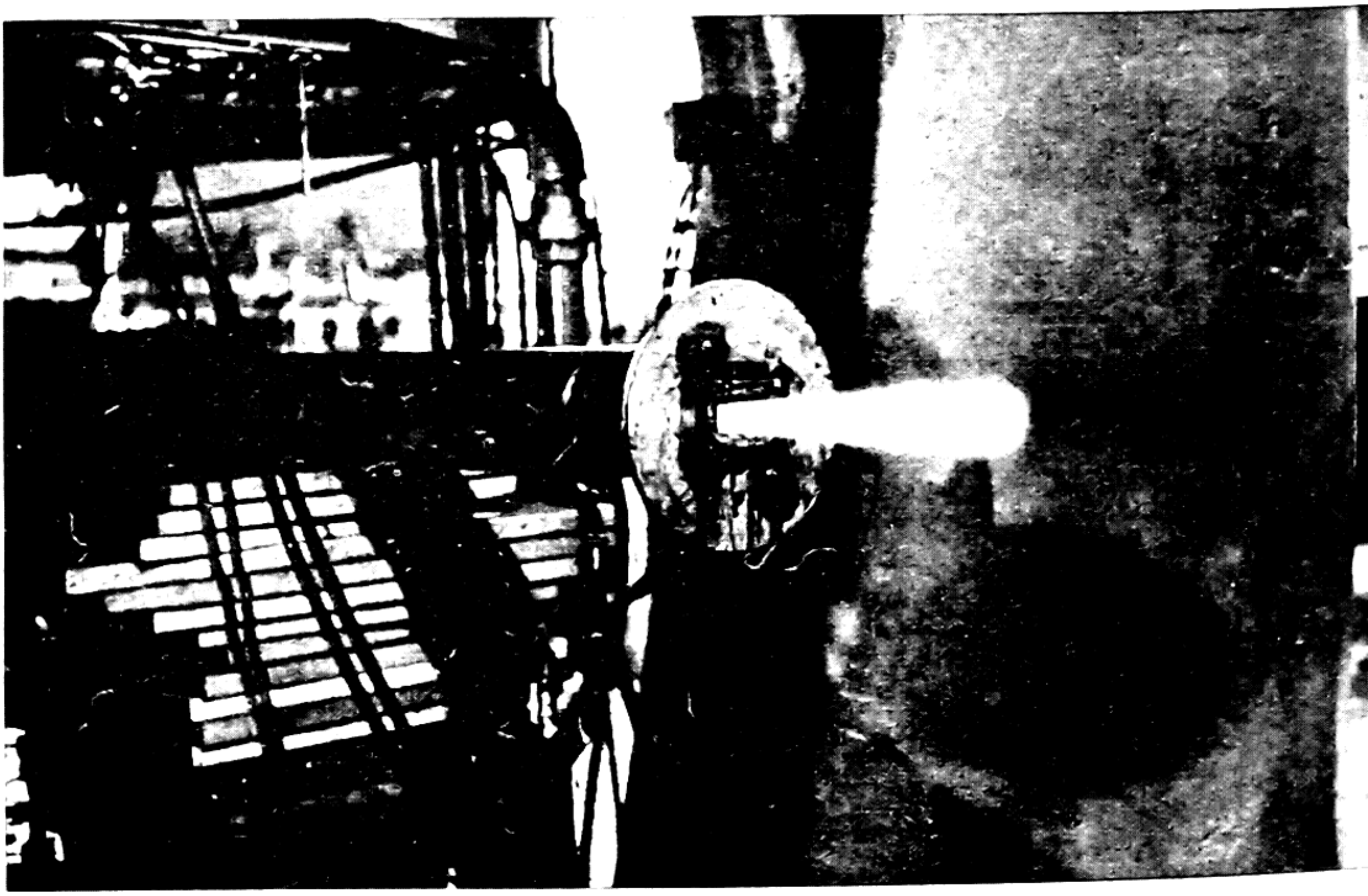


Fig. 326. Six-million volt beam of deuterons emerging through a platinum window from a cyclotron into the air.

On emerging at *A* the paths of the protons are straightened out into a beam by the attraction of the deflecting plate *B*.

The neutron

From the bombardment of beryllium with α-particles Sir James Chadwick discovered in 1932 a particle with the same mass as a proton but of zero charge, called the neutron:

$$_4Be^9 + {}_2He^4 = {}_6C^{12} + \underset{\text{neutron}}{{}_0n^1}.$$

The radiation from beryllium had been investigated by Bothe and Becker in Germany, and by M. and Madame Curie-Joliot in France and was thought by them to be γ-rays.

Induced radioactivity

Although they missed narrowly the discovery of the neutron, M. and Madame Curie-Joliot a little later discovered induced radioactivity, for which they were awarded a Nobel prize. They found that aluminium bombarded with α-particles gave rise to a radioactive isotope of phosphorus:

$$_{13}Al^{27} + {}_2He^4 = {}_{15}P^{30} + {}_0n^1.$$

The phosphorus breaks up into silicon and has a half-life period of 2 or 3 minutes.

About 250 artificially induced, radioactive isotopes are now known. One of the most important is radioactive sodium, $_{11}Na^{24}$, which has a half-life period of 15 hours and emits β- and γ-rays as it disintegrates into magnesium. This promises to have therapeutic value, as a substitute for radium, in diseases such as cancer. It can be made easily with a cyclotron by bombardment of ordinary sodium with deuterons:

$$_{11}Na^{23} + {}_1D^2 = {}_{11}Na^{24} + {}_1H^1.$$

Artificially prepared radioactive substances, particularly radioactive phosphorus, have already been used in biological research. The passage of these atoms through the body can be traced by their radioactive effects.

Fission

Before 1939 all known nuclear changes had consisted of the removal of a small mass from the nucleus in the form of an α-particle, a proton or a neutron. In that year Hahn and Strassmann discovered that the nucleus of the uranium isotope, U 235, when bombarded with slow neutrons, splits up into two nearly equal parts. Hahn and Strassmann did not realise what was happening; it was Frisch and Meitner who suggested nuclear fission. Several different nuclei have been identified as the result of the fission, and all that can be said is that the nucleus splits into parts with masses in the approximate ratio 5:7. A comparatively large amount of mass is annihilated in the process and hence a large amount of energy is released. At each fission about three neutrons are emitted. The phenomenon may be represented as in Fig. 327.

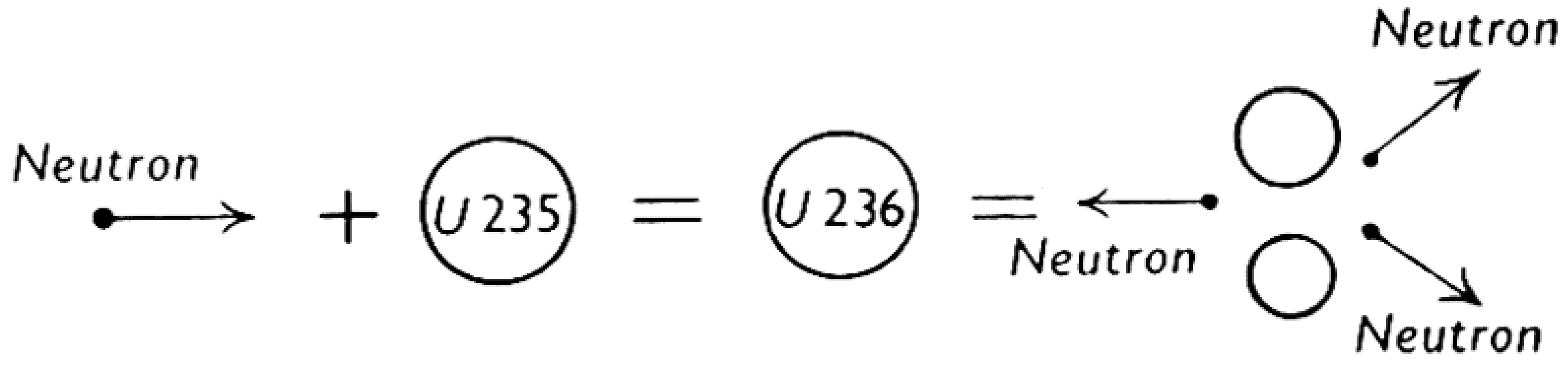


Fig. 327

The atomic bomb

It was at once realised that a chain process in a mass of pure U 235 is possible; once one nucleus has been split three others will be split by the neutrons released, then nine others and so on, providing an explosive of unparalleled violence. In November 1941 the American Government was advised by its scientific experts that an atomic bomb could be produced in three or four years.

Natural uranium contains only 0·7% of U 235; the much more abundant isotope is U 238. Two of the main problems connected with the manufacture of the bomb were the production of U 235 in quantity and the control of the explosion. The two isotopes of uranium may be separated by two methods: (1) the gaseous diffusion method depending on the slightly more rapid diffusion, through a porous barrier, of lighter isotopes than of heavier; (2) the electromagnetic method, in principle the same as that of the mass spectrograph described on p. 318. Both of these methods when used in the laboratory produce minute quantities of U 235. The idea of their application on an industrial scale would, in peace time, have been regarded as laughable; the production of the atomic bomb cost the U.S.A. £500,000,000.

U 235 does not explode as it is made, despite the fact that there are always stray neutrons present. Neutrons, being uncharged, are undeviated by the charges in the atom, and do not cause fission until they reach a nucleus in their direct path. They must travel a considerable distance, on an average, before being effective. There is therefore a critical size for a bomb, and until this size is reached neutrons escape from the mass of U 235 without causing cumulative fission. The explosion may be set off by bringing together two pieces of U 235 each too small to explode by itself.

The atomic pile

Under the bombardment of fairly fast neutrons the more abundant U238 absorbs a neutron to become U239; from this a β-particle is ejected giving a new element, neptunium, of atomic number 93, and then a further β-particle is ejected giving plutonium of atomic number 94:

$$_{92}U^{238}+{}_0n^1={}_{92}U^{239}={}_{93}Np^{239}+{}_{-1}e^0,$$

$$_{93}Np^{239}={}_{94}Pu^{239}+{}_{-1}e^0.$$

Plutonium exhibits nuclear fission like U235, and in so doing liberates a large amount of energy.

The chain reaction in a carefully purified natural mixture of U235 and U238 can be made to proceed slowly, so that an atomic pile, which gives out continuously large quantities of heat, can safely be built of this material.

Slow or 'thermal energy' neutrons are most effective in producing the fission of U235, while those emitted during fission are fast neutrons. Neutrons can be slowed down by elastic collision with nuclei which do not absorb them; such substances are called moderators. Since there is little change in speed and kinetic energy at an elastic collision with a heavy nucleus the light elements are the most effective moderators and the most practicable substances are graphite and heavy water.

The first atomic pile was built in Chicago in 1942. It consisted of a lattice of lumps of uranium (about 5 tons) and graphite bricks to act as moderators. Cadmium rods, which absorb neutrons readily and hence can be used to control the reaction, passed through ten slots in the pile. The pile stopped working when all the rods were pushed home; it was started again by drawing out all the rods except one and then by pulling out the last rod gradually.

During the working of the pile U235 undergoes fission and is gradually used up. However, plutonium is being formed and its concentration gradually increases. Since plutonium itself can undergo fission it compensates for the decrease in U235. The fission products tend to quench the chain reaction and slowly produce a 'poisoning' effect.

The plutonium produced in a pile can be separated chemically from the uranium since it is a different element. This should be a much easier process than the separation of isotopes, but the percentage of the plutonium present is very small and it must be handled by remote control because of the danger from radiation. The second bomb dropped on Japan was said to have been of a different type from the first, and was possibly constructed of plutonium.

It is likely that one of the first uses of atomic piles is in large electric generating stations in place of coal. The consumption of 1 kg. of uranium produces as much heat as 3000 tons of coal and the costs are approximately the same.

Thorium and protoactinium also undergo fission; thorium is more plentiful in nature than uranium, but protoactinium is scarce. Theoretically many nuclear reactions can be used to release energy, but methods of initiating and controlling them are not yet known.

In the words of Sir Henry Dale, 'The release of atomic energy, now an accomplished fact, can either destroy civilisation or immensely enrich its possibilities; the choice is clearly before mankind and those who guide its destinies.'

Cosmic rays

Experiments between the years 1909–1914 revealed a mysterious radiation, now known as cosmic rays, reaching the earth from outer space. The radiation can be detected through its power to cause the slow discharge of an electroscope by ionising the air.

The radiation increases in intensity with increasing height above the earth's surface. This was shown by Wulf, who did experiments on the top of the Eiffel Tower and by Glockel, who made a balloon ascent of nearly 3 miles. Millikan and Cameron sank electroscopes to a depth of 70 feet in lakes and found a decrease in intensity. In this way the radiation was shown to have its origin outside the earth.

The radiation was first thought to be X-rays of very short wavelength but is now believed to consist of charged particles. Charged particles should be deflected by the earth's magnetic field causing a smaller intensity at the equator than elsewhere, and this latitude effect has been observed experimentally. Since cosmic rays seem to arrive equally from all directions both by day and night, and since they are undiminished in the southern hemisphere when the Milky Way is invisible, it has been suggested that they come from the extra-galactic nebulae.

Several theories of their origin have been put forward. One suggests that they are emitted during the atomic transformations in the stars, but this explanation does not account for their huge energies. The energies may be acquired by the passage of the particles through a celestial electric field or the magnetic field of double stars, the latter acting like a huge cyclotron.

One of the methods of cosmic-ray research is to observe their effect on passing through matter. While experimenting on the passage of cosmic rays through a Wilson cloud chamber, Anderson, in America, observed among the secondary rays produced a particle with the mass of an electron but a positive charge. This positive electron has been termed the positron. Its existence was predicted in 1930 by Dirac, but it remained undiscovered until 1932 because it has a very short free life. When a positron and an electron combine they annihilate each other and produce γ-radiation equivalent in energy to the mass which disappears. The reverse effect, the production of an electron pair from γ-rays, has also been observed, when γ-rays of high energy pass through the intense electric field near the nucleus of a heavy atom.

SUMMARY

1895	Röntgen discovered X-rays.
1896	Becquerel discovered radioactivity.
1897	J. J. Thomson discovered the electron and measured e/m.
1898	Pierre and Marie Curie isolated radium.
1901	Planck's Quantum Theory.
1903	Rutherford and Soddy put forward Disintegration Theory.
1905	Einstein's special theory of relativity; showed equivalence of mass and energy.
1906	Millikan's accurate determination of the charge of the electron.
1910	Soddy suggested existence of isotopes.
1911	J. J. Thomson's positive-ray experiments.
1911	Rutherford's hypothesis of the nuclear atom.
1912	Von Laue proposed use of crystal for diffracting X-rays.
1912	C. T. R. Wilson's cloud chamber.
1913	Bragg's X-ray spectrograph.
1913	Moseley's experiments.
1913	Bohr applied quantum theory to Rutherford's atom.
1919	Rutherford produced artificial transmutation of the element nitrogen.
1919	Aston's mass spectrograph.
1922	de Broglie's theory of electron waves.
1925	Heisenberg's Matrix Mechanics.
1925	Blackett obtained first photographs, using a cloud chamber, of nuclear collisions involving transmutation.
1926	Schrödinger's Wave Mechanics.
1927	Davisson and Germer diffracted electron waves.
1928	G. P. Thomson diffracted electron waves.
1930	Dirac's theoretical prediction of positron.
1932	Cockcroft and Walton produced artificial disintegration of lithium by proton bombardment.
1932	Chadwick discovered the neutron.
1932	Lawrence built the first cyclotron.
1932	Urey discovered heavy hydrogen.
1932	Anderson discovered the positron.
1933	M. and Madame Curie-Joliot produced artificially radioactive elements.
1938	Hahn and Strassmann discovered nuclear fission; explained by Frisch and Meitner.
1942	First atomic pile.
1945	Atomic bomb.

QUESTIONS

1. Describe what happens when an electric discharge is passed between electrodes in a tube of gas which is gradually exhausted.

What are the principal properties of cathode rays? (C.)

2. If a stream of electrons each of mass *m*, charge *e*, and velocity 3×10^9 cm. per sec. is deflected 2 mm. in passing for 10 cm. through an electrostatic field of 18 volts per cm. perpendicular to their path, find e/m in e.s.u. per g. (1 e.s.u. potential = 300 volts.) (C.)

3. Calculate the radius of the circle described by an electron which is projected at right angles to a magnetic field of 100 dynes per unit pole with a velocity of 10^{10} cm. per sec. (Charge and mass of electron are $4{\cdot}774 \times 10^{-10}$ e.s.u. and 9×10^{-28} g. respectively.) (B.)

4. Show that the periodic time of a charged particle, moving in a circle in a magnetic field, is independent of its velocity.

Explain how this fact is utilised in the cyclotron.

5. Give the masses, to the nearest whole number, in terms of the mass of the proton, and the charges in terms of the charge of the electron, of the following particles: electron, positron, proton, neutron, deuteron, α-particle.

6. State the rule for the mechanical force on an element of a current-carrying conductor in a magnetic field. Apply the rule to the motion of an electron in a magnetic field. Discuss the nature of the motion when the electron enters the field at right angles.

Describe an experiment whereby the value of the ratio of charge to mass of an electron may be determined. (N.)

7. Find an equation to represent the path of an electron of charge *e* and mass *m* projected with velocity *v* at right angles (i) to a uniform electric field of strength *E*, (ii) to a uniform magnetic field of strength *H*.

A light flexible wire lies in a plane which is perpendicular to a non-uniform magnetic field. If *T* is the tension and *i* is the current in the wire, show that it lies along the possible path of an electron in the field, provided $T/i = mv/e$. (C. S.)

8. A charged particle of mass *m* and charge *e* is projected with a velocity *v* at an angle θ to the direction of (*a*) a uniform magnetic field, (*b*) a uniform electric field. Discuss the motion of the particle in each case. How may the velocity of such a particle be measured? (C. S.)

9. How does the velocity of an electron depend upon the accelerating voltage to which it is subjected?

Give a general account of three electronic devices employing electrons emitted from a heated cathode and having velocities corresponding to accelerating potentials of tens, hundreds, and many thousands of volts respectively. (N.)

10. Calculate the radius of a water drop which would just remain suspended in the earth's electric field of 3 volts per cm. when charged with 1 electron. ($e = 4{\cdot}774 \times 10^{-10}$ e.s.u.) (B.)

11. According to Stokes the resistance experienced by a small sphere of radius a, moving through a medium of viscosity η with velocity v, is given by the expression $6\pi a\eta v$. Use this to determine the terminal velocity of an oil drop of density 0·95 g.cm.$^{-3}$ and radius 10^{-4} cm. falling through air of density 0·0013 g.cm.$^{-3}$.

The drop carries a charge of $+E$ e.m.u. and is driven upwards by an electric field of 2000 volts cm.$^{-1}$ with a velocity of 0·036 cm.sec.$^{-1}$. Calculate the magnitude of E.

Give a brief description of the determination of the electronic charge (e) by experiments with oil drops. (The viscosity of air is 181×10^{-6} g.cm.$^{-1}$ sec.$^{-1}$; 1 volt $= 10^8$ e.m.u.) (N.)

12. Describe apparatus by means of which X-rays may be produced. Compare cathode rays and X-rays as regards their nature and properties. (D.)

13. Draw a labelled diagram of some form of X-ray tube and of its electrical connections when in actual use.

Electrons starting from rest and passed through a potential difference of 1000 volts are found to acquire a velocity of $1{\cdot}88 \times 10^9$ cm. per sec. Calculate the ratio of the charge to the mass of the electron in coulombs per g. (N.)

14. What is the shortest wave-length of X-rays produced by million-volt electrons? ($e = 4{\cdot}774 \times 10^{-10}$ e.s.u.; $h = 6{\cdot}55 \times 10^{-27}$ erg-sec.)

15. Discuss the properties of the three main types of radiation produced during radioactive changes. (B.)

16. Compare and contrast the conduction of electricity through (*a*) metals, (*b*) electrolytes, (*c*) ionised gases. (C. S.)

17. Write an essay on *one* of the following:

(*a*) The disintegration of the atom.
(*b*) The release of atomic energy.
(*c*) The structure of the atom.

ANSWERS TO QUESTIONS

CHAPTER 1 (*page* 15)

2. 1·26 e.s.u.

4. (*a*) 3·16 e.s.u. at 26° 34′ to *OE*, (*b*) 23·4 ergs.

5. (*a*) $7\frac{1}{7}$ cm. from the +5 charge between the charges, $16\frac{2}{3}$ cm. from the +5 charge and $6\frac{2}{3}$ cm. from the −2 charge, (*b*) 17·2 cm. from the −2 charge and 27·2 cm. from the +5 charge.

6. 5 ergs.

7. $1{\cdot}03 \times 10^9$ cm.sec.$^{-1}$.

9. 100 cm.

10. 710 microfarads.

11. 0·556 e.s.u., 2×10^{-9} sec.

12. $4{\cdot}87 \times 10^{-12}$ g.

14. 90 and 30 e.s.u., 0·20 and 0·60 e.s.u. per sq.cm.

15. 0·0053 erg.

16. $\dfrac{R^2}{a^2}\,v$.

CHAPTER 2 (*page* 34)

6. 283 sq.cm.

7. 0·212 microfarad, 0·849 microcoulomb.

8. 3.

9. (*a*) 50,000 ergs, (*b*) 10,000 ergs.

10. 1000 ergs.

11. 2 e.s.u., 250 ergs; 10 e.s.u., 1250 ergs.

13. 60,000 volts.

15. 90 e.s.u.; (*a*) $4\frac{4}{9}$ e.s.u., (*b*) $\frac{4}{9}$ e.s.u.

16. (*a*) $5\frac{5}{7}$ e.s.u., 875 ergs, (*b*) $13\frac{1}{3}$ e.s.u., 375 ergs.

18. 10 e.s.u.

20. 0·084 microfarad.

21. Original energy in *AB* and *CD*, 0·32 and 0·08 joules respectively; loss of energy on connection, 0·144 joules.

22. 17·7 dynes.

23. 900 volts, 5 volts, decreased error 3 mm.

25. (*a*) $12\frac{1}{2}$ ergs, (*b*) 125 ergs.

26. $40\sqrt{\pi}$ e.s.u.

27. $4{\cdot}17 \times 10^{-11}$ amp.

28. 115 sec.

29. $1\frac{5}{6}$ volts; $1\frac{5}{6}$, $4\frac{1}{3}$, $2\frac{1}{2}$ microcoulombs respectively.

30. $\dfrac{\epsilon A}{4\pi d\,(\epsilon+1)}$ e.s.u.,

$$\frac{\epsilon-1}{2\,(\epsilon+1)}\,\frac{AV}{4\pi d t}\,10^{-12}\text{ amp.}$$

CHAPTER 3 (*page* 53)

4. $1\frac{3}{7}$.

5. 5, 0·2.

6. 2 : 1.

8. 100 e.s.u. = 30,000 volts.

9. 0·069 watt.

10. 0·12 milliamp.

11. 150 e.s.u., 0·25 cm.

12. $2{\cdot}25 \times 10^{13}$ ohms.

13. 0·00265 coulomb.

CHAPTER 4 (*page* 80)

2. 37·8 cm.

3. 26·7 c.g.s. units.

4. 17·0 cm.

5. 88·6 c.g.s. units.

6. Field is $\frac{0{\cdot}317M}{BC^3}$ at 56° 32′ to AB, where M is the moment of the magnet.

7. 719 c.g.s. units.

8. 26·0 c.g.s. units.

9. 180 or 60 c.g.s. units.

10. 67° 36′.

12. 180°.

13. $MH \sin\theta$, $MH(1-\cos\theta)$.

16. $\frac{16\pi^2}{15}$ c.g.s. units.

17. 14·14, 11·9, 0, 11·9.

18. 0·2 oersted.

20. 172 c.g.s. units.

22. $1 : \sqrt{7} : \sqrt{\frac{7}{5}}$.

26. 74° 21′.

27. $\frac{1}{\tan^2\phi} = \frac{1}{\tan^2\alpha_1} + \frac{1}{\tan^2\alpha_2}$, where ϕ is the true angle of dip.

28. 69° 38′.

29. 4° 50′.

30. 0·85 : 1.

CHAPTER 5 (*page* 97)

1. 45°.

3. 28·2 %.

4. 0·235 amp.

5. 0·038 amp.

6. 0·175 oersted.

7. 0·184 oersted.

8. 0·178 oersted.

9. $2 \sin^{-1} \tan\theta$.

10. 28,800 dynes.

12. $3{\cdot}9 \times 10^{-4}$ volts.

13. 0·106 volt.

15. $7{\cdot}07 \times 10^{-5}$ volt.

19. If θ_1 and θ_2 are the deflections, $\frac{\tan\theta_1}{\tan\theta_2} = \frac{1}{\pi}$.

20. (*a*) 8π oersted, (*b*) 4π oersted; 540π c.g.s. units.

21. 0·0966 amp. or 0·116 amp.

22. (i) 0·419 oersted, (ii) 0·148 oersted.

23. 18.

CHAPTER 6 (*page* 119)

2. 2480.

3. 0·178 oersted.

5. 475 c.g.s. units, 157, 1980.

6. 796.

7. 1250π c.g.s. units.

8. $2{\cdot}25 \times 10^{-5}$ coulomb.

9. 91·2 turns; flux is reduced to 36,000 maxwells.

10. Reduces flux from 37,700 to 14,500 maxwells.

CHAPTER 7 (*page* 136)

2. 10 ohms, 1·60 volts.

3. 1 volt, 2 volts.

4. 3.

5. 4 groups, each of 3 cells in series, in parallel.

6. $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{3}$ amp.

7. $1\frac{7}{13}$ volts.

8. 1·31 volts.

9. $\frac{21}{82}$ amp. in *AB* and *BC*, $\frac{17}{82}$ amp. in *AD*, $\frac{29}{82}$ amp. in *DC*.

10. $E/8$ amp. in *AB*, *CB*, *DC* and *DA*; $E/4$ amp. in *BD*; zero in *AC*.

12. $\frac{5}{6}$.

13. 1590 g.

14. Yes; cost is $10\frac{1}{2}d$.

16. 4·8° C.

18. $\frac{8}{32}$ watt, $2\frac{1}{4}$ volts.

20. 4·18 joules per cal.

21. 31·9° C.

22. $\dfrac{\sigma i^2}{2J\pi^2 r^3}$, $\dfrac{1{\cdot}129}{1}$.

23. (*a*) 0·00444 cm., (*b*) 0·34, (*c*) 94·2 volts.

24. 31.

25. 58.

26. 22·7, 4·37 kilowatts.

27. 40·3 metres, 8*s.* $3\frac{1}{4}d$.

28. (*a*) 12·4 sq.cm.; (*b*) 29·8 H.P., 5290 cal. per sec.; (*c*) 11·1*d.* per hour. Ratio of costs, 1 : 6·25.

CHAPTER 8 (*page* 162)

1. 0·020 ohm shunt.

2. (*a*) 0·002 ohm shunt, 9980 ohms in series.

3. (*a*) 9988 ohms in series, (*b*) 0·0012 ohm shunt.

4. 980 ohms in series.

5. 10^{-9} amp. per scale division.

6. 120 dyne cm.

7. $1{\cdot}33\times10^{-4}$ microamp.

8. Deflection is 8·1 mm. per 10^{-9} amp.

11. 0·216 microfarad.

13. 0·5 ohm.

14. 75 cm., 90 cm., 39·5 cm.

17. 0·0675 volt.

18. $\frac{2}{145}$ amp.

20. 32·05 miles from *B*.

21. 2 : 3.

23. $8{\cdot}2\times10^{-5}$ ohm cm.

24. 2500° C.

25. 13·74 ohms.

27. 0·20 ohm.

CHAPTER 9 (*page* 183)

9. $a=2$, $b=2$, $c=4$, $g=-1$.

CHAPTER 10 (*page* 197)

1. 45 min.

2. 111 c.c.

3. $1{\cdot}80\times10^{6}$ coulombs.

4. 2·75 amp.

5. 0·19 oersted.

8. 10 %.

9. $6{\cdot}06\times10^{23}$, $1{\cdot}65\times10^{-24}$ g.

CHAPTER 11 (*page* 212)

1. $2{\cdot}95 \times 10^5$ coulombs.

2. 0·078 g., 0·115 amp.

4. 1·53 volts.

5. 2·5 ohms.

6. 0·95, 3·7 amp.

8. $\frac{22}{35}$.

9. 2·07 volts.

CHAPTER 12 (*page* 234)

1. 0·40 oersted.

2. 1·08 millicoulombs.

3. 300 ohms.

4. 7670 oersted.

5. *A* will be more sensitive; deflections of *A* and *B* for given change of flux are 101 : 15.

6. 176 microcoulombs.

7. 34 microamp.

8. 0·71 millicoulomb.

9. 0·467 amp.

10. 0·050 coulomb.

14. 4570 kilowatts, 22,850 amp.

15. 22,500 volts.

19. 2 millihenries.

20. ¼ joule.

23. 1 sec.

24. 69·3 sec.

25. 139 sec.

26. $CVe^{-t/RC}$.

27. $E(1-e^{-t/RC})$, $-RC\log_e\left(1-\frac{V_2}{V_1}\right)$.

32. 239 r.p.m.

33. 99·62 volts, 91·25 volts, 91·25 %.

34. (*a*) 152·5 volts, (*b*) 14,487·5 watts.

CHAPTER 13 (*page* 249)

1. 50·1 c.p.s.

2. $0{\cdot}8\pi$ volts.

3. $2\pi^2$ volts, $\pi \times 10^6$ dyne cm.

4. $\dfrac{\pi^2 r^2 nN\omega i \times 10^{-8}}{5R}$ volts, $\dfrac{\omega}{2\pi}$ c.p.s.

7. 0·502 amp., 283 volts.

10. (*a*) 100·8 ohms, (*b*) 1260 ohms.

11. (*a*) 3·18 amp., 90° lag; (*b*) 5·02 amp., 90° lead; (*c*) 3·03 amp., 72° 21′ lag; (*d*) 4·54 amp., 63° 0′ lead; (*e*) 6·47 amp., 57° 6′ lag.

12. $5{\cdot}03 \times 10^5$ c.p.s.

13. 0·16.

14. 8·39 volts, 16·8 watts, 0·95.

15. (*a*) 2·875 amp., (*b*) 452 volts on coil, 229 volts on condenser.

CHAPTER 14 (*page* 273)

4. 500° C.

7. Two groups in parallel, each of three thermocouples in series, 100 divisions.

16. 200 watts.

CHAPTER 17 (*p.* 348)

2. 6×10^{17} e.s.u. per g.

3. 5·66 cm.

10. $1{\cdot}05 \times 10^{-5}$ cm.

11. 0·0114 cm.sec.$^{-1}$, $8{\cdot}09 \times 10^{-20}$ e.m.u.

13. $1{\cdot}77 \times 10^8$ coulombs per g.

14. $1{\cdot}23 \times 10^{-10}$ cm.

INDEX

Absolute electrometer, 27
Acceptor circuit, 246, 298
Accumulator, 211
α-particles, 326
Alternating current, 239 et seq.
Ammeter, 147, 151
Ampère, 101
Ampere, the, 85, 168, 170
Amplification factor, 265, 300
Appleton layer, 289
Armature, 229
Arrhenius, 187
Artificial disintegration, 338
Artificial radioactivity, 343
Aston's mass spectrograph, 318
Atomic bomb, 344
Atomic nucleus, 332
Atomic number, 324
Atomic pile, 345
Atomic structure, 335
Attracted disc electrometer, 27
Aurora Borealis, 72
Avogadro's number, 188

B and H, 108, 110, 117, 180
Back E.M.F., in electrolysis, 210
  in inductance, 219
  in motor, 228
Ballistic galvanometer, 148
Balmer series, 334
Barkhausen effect, 104
Becquerel, 325
β-particles, 326
Blackett, 339
Bohr, 332
Bragg, 322

Callendar and Barnes, 129, 139
Capacitance, definition, 10
  of parallel plate condenser, 24
  of concentric cylinders, 33
  of concentric spheres, 33
  measurement of, 41, 43
  in A.C. circuit, 242
Cathode rays, 310
Cathode-ray oscillograph, 266, 291
Cells, primary, 199, 207
  in series and in parallel, 123
  internal resistance of, 123, 159
  secondary, 211
  Weston, 150, 173
Chadwick, 343
Characteristic of valve, 262, 265
Choke, 220
Circuital law, 91
Cloud chamber, 327
Cockcroft and Walton, 339
Coercive force, 105
Coil, rotating, 239
Commutator, 86, 228
Compass, 73
Concentration cell, 206
Concentration polarisation, 211
Condenser, defined, 21
  cylindrical, 33
  discharge through inductance, 224
  discharge through resistance, 223
  energy of, 26, 30
  force between plates of, 26
  in A.C. circuits, 242
  in series and in parallel, 29
  parallel plate, 24
  spherical, 33
Condensing electroscope, 45
Contact potential, 202
Cosmic rays, 346
Coulomb, 2, 101
Coulomb, the, 4, 175
Crompton potentiometer, 152
Crookes' dark space, 310
Curie, 326
Curie-Joliot, 343
Cyclotron, 340

Damping, 142, 143, 148
Daniell cell, 199, 200, 206, 207
Davisson and Germer, 260, 338
de Broglie, 337
Debye and Hückel, 191
Declination, 65
Degaussing, 74
Detection, 296
Deuterium, 317
Deuteron, 340
Diamagnetism, 113
Dielectric constant, 22, 178, 180, 277
  measurement of, 42
Dimensions, 175
Diode, 261, 299
Dip, 65–72
Dipole, 22
Discharge potential, 208
Disintegration, atomic, 329
Dissociation, electrolytic, 187
Dynamo, 230

Earth inductor, 68
Earth's electric field, 50
Earth's magnetic field, 65
Eddy currents, 142, 226, 227

Edison, 131
Einstein, 256, 318
Electrochemical equivalent, 185
Electrode potential, 205
Electrolysis, 184, 209
Electromagnetic induction, 89, 118, 214
Electrometer, attracted disc, 27
  quadrant, 39
Electromotive force, 9, 122
Electron, 18, 88
  charge, 188, 313
  diffraction, 338
  microscope, 270
Electronic work function, 204, 258, 260
Electrophorus, 43
Electroscope, 2, 19
Energy, electrical, 130
  and mass, 318
Equipotential surface, 9
Equivalent conductivity, 189, 192

Farad, 28, 175
Faraday, 6, 11, 20, 25, 185, 214, 310
Faraday, the, 185
  cylinder, 311
  dark space, 310
  ice pail experiment, 20
Fleming's left-hand rule, 87
Fleming's right-hand rule, 90
Flux, magnetic, 108, 214
Franklin, 46
Frequency, 231, 240, 286

Galvani, 201
Galvanometer, 85, 141, 158
γ-rays, 326
Gauss, the, 108
Gauss, 56, 166, 167
  proof of inverse square law, 58
Geophysical prospecting, 76
Gilbert, 1, 55
Growth and decay of current, 221
Guard ring, 24

Heating effect of current, 128
Heaviside layer, 289
Heisenberg, 336
Henry, the, 217
Hertz, 280
Hittorf, 194
Hydrogen atom, 332
Hydrogen electrode, 206
Hydrogen, heavy, 317
Hydrogen spectrum, 334
Hysteresis, 105, 267

Impedance, 244
Induced charge, 17
Induced current, 89, 118, 214
Inductance, 219, 241
Induction, coil, 225
  electric, 31
  magnetic, 107, 116
Intensity, electric, 4, 8
  magnetic, 56
  of magnetisation, 102
Internal resistance, 123, 159
International units, 168
Inverse square law, electrostatic, 3, 12, 30
  magnetic, 55, 58, 115
Ionic theory, 187
Ionosphere, 289
Isoclinic lines, 71
Isogonic lines, 70
Isotopes, 317

Joule, 126, 127

Kelvin, 25, 255
Kilowatt-hour, 130
Kirchhoff's laws, 126
Kohlrausch, 192

Langevin, 114
Laplace's law, 95
Laue, von, 321
Lenz's law, 90, 214
Leyden jar, 28
Lines of force, electric, 4
  magnetic, 56, 108, 117
Lines of induction, electric, 31
  magnetic, 108, 110, 117

Magnetic circuit, 110
Magnetic elements, 66
Magnetic meridian, 65
Magnetometer, deflection, 60
  vibration, 64
Magnetomotive force, 92, 111
Marconi, 287
Mass spectrograph, 318
Maxwell, Clerk, 31, 84, 174, 276, 278
Maxwell, unit defined, 108
Mechanical equivalent of heat, 128
Metre bridge, 153
Millikan, 256, 314
Mine, magnetic, 74
M.K.S. units, 180
Mobility of ions, 194
Modulation, 296
Moment, magnetic, 57, 63
Moseley, 324
Motors, 227
Moving-coil galvanometer, 141, 145
Mutual inductance, 217

Neutral points, 59
Neutron, 332, 343
Nucleus, 331

Oersted, the, 56
Ohm, 121, 124, 127
Ohm, the, 121, 168, 171
Ohmmeter, 160
Ohm's law, 121, 122
  verification of, 124
Oscillator, 303
Oscillatory circuit, 282, 298

Paramagnetism, 113
Peltier effect, 251
Pentode, 306
Permeability, 108, 109, 115, 178, 180
Permittivity, 180
Phase angle, 240
Photoelectric effect, 255
Photon, 256
Piezo-electricity, 23
Planck's constant, 256, 320, 332, 337
Platinum resistance thermometer, 157
Points, action of, 14
Polarisation, dielectric, 22, 31
  in electrolytes, 199, 209
Positive column, 310
Positive rays, 316
Positron, 347
Post Office box, 154
Potential, 6, 8, 11
Potentiometer, 150, 158
Power, electric, 129
  factor, 244
Proton, 332

Quadrant electrometer, 39
Quantum theory, 256, 332

Radar, 291
Radioactivity, 325
Reactance, 241, 243
Reaction, 302
Rectifiers, 248, 262
Reduction factor, 86
Rejector circuit, 246, 298
Reluctance, 111
Residual charge, 23
Residual magnetism, 105
Resistance, definition of, 121, 128
  measurement of, 153
Resistivity, 155
Resonant frequency, 246, 298
Röntgen, 319
Rutherford, 315, 331, 338

Saturation current, 261
Saturation, magnetic, 104
Schrödinger, 338
Secondary cells, 211
Secondary reactions, 184, 208
Seebeck effect, 251
Self-induction, 219
Shunt, 147
Simpson, Sir George, 47
Solenoid, 93, 116
Sound films, 259
Space charge, 261
Specific inductive capacity, 22
Specific resistance, 155
Standard cell, 150
Strength of electric field, 4, 8
Strength of magnetic field, 56, 216
Supraconductivity, 156
Surface density of charge, 5, 13
Susceptibility, 104, 108

Tangent galvanometer, 85, 166
Telephone, 233
Television, 268
Temperature coefficient of resistance, 156
Tetrode, 305
Thermionic effect, 260
Thermocouple, 145, 251
Thomson, G. P., 338
Thomson, J. J., 312, 315, 316
Thomson effect, 255
Transformer, 226
Transmission of power, 130
Transport numbers, 196
Triode, 264, 300

Unit charge, 4
Unit current, 85
Unit pole, 56
Units, 166 et seq.
Uranium, 326, 330, 343
Urey, 317

Virtual current and E.M.F., 240
Volt, 6, 128, 168
Volta, 199, 201, 202
Voltameter, 185
Voltmeter, current, 149, 151
  electrostatic, 41

Watt, 129
Wattmeter, 245
Wave-length, 286
Wave mechanics, 336
Weber, 166, 167
Weston cell, 150, 173
Wheatstone bridge, 153
Wilson, C. T. R., 49
Wilson cloud chamber, 327
Wimshurst machine, 44
Work done in threading a circuit, 92

X-rays, 319, 336

ALLAMA IQBAL LIBRARY
5391